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A GENERAL CLASS OF ESTIMATORS IN TWO PHASE SAMPLING USING MULTI-AUXILIARY VARIABLES IN THE PRESENCE OF NON-RESPONSE AT SECOND PHASE

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ABSTRACT. In this paper we have proposed a general class of estimators in two phase sampling using multi-auxiliary variables for estimating population mean of study variable in the presence of non-response for both study and auxiliary variables at second phase. Special cases of suggested class have also been deduced. The expressions of bias and mean square error have also been derived. Mathematical and empirical comparison has been made for comparing the efficiency of proposed estimators.

1. INTRODUCTION

The most common method of data collection in survey research is sending the questionnaire through mail. The reason may be the minimum cost involved in this method. But this method has a major disadvantage that, a large rate of non-response may occur which may result in an unknown bias at any assumption because of the fact that the estimate based only on responding units is representative of the both responding and non-responding units.

Personal interview is another method of data collection which generally may result in a complete response, but the cost involved in personal interviews is much higher than the mail questionnaire method. We may conclude from the above discussion that the advantage of one method is the disadvantage of other and vice versa. Hansen and Hurwitz (1946) combined the advantages of both procedures. They considered a problem to determine the number of mail questionnaires along with the number of personal interviews to take in following up non-response to the mail questionnaire in order to attain the required precision at minimum cost.

Let us assume that the population of size N is divided into two groups, one group consisting of the units who respond called response class and the other group consisting of the units who do not respond is called non-response class. Let N_1 and

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N_2 be the sizes of responding class and non-responding class of population respectively. A simple random sample of size n_1 is selected from population of size$N$, out of which n_{11}units respond and n_{12} do not. We can say that $n_{11}$is the sample selected from N_1 units and n_{12}be the number of units selected from N_2 units. Let r be the size of the sample selected from n_{12} where$r = \frac{n_{12}}{k}$, $k > 1$. Let $\bar{y}_1$ and $\bar{y}_{2r}$ denote the sample means based on n_{11} and r units respectively.

Hansen and Hurwitz (1946) proposed the following estimator to estimate the population mean$\bar{Y}$ based on $n_{11}$and r units:

$$\bar{y}_1^* = \frac{n_{11}\bar{y}_1 + n_{12}\bar{y}_{2r}}{n_1} \tag{1.1}$$

The estimator $\bar{y}_1^*$ is unbiased and has variance

$$Var\,(\bar{y}_1^*) = \lambda_1 S_y^2 + \theta S_{y_2}^2 \tag{1.2}$$

Where $\lambda_1 = \frac{1}{n_1} - \frac{1}{N}$ and $\theta = \frac{W_2(k_1-1)}{n_1}$; $W_2 = \frac{N_2}{N}$ and S_y^2 and $S_{y_2}^2$be the population variances based on N and N_2units respectively.

1.1. **Two-Phase Sampling in the Presence of Non-Response at Second Phase using Single Auxiliary Variable.** In two- phase sampling, a larger sample is selected from the population and information of auxiliary variable is recorded. There may also be the situation when cost of drawing large sample is too high, therefore, we take a sub sample at second phase, information on auxiliary variable and variable of interest is recorded for estimation. The two phase sampling may cause a loss of efficiency but this loss is compensated by the use of auxiliary information. We assume that the population of size N is divided into two groups, one group consisting of the units who respond called response class and the other group who do not respond is called non-response class. Let $N_1$and N_2 be the sizes of responding class and non-responding class of population respectively. A large first phase sample of size n_1 is selected from a population of N units by simple random sampling without replacement (SRSWOR), we assume that at the first phase, all the n_1 units provide information on auxiliary variablex_{1i}, a smaller second phase sample of size $n_2$observations is selected from n_1 first phase sample and information on study variable y is recorded. Let n_{21}units provide information on y and n_{22} units do not. Therefore, take a subsample of$r = \frac{n_{22}}{k}$, $k > 1$so that the r units can be re-contacted by personal interview. Let $\bar{x}_1$be the sample mean of auxiliary variable$x_1$based on the first-phase sample.

1.1.1. *Modified Hansen and Hurwitz' (1946)Estimator.* For the need of Hansen and Hurwitz (1946) estimator in two phase sampling, it is modified as:

$$\bar{y}_2^* = \frac{n_{21}\bar{y}_2 + n_{22}\bar{y}_{2r}}{n_2}. \tag{1.3}$$

The estimator $\bar{y}_2^*$ is unbiased and has variance

$$Var\,(\bar{y}_2^*) = \lambda_2 S_y^2 + \theta S_{y_2}^2, \tag{1.4}$$

where $\lambda_1 = \frac{1}{n_1} - \frac{1}{N}$, $\lambda_2 = \frac{1}{n_2} - \frac{1}{N}$, $\lambda_3 = \frac{1}{n_2} - \frac{1}{n_1}$ and $\theta = \frac{W_2(k-1)}{n_2}$; $W_2 = \frac{N_2}{N}$ and S_y^2 and $S_{y_2}^2$be the population variances based on N and N_2units respectively.

1.1.2. *Khare and Srivastava (1993, 1995) Estimators:* Khare and Srivastava (1993, 1995) proposed the following ratio, product and regression type estimators using two-phase sampling where non-response is considered at second phase. The ratio estimator is

$$T_{R1d} = \frac{\bar{y}_2^*}{\bar{x}_2^*}\bar{x}_1, \tag{1.5}$$

with

$$Bias\left(T_{R1d}\right) = \left(\frac{1}{\bar{X}}\right)\left\{\lambda_3\left(RS_x^2 - S_{xy}\right) + \theta\left(RS_{x_2}^2 - S_{xy_{(2)}}\right)\right\}, \tag{1.6}$$

and

$$MSE\left(T_{R1d}\right) = \lambda_3\left\{S_y^2 + R^2S_x^2\left(1-2C\right)\right\} + \lambda_1 S_y^2 + \theta\left\{S_{y2}^2 + R^2S_{x2}^2\left(1-2C_{(2)}\right)\right\}, \tag{1.7}$$

where$C = \frac{\beta}{R}$,$C_{(??)} = \frac{\beta_{(??)}}{R}$,$\beta = \frac{S_{xy}}{S_x^2}$, $\beta_{(??)} = \frac{S_{xy(??)}}{S_{x(??)}^2}$,$R = \frac{\bar{Y}}{\bar{X}}$,$\lambda_1 = \frac{1-f_1}{n_1}$, $\lambda_2 = \frac{1-f_2}{n_2}$, $\lambda_3 = \lambda_2 - \lambda_1$, $\theta = \frac{W_2(k-1)}{n_2}$.

The product estimator is

$$T_{P1d} = \frac{\bar{y}_2^*}{\bar{x}_1}\bar{x}_2^*, \tag{1.8}$$

with

$$Bias\left(T_{P1d}\right) = \frac{1}{\bar{X}}\left\{\lambda_3 S_{xy} + \theta S_{xy_2}\right\}, \tag{1.9}$$

and

$$MSE\left(T_{P1d}\right) = \lambda_3\left\{S_y^2 + R^2S_x^2\left(1+2C\right)\right\} + \lambda_1 S_y^2 + \theta\left\{S_{y2}^2 + R^2S_{x2}^2\left(1+2C_{(2)}\right)\right\}. \tag{1.10}$$

The regression estimator is

$$T_{LR1d} = \bar{y}_2^* + b^*(\bar{x}_1 - \bar{x}_2^*), \tag{1.11}$$

where $b^* = \frac{S_{xy}}{S_x^2}$.

The bias is

$$Bias\left(T_{LR1d}\right) \approx \beta_{xy}\left(\frac{N\left(N-n_1\right)}{\left(N-1\right)\left(N-2\right)}\frac{\mu_{21}}{n_1\bar{X}S_{yx}} - \frac{N\left(N-n_1\right)}{\left(N-1\right)\left(N-2\right)}\frac{\mu_{30}}{n_1\bar{X}S_x^2} + \theta\frac{\mu_{30(2)}}{\bar{X}S_x^2} + \frac{N\left(N-n_2\right)}{\left(N-1\right)\left(N-2\right)}\frac{\mu_{30}}{n_2\bar{X}S_x^2} - \frac{N\left(N-n_2\right)}{\left(N-1\right)\left(N-2\right)}\frac{\mu_{21}}{n_2\bar{X}S_{yx}} - \theta\frac{\mu_{21(2)}}{\bar{X}S_{yx}}\right)$$

and

$$MSE\left(T_{LR1d}\right) = \lambda_3\left(1-\rho^2\right)S_y^2 + \lambda_1 S_y^2 \tag{1.12}$$

$$+\theta\left\{S_{y2}^2+R^2CS_{x2}^2\left(1-2C_{(2)}\right)\right\}.$$

Tabasum and Khan (2004) revisited the ratio estimator T_{R1d} of Khare and Srivastava (1993) and found that the cost of these estimators is less than the cost gained by Hansen and Hurwitz' (1946) estimator.

