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EFFECT OF SHAPE TRANSFORMATION ACCOMPANIED BY M1 TRANSITIONS ON THE ENERGY WEIGHT SUM RULE

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Abstract. In the Random Phase Approximation (RPA), using the analytic properties of the nucleus transition matrix elements and by means of contour integrals and residue theorem, we obtained an analytic formula for the Energy Weighted Sum Rule (EWSR) of the M1 transitions as a function of the deformation parameters of the excited states of the nucleus. It is shown that an essential decrease of the experimental M1 transitions rates may be due to the change of nuclear shape caused by the transitions between different energy levels. The latter may be also responsible for the observed quenching of the M1 sum rules. The numerical calculations are carried out, and the deformed dependence of the sum rules for the ^{140}Ce, ^{154}Sm, ^{156}Gd and ^{196}Pt is analyzed.

1. Introduction

In quantum mechanics, the transition probability of the system from one state to the other is restricted to the sum rules which are independent from the model and subject to transitions matrix elements. The sum rules in nuclear physics are very important to finding parameters and understanding the reliability of used models [1]. Microscopic nuclear models are used to investigate the properties of nuclear collective excitations [2]. Approximate calculation methods are used to investigate the structure of nucleus within the framework of assumed models. In these models the RPA has been extensively exploited to calculate intensities of the various nuclear processes, probabilities of electromagnetic, beta and double beta decay transitions and corresponding sum rules by taking into account ground state correlations. Numerical calculations of the sum rules within framework of the modern microscopic models of nucleus are simple with a small number of the phonon states in spherical nuclei. However, in deformed nuclei the spectrum of such states is characterized by high density. This gives rise to considerable difficulties in exact calculations of all the eigenvalues ω_n and eigenfunctions ψ_n of the model Hamiltonian and in the correct evaluation nuclear matrix elements of the different processes. The analytical

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solutions of this problem for calculation beta decay sum rules are given in [3]. Later, the developed method developed in [3] successfully applied to calculate the Non-Energy Weighted Sum Rule (NEWSR) and Energy Weighted Sum Rule (EWSR) of the electric and magnetic dipole transitions and beta decay matrix elements on the ground state basis [4]. This approach based on the analytical properties of nuclear matrix elements makes it possible to describe the integral quantities as sum rules without a preliminary determination of the wide spectrum of 1^+ states and wave functions. The experimental spin-magnetic strength was found to be quenched more than 1.5 times in heavy nuclei as compared with prediction of the Quasi Random Phase Approximation (QRPA) (see, e.g., [5] and references therein). Up to now the reason for this disagreement between the QRPA calculation and the experiments is not exactly explained. So, investigation of this disagreement is very important. The main reason of these disagreements may be due to the change of nuclear shape caused by the transitions between different energy levels. The phenomena associated with shape coexistence and intruder states in heavy and medium nuclei are discussed in [6,7]. It is experimentally known that in some nuclei, the rate transitions between levels having different shape and structure decreases [8,9]. So, one of the reason of these disagreements should be the differences of the shape of the excitations and ground state participating in transitions. For this reason, calculating the sum rules for the transition matrix elements of levels, which have deformation parameters different from the ground state is very important. In this study, the method developed in [3] has been applied successfully for investigating magnetic dipole transitions between states having different shape. We obtained an analytical formula for the energy weight sum rule of the magnetic dipole transition matrix elements containing the excited and ground state deformation parameters of nucleus. It is shown that an essential decrease of the rate M1 excitations of the 1^+ states may be due to the change of nuclear shape caused by the M1 transitions. The numerical calculations are carried out, and the shape dependence of the EWSR for the ^{140}Ce, ^{154}Sm, ^{156}Gd and ^{196}Pt is analyzed.

2. The Analytical Calculations

The key problem in the program for investigating deformation dependence of the sum rules is the calculation of the EWSR of the M1 transitions to the states which have shapes different from the ground state ones. The sum rules for the transition matrix elements from one state to the other one are obtained by using commutation relations of the transition operators and their hermitic conjugates with each other, and with the system model Hamiltonian by making explicit use of the closure relation of exact eigenstates of the system. There are two widely used types of sum rules: none energy-weighted sum rule (NEWSR) and linear energy-weighted sum rule (EWSR). For any one-body operator M, the transition matrix elements from the ground state to the excited states of the system is given by the NEWSR

$$\sum_{k>0} |<k|M|0>|^2 =<0\left|[M,M^+]\right|0> . \tag{2.1}$$

The energy-weighted sum rule is widely used in the nuclear physics can be written

$$\sum_{k>0}(E_k - E_0)|< k|\, M\, |0>|^2 = \frac{1}{2} < 0\left|[M^+, [H, M]]\right| 0 > . \tag{2.2}$$

Here in (1.1) and (1.2) energy and wave function are eigenvalues and eigenfunctions of Hamiltonian operator of nucleus, respectively. For the transition operator M in quasi-boson approximation of the RPA the double commutator in (1.2) is a c-number. E0 and also denote the energy and wave function of the ground state, respectively. Thouless [10] showed that left-hand side of (1.2) calculated with RPA is equal to the right-hand side of (1.2) calculated using the Hartree-Fock ground state wave function. Since the right-hand side of the sum rule (1.2) does not contain any parameters of the effective interactions of the model used for description nuclear excitations. On the other hand, the left-hand side of the (1.1) contains wave functions and energy levels of nucleus, its values depend on accuracy of the methods and models used. Thus, the calculation of the EWSR allows one to make some conclusions about the accuracy of methods and approximations.

2.1. **Model Hamiltonian and description of 1^+ states.** Let us consider the system of nucleons in the axially symmetric average field interacting via pairing and spin-spin residual forces. We neglect for simplicity the restoring rotational invariance forces which have a minor effect in the deformation dependence of the EWSR. Then the model Hamiltonian of the intrinsic motion (for a fixed orientation of the nucleus) can be written in the quasiparticle representation [11]:

$$H = H_{sqp} + V_{\sigma\tau}. \tag{2.3}$$

Here, H_{sqp} represents the Hamiltonian of the single-quasiparticle motion. The term $V_{\sigma\tau}$ takes into account the spin-isospin interaction, which produces the 1^+ states in deformed nuclei and has the form

$$V_{\sigma\tau} = \frac{1}{2}\chi_{\sigma\tau}\sum_{i\neq j}\vec{\sigma}_i\vec{\sigma}