1.1.3. *Singh et al. (2010) Estimators:* Singh et al. (2010) suggested the following generalized version of modified exponential ratio and exponential product type estimators using two-phase sampling.

$$t_{(a)}^{(1)}=\bar{y}_2^*\exp\left[\frac{a\left(\bar{x}_2^*-\bar{x}_1\right)}{\left(\bar{x}_2^*+\bar{x}_1\right)}\right], \tag{1.13}$$

with

$$Bias\left(t_{(a)}^{(1)}\right)=\frac{a}{2\bar{X}}\left[\lambda_3\left(S_{xy}-\frac{R}{2}S_x^2\right)+\theta_2\left(S_{xy_{(2)}}-\frac{R}{2}S_{x_2}^2\right)\right] \tag{1.14}$$

and

$$MSE(t_{(a)}^{(1)})=\lambda_3\left[S_y^2+\left(\frac{aR^2S_x^2}{4}\right)(1+4C)\right]+\lambda_1S_y^2 \tag{1.15}$$

$$+\theta\left[S_{y_2}^2+\left(\frac{aR^2S_{x_2}^2}{4}\right)\left(1+4C_{(2)}\right)\right].$$

Remark 1.1. When a = -1 and 1, the expression given in (1.27) becomes Exponential Ratio Estimator and Exponential Product Estimator along with their Bias and mean square error respectively.

Quite often, we possess information on several variates and it may be considered important to make use of the whole of the available material to improve the precision. Several authors have used the multi-auxiliary characters for estimating the population mean using the known values of population means of auxiliary variables without considering the problem of non-response. Some of the names among them may be Olkin (1958), Shukla (1965, 66), Raj (1965), Rao and Mudholkar (1967), Mohanty (1967, 70), Srivastava (1971), Khare and Srivastava (1980, 81), Khare (1983), Srivastava and Jhajj (1983) and Sahoo (1986).

However, it is a common practice in sample surveys that the data may not be obtained for all the units selected in a sample due to some problem of non-response. Tripathi and Khare (1997) considered the simultaneous estimation of several population means for two types of non-responses (partial and complete) using single phase sampling. Khare and Sinha (2009) proposed two classes of estimators for estimating the population mean of study character using multi-auxiliary characters in the presence of non-response.

In the following section we propose a general class of estimators in two phase sampling using multi-auxiliary variables for estimating population mean of study variable, in the presence of non-response for both study and auxiliary variables at second phase.

2. PROPOSED CLASS OF ESTIMATORS

In this section, we will suggest a general class of estimators for two-phase sampling using multi-auxiliary variables when we consider non response on study variable and all the auxiliary variables.

Considering the sample design discussed in section 1. Let we have q auxiliary variables $x_1, x_2, ..., x_q$ to estimate the mean of study variable$y$. Further, let $\bar{X}_i$ denote the known population mean of i^{th}auxiliary variable, $\bar{x}_{1(i)}$ and $\bar{x}_{2(i)}$ denote the sample means of i^{th}auxiliary variables from first and second phase samples respectively, $\bar{x}^*_{2i}$ and $\bar{y}^*_2$ be the sample means for auxiliary variables and study variable respectively defined by Hansen and Hurwitz (1946) based on n_{21}+r sampling units.

Also, let the sampling errors of study and i^{th}auxiliary variable are

$$\bar{e}_{\bar{y}^*_2} = \bar{y}^*_2 - \bar{Y}, \tag{2.1}$$

$$\bar{e}_{x^*_{2i}} = \bar{x}^*_{2i} - \bar{X}_i \tag{2.2}$$

and

$$\bar{e}_{x_{1i}} = \bar{x}_{1i} - \bar{X}_i. \tag{2.3}$$

Let, $d^{'}_{q_1} = \begin{bmatrix} d_1 & d_2 & \cdots & d_{q_1} \end{bmatrix}$, $d^{'}_{q_2} = \begin{bmatrix} d_{q_1+1} & d_{q_1+2} & \cdots & d_{q_2} \end{bmatrix}andd^{'}_{q_3} = \begin{bmatrix} d_{q_2+1} & d_{q_2+2} & \cdots & d_{q_3} \end{bmatrix}$; where$d_i = \left(\bar{e}_{x^*_{2i}} - \bar{e}_{x_{1i}}\right)$; $i = 1, 2, ..., q$and$q = q_1 + q_2 + q_3$. Further, suppose the vector of unknown constants are$\alpha^{'}_{q_1} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{q_1} \end{bmatrix}$, $\alpha^{'}_{q_2} = \begin{bmatrix} \alpha_{q_1+1} & \alpha_{q_1+2} & \cdots & \alpha_{q_2} \end{bmatrix}$ and$\alpha^{'}_{q_3} = \begin{bmatrix} \alpha_{q_2+1} & \alpha_{q_2+2} & \cdots & \alpha_{q_3} \end{bmatrix}$. Also, suppose, $X_{q_1} = \left[\bar{X}^{-1}_i\right]_{(q_1\times q_1)}$;$i = 1, ..., q_1$, $X_{q_2} = \left[\bar{X}^{-1}_i\right]_{(q_2\times q_2)}$;$i = q_1+1,\ q_1+2..., q_2$ and $X_{q_3} = \left[\bar{X}^{-1}_i\right]_{(q_3\times q_3)}$;$i = q_2 + 1,\ q_2 + 2..., q_3$are the diagonal matrixes and $q = q_1 + q_2 + q_3$.

2.1. **Mathematical Expectations:** The following expectations will be used to derive the expressions for bias and mean square error of proposed class of estimators

$$E\left(\bar{e}_{y^*_2}\right) = E\left(\bar{e}_{x^*_{2i}}\right) = E\left(\bar{e}_{x_{1i}}\right) = 0$$

$$E\left(d_{q_1} e_{\bar{y}^*_2}\right) = \left[\lambda_3 S_{yx_i} + \theta S_{y(2)x_i}\right]_{q_1\times 1} = \varphi_{yx_{q_1}};\ i = 1, 2, ..., q_1$$

$$E\left(d_{q_2} e_{\bar{y}^*_2}\right) = \left[\lambda_3 S_{yx_i} + \theta S_{y(2)x_i}\right]_{q_2\times 1} = \varphi_{yx_{q_2}};\ i = q_1 + 1, q_1 + 2, ..., q_2$$

$$E\left(d_{q_3} e_{\bar{y}^*_2}\right) = \left[\lambda_3 S_{yx_i} + \theta S_{y(2)x_i}\right]_{q_3\times 1} = \varphi_{yx_{q_3}};\ i = q_2 + 1, q_2 + 2, ..., q_3$$

$$E\left(d_{q_1} d'_{q_1}\right) = \left[\lambda_3 S_{x_ix_j} + \theta S_{x_ix_{j(2)}}\right]_{q_1\times q_1} = \Delta_{q_1};\ i, j = 1, 2, ..., q_1$$

$$E\left(d_{q_2} d'_{q_2}\right) = \left[\lambda_3 S_{x_ix_j} + \theta S_{x_ix_{j(2)}}\right]_{q_2\times q_2} = \Delta_{q_2};\ i, j = q_1 + 1, q_1 + 2, ..., q_2$$

$$E\left(d_{q_3} d'_{q_3}\right) = \left[\lambda_3 S_{x_ix_j} + \theta S_{x_ix_{j(2)}}\right]_{q_3\times q_3} = \Delta_{q_3};\ i, j = q_2 + 1, q_2 + 2, ..., q_3$$

$$E\left(\bar{e}_{x_{1q_1}} \bar{e}^t_{\bar{x}_{1q_1}}\right) = \left[\lambda_1 S_{x_ix_j}\right]_{q_1\times q_1} = \Sigma_{q_1};\ i, j = 1, 2, ..., q_1,$$

$$E\left(\bar{e}_{x_{1q_2}} \bar{e}^t_{\bar{x}_{1q_2}}\right) = \left[\lambda_1 S_{x_ix_j}\right]_{q_2\times q_2} = \Sigma_{q_2};\ i, j = q_1 + 1, q_1 + 2, ..., q_2$$

$$E\left(\bar{e}_{x_{1q_3}} \bar{e}^t_{\bar{x}_{1q_3}}\right) = \left[\lambda_1 S_{x_ix_j}\right]_{q_3\times q_3} = \Sigma_{q_3};\ i, j = q_2 + 1, q_2 + 2, ..., q_3$$

$$E\left(\bar{e}^2_{x^*_{2q_1}}\right) = \left[\lambda_2 S^2_{x_i} + \theta S^2_{x_i(2)}\right]_{q_1\times 1} = s^*_{q_1}; i, j = 1, 2, ..., q_1,$$

$$E\left(\bar{e}^2_{x^*_{2q_2}}\right) = \left[\lambda_2 S^2_{x_i} + \theta S^2_{x_i(2)}\right]_{q_2\times 1} = s^*_{q_2}; i, j = q_1 + 1, q_1 + 2, ..., q_2$$

$$E\left(\bar{e}^2_{x^*_{2_{q_3}}}\right) = \left[\lambda_2 S^2_{x_i} + \theta S^2_{x_i(2)}\right]_{q_3\times 1} = s^*_{q_3}; i,j = q_2+1, q_2+2, ..., q_3$$

$$E\left(\bar{e}^2_{x_{1_{q_1}}}\right) = \left[\lambda_1 S^2_{x_i}\right]_{q_1\times 1} = s_{q_1}; i,j = 1,2,...,q_1$$

$$E\left(\bar{e}^2_{x_{1_{q_2}}}\right) = \left[\lambda_1 S^2_{x_i}\right]_{q_2\times 1} = s_{q_2}; i,j = q_1+1, q_1+2, ..., q_2$$

$$E\left(\bar{e}^2_{x_{1_{q_3}}}\right) = \left[\lambda_1 S^2_{x_i}\right]_{q_3\times 1} = s_{q_3}; i,j = q_2+1, q_2+2, ..., q_3$$

2.2. **Proposed Class of Regression-Cum-Ratio-Exponential Estimators.** We have proposed the following general class of estimator using multi-auxiliary variables:

$$t_q = \left(\bar{y}^*_2 + a\sum_{i=1}^{q_1}\alpha_i\left(\bar{x}_{1i} - \bar{x}^*_{2i}\right)\right) \tag{2.4}$$

$$\left[b\prod_{i=q_1+1}^{q_2}\left(\frac{\bar{x}_{1i}}{\bar{x}^*_{2i}}\right)^{c\alpha_i} + d\exp\sum_{i=q_2+1}^{q_3}\alpha_i\left(\frac{g\left(\bar{x}^*_{2i} - \bar{x}_{1i}\right)}{\left(\bar{x}^*_{2i} + \bar{x}_{1i}\right)}\right)\right]$$

where a, b, c and d are suitable constants to be chosen for generating members of this class such that $b+d=1$; and α_i, β_i and γ_i are unknown constants to be determined by minimizing the mean square error of t^m_1 given in (2.4).

Using the result from (2.1), (2.2) and (2.3), we can write (2.4) after ignoring the third and higher order terms as:

$$t_q \approx \left(\bar{Y} + \bar{e}_{y^*_2} + a\sum_{i=1}^{q_1}\alpha_i\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x^*_{2i}}\right)\right)\left[b\prod_{i=q_1+1}^{q_2}\left(1 + c\alpha_i\frac{\bar{e}_{x_{(1)i}} - \bar{e}_{x^*_{(2)i}}}{\bar{X}_i}\right.\right.$$

$$\left. + \frac{c\alpha_i\left(c\alpha_i+1\right)}{2\bar{X}^2_i}\bar{e}^2_{x^*_{(2)i}} + \frac{c\alpha_i\left(c\alpha_i-1\right)}{2\bar{X}^2_i}\bar{e}^2_{x_{(1)i}} - c^2\alpha^2_i\frac{\bar{e}_{x_{(1)i}}\bar{e}_{x^*_{(2)i}}}{\bar{X}^2_i}\right)$$

$$+d\left(1 + \sum_{i=q_2+1}^{q_3}\alpha_i\frac{g}{2\bar{X}_i}\left(\bar{e}_{x^*_{(2)i}} - \bar{e}_{x_{(1)i}}\right) - \sum_{i=q_2+1}^{q_3}\alpha_i\frac{g\left(\bar{e}^2_{x^*_{(2)i}} - \bar{e}^2_{x_{(1)i}}\right)}{4\bar{X}^2_i}\right.$$

$$\left.\left. + \left(\sum_{i=q_2+1}^{q_3}\alpha_i\right)^2\left(\frac{g^2}{8\bar{X}^2_i}\left(\bar{e}^2_{x^*_{(2)i}} + \bar{e}^2_{x_{(1)i}} - 2\bar{e}_{x^*_{(2)i}}\bar{e}_{x_{(1)i}}\right)\right)\right)\right].$$

Simplifying and writing in matrix notation, we get:

$$t_q \approx \left(\bar{Y} + e_{\bar{y}^*_2} - a\alpha'_{q_1}d_{q_1}\right)\left[1 + bc\sum_{i=q_1+1}^{q_2}\frac{\alpha_i}{\bar{X}_i}\left(\bar{e}_{x_{1i}} - \bar{e}_{x^*_{2i}}\right)\right.$$

$$+bc\sum_{i=q_1+1}^{q_2}\frac{\alpha_i\left(c\alpha_i+1\right)}{2\bar{X}^2_i}\bar{e}^2_{x^*_{2i}} + d\sum_{i=q_2+1}^{q_3}\frac{g\alpha_i}{2\bar{X}_i}\left(\bar{e}_{x^*_{2i}} - \bar{e}_{x_{1i}}\right)$$

$$+bc\sum_{i=q_1+1}^{q_2}\alpha_i\frac{\left(c\alpha_i-1\right)}{2\bar{X}^2_i}\bar{e}^2_{x_{(1)i}} - bc^2\sum_{i=q_1+1}^{q_2}\alpha^2_i\frac{\bar{e}_{x_{(1)i}}\bar{e}_{x^*_{(2)i}}}{\bar{X}^2_i} - d\sum_{i=q_2+1}^{q_3}\alpha_i\frac{g\left(\bar{e}^2_{x^*_{(2)i}} - \bar{e}^2_{x_{(1)i}}\right)}{4\bar{X}^2_i}$$

$$+d\left(\sum_{i=q_2+1}^{q_3}\alpha_i\right)^2\left(\frac{g^2}{8\bar{X}_i^2}\left(\bar{e}^2_{x^*_{(2)i}}+\bar{e}^2_{x_{(1)i}}-2\bar{e}_{x^*_{(2)i}}\bar{e}_{x_{(1)i}}\right)\right)\Bigg].$$

Again ignoring third and higher order terms for each expansion of product and simplifying, we can write the expression as:

$$t_q \approx \bar{Y}+\bar{e}_{y_2^*}+a\sum_{i=1}^{q_1}\alpha_i\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x^*_{(2)i}}\right)+\bar{Y}bc\sum_{i=q_1+1}^{q_2}\frac{\alpha_i}{\bar{X}_i}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x^*_{(2)i}}\right)$$

$$+\bar{Y}d\sum_{i=q_1+1}^{q_3}\alpha_i\frac{g}{2\bar{X}_i}\left(\bar{e}_{x^*_{(2)i}}-\bar{e}_{x_{(1)i}}\right)+abc\sum_{i=q_1+1}^{q_2}\frac{\alpha_i}{\bar{X}_i}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x^*_{(2)i}}\right)\sum_{i=1}^{q_1}\alpha_i\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x^*_{(2)i}}\right)$$

$$+\bar{Y}bc\sum_{i=q_1+1}^{q_2}\alpha_i\frac{\bar{e}^2_{x^*_{(2)i}}}{2\bar{X}_i^2}+\bar{Y}bc^2\sum_{i=q_1+1}^{q_2}\alpha_i^2\frac{\bar{e}^2_{x_{(1)i}}}{2\bar{X}_i^2}-\bar{Y}bc\sum_{i=q_1+1}^{q_2}\alpha_i\frac{\bar{e}^2_{x_{(1)i}}}{2\bar{X}_i^2}$$

$$-\bar{Y}bc^2\sum_{i=q_1+1}^{q_2}\alpha_i^2\frac{\bar{e}_{x_{(1)i}}\bar{e}_{x^*_{(2)i}}}{\bar{X}_i^2}-2\bar{Y}d\frac{g^2}{8\bar{X}_i^2}\left(\sum_{i=q_2+1}^{q_3}\alpha_i\right)^2\bar{e}_{x^*_{(2)i}}\bar{e}_{x_{(1)i}}$$

$$+\bar{Y}dg\sum_{i=q_2+1}^{q_3}\alpha_i\frac{\bar{e}^2_{x_{(1)i}}}{4\bar{X}_i^2}+\bar{Y}d\frac{g^2}{8\bar{X}_i^2}\left(\sum_{i=q_2+1}^{q_3}\alpha_i\right)^2\bar{e}^2_{x^*_{(2)i}}+\bar{Y}d\frac{g^2}{8\bar{X}_i^2}\left(\sum_{i=q_2+1}^{q_3}\alpha_i\right)^2\bar{e}^2_{x_{(1)i}}$$

$$-\bar{Y}dg\sum_{i=q_2+1}^{q_3}\alpha_i\frac{\bar{e}^2_{x^*_{(2)i}}}{4\bar{X}_i^2}+\bar{Y}bc^2\sum_{i=q_1+1}^{q_2}\alpha_i^2\frac{\bar{e}^2_{x^*_{(2)i}}}{2\bar{X}_i^2}+bc\sum_{i=q_1+1}^{q_2}\alpha_i\frac{\bar{e}_{x_{(1)i}}-\bar{e}_{x^*_{(2)i}}}{\bar{X}_i}\bar{e}_{y_2^*}$$

$$+d\sum_{i=q_2+1}^{q_3}\alpha_i\frac{g}{2\bar{X}_i}\left(\bar{e}_{x^*_{(2)i}}-\bar{e}_{x_{(1)i}}\right)\bar{e}_{y_2^*}+d\sum_{i=q_2+1}^{q_3}\alpha_i\frac{g}{2\bar{X}_i}\left(\bar{e}_{x^*_{(2)i}}-\bar{e}_{x_{(1)i}}\right)a\sum_{i=1}^{q_1}\alpha_i\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x^*_{(2)i}}\right).$$

We can write the above expression in matrix notation as:

$$t_q-\bar{Y}\approx\bar{e}_{y_2^*}-a\alpha^t_{q_1}d^*_{q_1}-\bar{Y}bc\alpha^t_{q_2}X_{q_2}d^*_{q_2}$$
$$+\frac{\bar{Y}}{2}dg\alpha^t_{q_3}X_{q_3}d^*_{q_3}+\frac{\bar{Y}}{2}bc^2\alpha^t_{q_2}X^2_{q_2}\alpha_{q_2}1_{q_2\times 1}\bar{e}^2_{x^*_{2q_2}}$$
$$+\frac{\bar{Y}}{2}bc\alpha^t_{q_2}X^2_{q_2}\bar{e}^2_{x^*_{2q_2}}+\frac{\bar{Y}}{2}bc^2\alpha^t_{q_2}X^2_{q_2}\alpha_{q_2}1_{q_2\times 1}\bar{e}^2_{x_{1q_2}}$$
$$-\bar{Y}bc^2\alpha^t_{q_2}X^2_{q_2}\bar{e}_{x_{1q_2}}\bar{e}^t_{x^*_{2q_2}}\alpha_{q_2}-\frac{\bar{Y}}{2}bc\alpha^t_{q_2}X^2_{q_2}\bar{e}^2_{x_{1q_2}}$$
$$-bc\alpha^t_{q_2}X_{q_2}d^*_{q_2}\bar{e}_{y_2^*}+abc\alpha^t_{q_2}X_{q_2}d^*_{q_2}d^{*'}_{q_1}\alpha_{q_1}$$
$$-\frac{\bar{Y}}{4}dg\alpha^t_{q_3}X^2_{q_3}\bar{e}^2_{x^*_{2q_3}}+\frac{\bar{Y}}{4}dg\alpha^t_{q_3}X^2_{q_3}\bar{e}^2_{x_{1q_3}}+\frac{dg}{2}\alpha^t_{q_3}X_{q_3}d^*_{q_3}e_{\bar{y}_2^*}$$
$$+\frac{\bar{Y}}{8}dg^2\alpha^t_{q_3}X^2_{q_3}\alpha_{q_3}1^t_{q_3}\bar{e}^2_{x^*_{2q_3}}+\frac{\bar{Y}}{8}dg^2\alpha^t_{q_3}X^2_{q_3}\alpha_{q_3}1^t_{q_3}\bar{e}^2_{x_{1q_3}}$$

$$-\frac{\bar{Y}}{4}dg^2\alpha^t_{q_3}X^2_{q_3}\bar{e}_{x_{1q_3}}e^t_{\bar{x}_{1q_3}}\alpha_{q_3}-\frac{adg}{2}\alpha^t_{q_3}X_{q_3}d^*_{q_3}d^{*t}_{q_1}\alpha_{q_1}. \tag{2.5}$$

Ignoring second and higher order terms, we can write

$$t_q-\bar{Y}\approx\bar{e}_{y_2^*}-a\alpha^t_{q_1}d^*_{q_1}-\bar{Y}bc\alpha^t_{q_2}X_{q_2}d^*_{q_2}+\frac{\bar{Y}}{2}dg\alpha^t_{q_3}X_{q_3}d^*_{q_3}.$$

Squaring and taking expectation

$$MSE\left(t_q\right)=E\left[\bar{e}_{\bar{y}_2^*}-h_{1\times q}^{'}m_{q\times 1}\right]^2, \tag{2.6}$$

where $h_q'=\left[\begin{array}{ccc}\alpha_{q_1}^{'} & \alpha_{q_2}^{'} & \alpha_{q_3}^{'}\end{array}\right]_{1\times q}$ and $m_q'=\left[\begin{array}{ccc}ad_{q_1}^{*'} & \bar{Y}bcX_{q_2}d_{q_2}^{*'} & -\frac{\bar{Y}dg}{2}X_{q_3}d_{q_3}^{*'}\end{array}\right]_{1\times q}$.
To find the optimum value of$h$, differentiating the equation (2.6) w.r.t h and equating to zero, we get:

$$2m_qE\left[\bar{e}_{\bar{y}_2^*}-m_q^{'}h_q\right]=0$$

or

$$E\left(m_q\bar{e}_{\bar{y}_2^*}\right)-E\left(m_qm_q^{'}\right)h_q=0$$

or

$$\Phi_{yx_q}-\mathrm{T}_qh_q=0 \tag{2.7}$$

where

$$T_q=\left[\begin{array}{ccc}a^2\Delta_{q_1} & \bar{Y}abc\Delta_{q_{12}}X_{q_2} & -\frac{\bar{Y}adg}{2}\Delta_{q_{13}}X_{q_3}\\ & \bar{Y}^2b^2c^2X_{q_2}\Delta_{q_2}X_{q_2} & -\frac{\bar{Y}^2bcdg}{2}X_{q_2}\Delta_{q_{23}}X_{q_3}\\ & & \frac{\bar{Y}^2d^2g^2}{4}X_{q_3}\Delta_{q_3}X_{q_3}\end{array}\right]$$

and

$$\Phi_{yx_q}'=\left[\begin{array}{ccc}a\Phi_{yx_{q_1}} & \bar{Y}bcX_{q_2}\Phi_{yx_{q_2}} & -\frac{\bar{Y}dg}{2}X_{q_3}\Phi_{yx_{q_3}}\end{array}\right].$$

Now from (2.7)

$$h_{q\times 1}=\mathrm{T}^{-1}\Phi_{yx_{q\times 1}}. \tag{2.8}$$

Now, from (2.5), ignoring the third and higher order terms for each expansion of product, we get

$$\begin{aligned}E\left(t_1^m-\bar{Y}\right)\approx E\Big(&\bar{e}_{y_2^*}-a\alpha_{q_1}^td_{q_1}^*-\bar{Y}bc\alpha_{q_2}^tX_{q_2}d_{q_2}^*+\frac{\bar{Y}}{2}dg\alpha_{q_3}^tX_{q_3}d_{q_3}^*\\&+\frac{\bar{Y}}{2}bc^2\alpha_{q_2}^tX_{q_2}^2\alpha_{q_2}1_{q_2}\bar{e}_{x_{2q_2}^*}^2+\frac{\bar{Y}}{2}bc\alpha_{q_2}^tX_{q_2}^2\bar{e}_{x_{2q_2}^*}^2\\&+\frac{\bar{Y}}{2}bc^2\alpha_{q_2}^tX_{q_2}^2\alpha_{q_2}1_{q_2}\bar{e}_{x_{1q_2}}^2-\bar{Y}bc^2\alpha_{q_2}^tX_{q_2}^2\bar{e}_{x_{1q_2}}\bar{e}_{x_{2q_2}^*}^t\alpha_{q_2}\\&-\frac{\bar{Y}}{2}bc\alpha_{q_2}^tX_{q_2}^2\bar{e}_{x_{1q_2}}^2-bc\alpha_{q_2}^tX_{q_2}d_{q_2}^*\bar{e}_{y_2^*}\\&+abc\alpha_{q_2}^tX_{q_2}d_{q_2}^*d_{q_1}^{*'}\alpha_{q_1}-\frac{\bar{Y}}{4}dg\alpha_{q_3}^tX_{q_3}^2\bar{e}_{x_{2q_3}^*}^2\\&+\frac{\bar{Y}}{4}dg\alpha_{q_3}^tX_{q_3}^2\bar{e}_{x_{1q_3}}^2+\frac{\bar{Y}}{8}dg^2\alpha_{q_3}^tX_{q_3}^2\alpha_{q_3}1_{q_3}^t\bar{e}_{x_{2q_3}^*}^2\\&+\frac{\bar{Y}}{8}dg^2\alpha_{q_3}^tX_{q_3}^2\alpha_{q_3}1_{q_3}^t\bar{e}_{x_{1q_3}}^2-\frac{\bar{Y}}{4}dg^2\alpha_{q_3}^tX_{q_3}^2\bar{e}_{x_{1q_3}}e_{\bar{x}_{1q_3}}^t\alpha_{q_3}\\&-\frac{adg}{2}\alpha_{q_3}^tX_{q_3}d_{q_3}^*d_{q_1}^{*t}\alpha_{q_1}+\frac{dg}{2}\alpha_{q_3}^tX_{q_3}d_{q_3}^*e_{\bar{y}_2^*}\Big).\end{aligned}$$

Now taking expectation, bias can be written as:

$$\begin{aligned}Bias\left(t_q\right)\approx bc\Big\{&a\alpha_{q_2}^tX_{q_2}\Delta_{q_{21}}\alpha_{q_1}-\alpha_{q_2}^tX_{q_2}\Phi_{yx_{q_2}}+\frac{\bar{Y}}{2}\alpha_{q_2}^tX_{q_2}^2\left(s_{2_{q_2}}^*-s_{1_{q_2}}^*\right)\\&+c\left(\frac{\bar{Y}}{2}\alpha_{q_2}^tX_{q_2}^2\alpha_{q_2}1_{q_2}\left(s_{1_{q_2}}^*+s_{2_{q_2}}^*\right)-\bar{Y}\alpha_{q_2}^tX_{q_2}^2s_{1_{q_{22}}}\alpha_{q_2}\right)\Big\}\end{aligned}$$

$$+dg\left\{-\frac{a}{2}\alpha_{q3}^{t}X_{q3}\Delta_{3_{q31}}\alpha_{q1}+\frac{\bar{Y}}{4}\alpha_{q3}^{t}X_{q3}^{2}\left(s_{1_{q3}}^{*}-s_{2_{q3}}^{*}\right)+\frac{1}{2}\alpha_{q3}^{t}X_{q3}\Phi_{yx_{q3}}\right.$$

$$(2.9)\qquad \left.+g\left(\frac{\bar{Y}}{8}\alpha_{q3}^{t}X_{q3}^{2}\alpha_{q3}1_{q3}^{t}\left(s_{1_{q3}}^{*}+s_{2_{q3}}^{*}\right)-\frac{\bar{Y}}{4}\alpha_{q3}^{t}X_{q3}^{2}s_{1_{q33}}\alpha_{q3}\right)\right\}.$$

The optimum values of unknowns are given in equation (2.8). By using the normal equations those are used to find the optimum values given in equation (2.8), equation (2.6) can be written as:

$$MSE\left(t_{q}\right)=E\left(\bar{e}_{y_{2}^{*}}\left(\bar{e}_{y_{2}^{*}}-h_{q}^{'}m_{q}\right)\right)$$

or

$$MSE\left(t_{q}\right)=E\left(\bar{e}_{y_{2}^{*}}^{2}\right)-h_{q}^{'}E\left(m_{q}\bar{e}_{y_{2}^{*}}\right)$$

or

$$MSE\left(t_{q}\right)=\lambda_{2}S_{y}^{2}+\theta S_{y(2)}^{2}-h_{1\times q}^{'}\Phi_{yx_{q\times1}}.$$

Now, using equation (2.8), we get:

$$(2.10)\qquad MSE\left(t_{q}\right)=\lambda_{2}S_{y}^{2}+\theta S_{y(2)}^{2}-\Phi_{yx_{q}}^{'}\mathrm{T}_{q}^{-1}\Phi_{yx_{q}}$$

3. SPECIAL CASES OF PROPOSED ESTIMATORS

As the class is generalized in nature, so special cases can be deduced for different values of a, b, c, d and g. Different special cases, their biases abs mean square errors are given in the following tables.

Table-1: Special Cases of Proposed Class of Estimators

Sr. #	**(a,b,c,d,g)**	**Estimator**
1	(0,1,1,0,0) Ratio	$t_r = \bar{y}_2^* \prod_{i=q_1+1}^{q_2} \left(\frac{\bar{x}_{1i}}{\bar{x}_{2i}^*}\right)^{\alpha_i}$
2	(0,1,-1,0,0) Product	$t_p = \bar{y}_2^* \prod_{i=q_1+1}^{q_2} \left(\frac{\bar{x}_{2i}^*}{\bar{x}_{1i}}\right)^{\alpha_i}$
3	(1,0,0,1,0) Regression	$t_{lr} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right)$
4	(-1,0,0,1,0) Difference	$t_d = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right)$
5	(0,0,0,1,1) Exponential Ratio	$t_{er} = \bar{y}_2^* \exp\left[\sum_{i=q_2+1}^{q_3} \alpha_i \left(\frac{\bar{x}_{1i}-\bar{x}_{2i}^*}{\bar{x}_{1i}+\bar{x}_{2i}^*}\right)\right]$
6	(0,0,0,1,-1) Exponential Product	$t_{ep} = \bar{y}_2^* \exp\left[\sum_{i=q_2+1}^{q_3} \alpha_i \left(\frac{\bar{x}_2^*-\bar{x}_{1i}}{\bar{x}_2^*+\bar{x}_{1i}}\right)\right]$
7	(1,1,1,0,0) Regression-Cum-Ratio	$t_{rcr} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \prod_{i=q_1+1}^{q_2} \left(\frac{\bar{x}_{1i}}{\bar{x}_{2i}^*}\right)^{\alpha_i}$
8	(1,1,-1,0,0) Regression-Cum-Product	$t_{rcp} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \prod_{i=q_1+1}^{q_2} \left(\frac{\bar{x}_{2i}^*}{\bar{x}_{1i}}\right)^{\alpha_i}$
9	(1,0,0,1,1) Regression-Cum-Exponential-Ratio	$t_{rcer} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \exp \sum_{i=q_2+1}^{q_3} \alpha_i \left(\frac{\bar{x}_{1i}-\bar{x}_{2i}^*}{\bar{x}_{1i}+\bar{x}_{2i}^*}\right)$
10	(1,0,0,1,-1) Regression-Cum-Exponential-Product	$t_{rcep} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \exp \sum_{i=q_2+1}^{q_3} \alpha_i \left(\frac{\bar{x}_{1i}-\bar{x}_{2i}^*}{\bar{x}_{2i}^*+\bar{x}_{1i}}\right)$
11	(-1,1,1,0,0) Difference -Cum-Ratio	$t_{dcr} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \prod_{i=q_1+1}^{q_2} \left(\frac{\bar{x}_{1i}}{\bar{x}_{2i}^*}\right)^{\alpha_i}$
12	(-1,1,-1,0,0) Difference -Cum-Product	$t_{dcp} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \prod_{i=q_1+1}^{q_2} \left(\frac{\bar{x}_{2i}^*}{\bar{x}_{1i}}\right)^{\alpha_i}$
13	(-1,0,0,1,1) Difference -Cum-Exponential-Ratio	$t_{dcer} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \exp \sum_{i=q_2+1}^{q_3} \alpha_i \left(\frac{\bar{x}_{1i}-\bar{x}_{2i}^*}{\bar{x}_{1i}+\bar{x}_{2i}^*}\right)$
14	(-1,0,0,1,-1) Difference -Cum-Exponential-Product	$t_{dcep} = \left(\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_i \left(\bar{x}_{1i} - \bar{x}_{2i}^*\right)\right) \exp \sum_{i=q_2+1}^{q_3} \alpha_i \left(\frac{\bar{x}_{1i}-\bar{x}_{2i}^*}{\bar{x}_{2i}^*+\bar{x}_{1i}}\right)$

The expressions of bias MSE of above special cases can be obtained from (2.9) and (2.10) by using suitable combination of (a, b, c, d, g). The empirical values of these expressions are given in empirical section. The following table contains the some existing estimators that are special cases of proposed class.

Table-3: Some existing estimators as special cases of proposed class

Sr. #	(a,b,c,d,g,q$_1$,q$_2$,q$_3$)	Estimator	Author
1	(0,0,0,1,0,0,0,0)	$\bar{y}_2^*$	Hansen and Hurwitz (1946)
2	(0,1,1,0,0,0,1,0)	$\bar{y}_2^*\left(\frac{\bar{x}_1}{\bar{x}_2^*}\right)$	Khare and Srivastava (1993, 1995)
3	(0,1,-1,0,0,0,1,0	$\bar{y}_2^*\left(\frac{\bar{x}_2^*}{\bar{x}_1}\right)$	Khare and Srivastava (1993, 1995)
4	(1,0,0,1,0,1,0,0)	$\bar{y}_2^*+\alpha\left(\bar{x}_1-\bar{x}_2^*\right)$	Khare and Srivastava (1993, 1995)
5	(0,0,0,1,1,0,0,1)	$\bar{y}_2^*\exp\left(\frac{\bar{x}_1-\bar{x}_2^*}{\bar{x}_1+\bar{x}_2^*}\right)$	Singh et al. (2010)
6	(0,0,0,1,-1,0,0,1)	$\bar{y}_2^*\exp\left(\frac{\bar{x}_2^*-\bar{x}_1}{\bar{x}_2^*+\bar{x}_1}\right)$	Singh et al. (2010)

4. THEORETICAL COMPARISON FOR THE PROPOSED CLASS

In this section we compare our proposed class of estimators with Hansen and Hurwitz (1946).

The comparison of proposed class of estimators with Hansen and Hurwitz (1946) is:

$$MSE\left(\bar{y}_2^*\right)-MSE\left(t_1^m\right)=\phi'_{yx_{1\times3}}\mathrm{T}^{-1}_{x_{3\times3}}\phi_{yx_{3\times1}}>0$$

We can observe that our proposed class of estimators is efficient than Hansen and Hurwitz (1946).

5. EMPIRICAL COMPARISON

For empirical comparison, we have used the data set of Census report of Faisalabad district (1998) Pakistan, used by Ahmad, et al. (2009b). The size of this population is $N = 283$. The first 28% of the observations are considered as non-response $N_2 = 80$and then$N_1 = 203$. The detail of study and auxiliary variables is given in Table A2 of Appendix A. The necessary matrices required for the calculation of bias and MSE are given in Table A3 and Table A4 respectively. The values of bias and percent relative efficiency of the proposed and existing estimators for different values of n_1,n_2 and$k$ are given in Appendix A.

We have developed a general class of estimators using multi-auxiliary variables consisting fourteen types of estimators.

The relative efficiency and bias of existing and suggested estimators are given in Table A4 and Table A5 respectively. From Table A4, the MSE of six estimators (ratio, product, regression, difference, exponential, exponential ratio and exponential product) is same and similarly, of eight remaining estimators is same. The group of eight estimators is more efficient because the PRE of this group is greater than the group consisting of six estimators. To check the performance within these two groups, we need to observe the bias of these from Table A5. This group of eight estimators having same MSE can be further divided in to two groups on the basis of biases, each consisting of four estimators. The first group of four estimators (Regression-cum-exponential-ratio, Regression-cum-exponential-product, difference-cum-exponential-ratio and difference-cum-exponential-product) is preferred because it has less bias as compared to the second group of estimators (Regression-cum -ratio, Regression-cum -product, difference-cum -ratio and difference-cum-product).

Now we want to check the rate of increase/decrees in Bias/MSE of different estimators by increasing the values ofn_1,n_2 and$k$under the assumption that the trend will be linear. For this purpose we considered two values for eachn_1,n_2 and$k$and calculated slopes for Bias/MSE for all estimators, the results are given in Table A6. Empty cell in the column of$n_1$shows that the expression of MSE of$\bar{y}_2^*$does not involve the value ofn_1.

From Table A.6, by comparing our suggested and existing estimators, we can observe that when we increase the value of$k$, the value of PRE oft_R and T_{lr1d} highly increase, but the PRE of the class consisting of eight estimators increase least. When we increase the value of k, bias of T_{R1d}increases the most and the value of exponential product and exponential ratio is almost stable.
When we change the value ofn_1, the increase in the values of PRE of T_{lr1d} is the most and in the value of PRE of$t^{(1)}_{R(-1)d}$ is least. In the case of bias, almost all of the estimators are stable and the stability of the class of four estimators (Regression-cum-exponential-ratio, Regression-cum-exponential-product, difference-cum-exponential-ratio and difference-cum-exponential-product) is more than all other considered estimators.

We can conclude from the above discussion regarding Bias, Relative efficiency and slopes that the class of four estimators (Regression-cum-exponential-ratio, Regression-cum-exponential-product, difference-cum-exponential-ratio and difference-cum-exponential-product) has less Bias and MSE than all suggested and existing estimators, and is also less sensitive for increasing the values ofn_1,n_2 and$k$. Hence this group is preferred over all other considered estimators.

References

[1] Cochran, W. G., Sampling Techniques. 3rd ed., Wiley, New York, 1977.

[2] Hansen, M. H. and Hurwitz, W. N., The Problem of Non Response in Sample Surveys. *Journal of American Statistical Association,* 41 (1946), 517-529.

[3] Khare, B. B., Some Problems of Estimators using Auxiliary Character. Ph.D. Thesis submitted to B.H.U., Varanasi, India (1983).

[4] Khare, B. B. and Sinha, B. B., On Class of Estimators for Population Mean using Multi-Auxiliary Characters in the Presence of Non Response. *Statistics in Transition*, 10 (2009), no. 1, 3-14.

[5] Khare, B. B. and Srivastava, S.R., On an Efficient Estimator of Population Mean using Two Auxiliary Variables. Proceedings of the National Academy of Science. India, 50 A (1980), 209-214

[6] Khare, B. B. and Srivastava, S.R., A Generalized Regression Ratio Estimator for the Population Mean using Two Auxiliary Variables. *Aligarh Journal of Statistics*, 1 (1981), no. 1, 43-51.

[7] Khare, B. B. and Srivastava, S., Estimation of Population Mean using Auxiliary Character in the Presence of Non Response. *National Academy Sciences Letters*, 16 (1993), no. 3, 111-114.

[8] Khare, B. B. and Srivastava, S., Study of Conventional and Alternative Two Phase Sampling Ratio, Product and Regression Estimators in the Presence of Non Response. Proceedings of the National Academy of Science. India, 65A (1995), no. 2, 195-203.

[9] Khare, B. B. and Srivastava, S., Transformed Ratio Type Estimators for the Population Mean in the Presence of Non Response. *Communication in Statistics-Theory and Methods,* 26 (1997), no. 7, 1779-1791.

[10] Miller, K. S., On the Inverse of the Sum of Matrices. *Mathematics Magazine,* 54 (1981), no. 2, 67-72.

[11] Mohanty, S., Combination of Regression and Ratio Estimator. *Journal of Indian Statistical Association*, 5 (1967), 16-29

[12] Mohanty, S., Contribution to the Theory of Sampling. Ph.D thesis submitted to I.A.S.R.I., New Delhi (1970).

[13] Olkin, I., Multivariate Ratio Estimator for Finite Populations. *Biometrika*, 45 (1958), 154-165.

[14] , Raj, D., On a Method of using Multi-Auxiliary Information in Sample Surveys. *Journal of the American Statistical Association*, 60(1965), no, 309, 270-277.

[15] Rao, P.S.R.S., Ratio Estimation with Sub Sampling the Non Respondents. *Survey Methodology.* 12 (1986), no. 2, 217-230.

[16] Rao. P.S.R.S. and Madholkar, G.S., Generalized Multivariate Estimators for the Mean of a Finite Population. *Journal of the American Statistical Association*, 62 (1967), 1009-1012.

[17] Sahoo, L.N., On a Class of Unbiased Estimators using Multi-auxiliary Information. *Journal of Indian Society of Agricultural Statistics*, 38 (1986), 379-382.

[18] Shukla, G. K., Multivariate Regression Estimate. *Jounal of Indian Statistical Association,* 3(1965), 202-211.

[19] Shukla, G. K., An Alternate Multivariate Ratio Estimate for finite population. *Calkutta Statistical Association Bulletin,* 15 (1966),127-134.

[20] Singh, H. P. and Kumar, S., Estimation of Mean in Presence of Non Response using Two-Phase Sampling Scheme. *Statist. Pap*, (2008a), DOI 10.1007/s00362-008-040-5.

[21] Singh, H. P. and Kumar, S., A Regression Approach to the Estimation of Finite Population Mean in Presence of Non Response. *Australian & New Zealand Journal of Statistics*, 50(2008b), 395-408.

[22] Singh, R., Kumar, M., Chaudhary, M.K. and Smarandache, F., Estimation of Mean in Presence of Non-Response Using Exponential Estimator. Unpublished manuscript, (2009).

[23] Singh, H. P. and Kumar, S. , A General Procedure of Estimating the Population Mean in the Presence of Non Response under Sampling using Auxiliary Information. *Statistics and Operations Research Transactions,* 33(2009), 71-84.

[24] Singh, H. P., Kumar, S. and Kozak, M., Improved Estimator of Finite Population Mean using Sub-Sampling to Deal with Non-Response in Two-Phase Sampling Scheme. *Communication in Statistics-Theory and Methods,* 39(2010), 791-802.

[25] Srivastava, S.K., An estimator using auxiliary information in sample surveys. *Calcutta Statistical Association Bulletin*, 16,(1967),121-132.

[26] Srivastava, S.K., A Generalized Estimator for the Mean of a Finite Population using Multi-auxiliary Information. *Journal of the American Statistical Association*, 66(1971), 404-407.

[27] Srivastava, S.K, and Jhajj, H.S., A Class of Estimators of the Population Mean using Multi-auxiliary Information. *Calkutta Statistical Association Bulletin*, 15 (1983), 127-134.

Appendix A

Table A.1: Description of Variables and mean w. r. t $N = 283$ and $N_2 = 80$

Name	Description of Variables	Mean for $N = 283$	Mean for $N_2 = 80$
Y_2	Population of currently married	1511.261484	1825.9
X_1	Population of both sexes	10931.4417	13226.025
X_2	Population of primary but below matric	1969.286219	2622.5625
X_3	Population of matric and above	754.360424	1048.275
X_4	Population of 18 years old and above	6173.162544	8146.425
X_5	Population of women 15-49 years old	2457.681979	2953.7375

Table A2: Variance-covariance matrix w. r. t N

	Y_2	X_1	X_2	X_3	X_4	X_5
Y_2	621542.7328	3963614.8	792130.418	303104.0047	2481864.876	823818.0161
X_1	3963614.842	29441849	5836183.44	2230311.546	17507723.7	6588651.038
X_2	792130.4178	5836183.4	2815406.61	624113.7475	3788276.829	1390088.258
X_3	303104.0047	2230311.5	624113.748	276331.9476	1795351.395	533963.3136
X_4	2481864.876	17507724	3788276.83	1795351.395	39560446.26	4060606.428
X_5	823818.0161	6588651	1390088.26	533963.3136	4060606.428	2196836.707

Table A3: Variance-covariance matrix w. r. t N_2

	Y_2	X_1	X_2	X_3	X_4	X_5
Y_2	915368.5722	5470158.3	1207503.21	429915.8	3939985.41	1236879.163
X_1	5470158.306	43396500	8701020.14	3059560.347	27495437.31	8863944.5
X_2	1207503.209	8701020.1	2605859.46	788441.1218	6822705.087	1983612.909
X_3	429915.8	3059560.3	788441.122	382683.9487	3553687.451	716956.6554
X_4	3939985.41	27495437	6822705.09	3553687.451	120333017.3	6217267.278
X_5	1236879.163	8863944.5	1983612.91	716956.6554	6217267.278	2000402.069

Table A4: Percent Relative Efficiency w. r. t$\bar{y}_2^*$ of Existing and Proposed Estimators

k	n_1	n_2	T_{R1d}	T_{lr1d}	$t^{(1)}_{R(-1)d}$	t_r,t_p,t_{lr}, t_d,t_{er},t_{ep}	t_{rcr},t_{rcp}, t_{rcer}, t_{rcep}, t_{dcr},t_{dcp}, t_{dcer},t_{dcep}
2	130	90	114.22	123.83	152.85	179.18	181.28
2	130	100	113.17	119.05	146.78	160.91	162.37
2	130	110	112.06	114.33	140.81	143.66	144.64
2	130	120	110.90	109.68	134.93	125.69	126.34
2	140	90	115.23	127.66	158.66	195.61	198.39
2	140	100	114.31	123.12	152.97	176.91	178.93
2	140	110	113.33	118.62	147.31	159.64	161.06
2	140	120	112.31	114.14	141.70	143.00	143.99
2	150	90	116.12	131.16	164.08	212.18	215.75
2	150	100	115.31	126.88	158.77	193.01	195.68
2	150	110	114.46	122.60	153.46	175.40	177.36
2	150	120	113.56	118.32	148.15	158.85	160.26
2	160	90	116.91	134.39	169.13	228.96	233.42
2	160	100	116.21	130.36	164.21	209.36	212.78
2	160	110	115.46	126.31	159.26	191.33	193.92
2	160	120	114.68	122.23	154.28	174.52	176.45
3	130	90	117.13	120.53	164.97	170.34	172.23
3	130	100	116.40	117.16	160.14	154.85	156.30
3	130	110	115.64	113.82	155.36	139.15	140.25
3	130	120	114.86	110.51	150.64	121.69	122.55
3	140	90	117.90	123.14	169.85	182.75	185.08
3	140	100	117.27	119.97	165.39	167.78	169.61
3	140	110	116.61	116.81	160.95	153.07	154.49
3	140	120	115.92	113.67	156.53	137.92	139.02
3	150	90	118.57	125.49	174.32	194.56	197.35
3	150	100	118.02	122.51	170.23	179.94	182.18
3	150	110	117.45	119.54	166.13	165.76	167.54
3	150	120	116.86	116.57	162.03	151.63	153.04
3	160	90	119.17	127.62	178.43	205.89	209.17
3	160	100	118.70	124.83	174.70	191.57	194.25
3	160	110	118.21	122.03	170.94	177.75	179.93
3	160	120	117.69	119.22	167.16	164.18	165.93

Table A4: Cont...

k	n_1	n_2	T_{R1d}	T_{lr1d}	$t^{(1)}_{R(-1)d}$	t_r,t_p,t_{lr}, t_d,t_{er},t_{ep}	t_{rcr},t_{rcp}, t_{rcer},t_{rcep}, t_{dcr},t_{dcp}, t_{dcer},t_{dcep}
4	130	90	118.86	118.73	172.79	160.96	162.71
4	130	100	118.30	116.12	168.77	147.43	148.85
4	130	110	117.73	113.54	164.79	133.31	134.46
4	130	120	117.14	110.97	160.85	117.50	118.45
4	140	90	119.48	120.70	176.97	171.03	173.09
4	140	100	119.00	118.27	173.30	158.33	160.02
4	140	110	118.50	115.84	169.64	145.45	146.85
4	140	120	117.99	113.42	166.01	131.88	133.03
4	150	90	120.02	122.47	180.76	180.28	182.65
4	150	100	119.61	120.19	177.42	168.20	170.18
4	150	110	119.18	117.91	174.09	156.13	157.79
4	150	120	118.74	115.63	170.75	143.78	145.16
4	160	90	120.50	124.06	184.22	188.87	191.56
4	160	100	120.15	121.92	181.20	177.31	179.59
4	160	110	119.78	119.78	178.17	165.84	167.78
4	160	120	119.41	117.64	175.13	154.28	155.91
5	130	90	120.00	117.59	178.24	153.04	154.69
5	130	100	119.55	115.47	174.81	141.03	142.42
5	130	110	119.09	113.36	171.40	128.35	129.53
5	130	120	118.62	111.26	168.02	114.24	115.24
5	140	90	120.52	119.18	181.89	161.58	163.46
5	140	100	120.13	117.20	178.78	150.47	152.08
5	140	110	119.74	115.23	175.67	139.05	140.42
5	140	120	119.34	113.26	172.58	126.91	128.08
5	150	90	120.97	120.59	185.18	169.25	171.36
5	150	100	120.64	118.75	182.37	158.84	160.66
5	150	110	120.31	116.90	179.55	148.27	149.85
5	150	120	119.96	115.05	176.74	137.33	138.69
5	160	90	121.37	121.86	188.16	176.23	178.57
5	160	100	121.09	120.13	185.63	166.40	168.44
5	160	110	120.81	118.40	183.09	156.50	158.28
5	160	120	120.51	116.67	180.54	146.38	147.93

Table A5: Results of absolute bias of existing and proposed estimators

k	n_1	n_2	T_{R1d}	$t^{(1)}_{R(-1)d}$	t_r,t_p	t_{er},t_{ep}	t_{rcr},t_{rcp}, t_{dcr},t_{dcp}	t_{rcer},t_{rcep}, t_{dcer},t_{dcep}
2	130	90	2.52	9.03	0.10	2.46	0.0912	0.0906
2	130	100	2.02	7.16	0.12	1.88	0.0703	0.0698
2	130	110	1.60	5.64	0.13	1.38	0.0538	0.0533
2	130	120	1.25	4.36	0.13	0.88	0.0419	0.0415
2	140	90	2.71	9.72	0.08	2.68	0.0996	0.0991
2	140	100	2.20	7.85	0.10	2.12	0.0787	0.0782
2	140	110	1.78	6.33	0.11	1.64	0.0617	0.0612
2	140	120	1.44	5.05	0.12	1.22	0.0480	0.0476
2	150	90	2.86	10.32	0.07	2.88	0.1069	0.1064
2	150	100	2.36	8.45	0.08	2.32	0.0860	0.0855
2	150	110	1.94	6.92	0.09	1.85	0.0688	0.0684
2	150	120	1.60	5.65	0.10	1.45	0.0547	0.0544
2	160	90	3.00	10.85	0.06	3.04	0.1132	0.1128
2	160	100	2.50	8.98	0.07	2.49	0.0923	0.0919
2	160	110	2.08	7.45	0.08	2.03	0.0752	0.0748
2	160	120	1.73	6.17	0.09	1.64	0.0610	0.0606
3	130	90	3.91	13.78	0.31	3.37	0.1308	0.1303
3	130	100	3.27	11.43	0.30	2.62	0.1078	0.1074
3	130	110	2.74	9.51	0.29	1.92	0.0909	0.0905
3	130	120	2.30	7.92	0.23	1.17	0.0812	0.0808
3	140	90	4.10	14.47	0.30	3.65	0.1385	0.1381
3	140	100	3.45	12.12	0.29	2.92	0.1147	0.1143
3	140	110	2.92	10.20	0.28	2.29	0.0962	0.0959
3	140	120	2.48	8.61	0.26	1.69	0.0826	0.0823
3	150	90	4.25	15.06	0.28	3.87	0.1454	0.1450
3	150	100	3.61	12.72	0.27	3.17	0.1212	0.1209
3	150	110	3.08	10.80	0.27	2.56	0.1020	0.1017
3	150	120	2.64	9.20	0.26	2.03	0.0868	0.0866
3	160	90	4.39	15.59	0.27	4.06	0.1515	0.1512
3	160	100	3.75	13.24	0.26	3.37	0.1272	0.1269
3	160	110	3.22	11.33	0.25	2.78	0.1075	0.1072
3	160	120	2.77	9.73	0.25	2.27	0.0917	0.0914

Table A5: Cont...

k	n_1	n_2	T_{R1d}	$t^{(1)}_{R(-1)d}$	t_r,t_p,	t_{er},t_{ep}	t_{rcr},t_{rcp}, t_{dcr},t_{dcp}	t_{rcer},t_{rcep}, t_{dcer},t_{dcep}
4	130	90	5.30	18.52	0.51	4.11	0.1744	0.1740
4	130	100	4.52	15.70	0.47	3.18	0.1497	0.1493
4	130	110	3.87	13.39	0.41	2.29	0.1324	0.1320
4	130	120	3.34	11.47	0.29	1.31	0.1237	0.1232
4	140	90	5.48	19.21	0.50	4.44	0.1807	0.1804
4	140	100	4.70	16.39	0.47	3.56	0.1546	0.1543
4	140	110	4.06	14.08	0.43	2.77	0.1349	0.1346
4	140	120	3.52	12.16	0.37	2.01	0.1213	0.1209
4	150	90	5.64	19.80	0.49	4.70	0.1867	0.1864
4	150	100	4.86	16.99	0.46	3.85	0.1598	0.1595
4	150	110	4.21	14.68	0.43	3.12	0.1389	0.1386
4	150	120	3.68	12.76	0.39	2.45	0.1229	0.1226
4	160	90	5.78	20.33	0.48	4.92	0.1922	0.1919
4	160	100	4.99	17.51	0.45	4.09	0.1648	0.1646
4	160	110	4.35	15.20	0.42	3.39	0.1431	0.1429
4	160	120	3.82	13.28	0.39	2.76	0.1260	0.1257
5	130	90	6.69	23.26	0.70	4.71	0.2216	0.2212
5	130	100	5.77	19.97	0.62	3.60	0.1948	0.1944
5	130	110	5.01	17.27	0.51	2.55	0.1767	0.1762
5	130	120	4.38	15.03	0.33	1.37	0.1677	0.1671
5	140	90	6.87	23.95	0.70	5.10	0.2263	0.2260
5	140	100	5.95	20.66	0.63	4.07	0.1978	0.1974
5	140	110	5.19	17.96	0.55	3.14	0.1766	0.1762
5	140	120	4.56	15.72	0.46	2.22	0.1623	0.1619
5	150	90	7.03	24.55	0.69	5.41	0.2311	0.2308
5	150	100	6.11	21.25	0.63	4.42	0.2015	0.2012
5	150	110	5.35	18.56	0.57	3.56	0.1787	0.1784
5	150	120	4.72	16.32	0.50	2.76	0.1616	0.1613
5	160	90	7.17	25.07	0.68	5.66	0.2357	0.2354
5	160	100	6.24	21.78	0.62	4.70	0.2054	0.2051
5	160	110	5.49	19.08	0.57	3.88	0.1816	0.1813
5	160	120	4.86	16.84	0.52	3.15	0.1630	0.1627

TableA6: Slopes of Proposed and Existing Estimators

	Estimators	k	n_1	n_2
PREs	T_{R1d}	-4.51	0.08	-0.07
	T_{lr1d}	-1.50	0.31	-0.37
	$t^{(??)}_{R(-1)d}$	-9.28	0.51	-0.49
	t_r,t_p,t_{lr}, t_d,t_{er},t_{ep}	2.19	1.42	-1.69
	t_{rcr},t_{rcp},t_{rcer},t_{rcep}, t_{dcr},t_{dcp},t_{dcer},t_{dcep}	2.33	1.48	-1.76
Bias	T_{R1d}	-3.61	0.02	0.02
	$t^{(1)}_{R(-1)d}$	-5.08	0.06	0.06
	t_r,t_p	-2.91	0.00	0.00
	t_{er},t_{ep}	-2.63	-0.02	-0.02
	t_{rcr},t_{rcp},t_{dcr},t_{dcp}	-3.02	0.00	0.00
	t_{rcer},t_{rcep},t_{dcer},t_{dcep}	-3.02	0.00	0.00

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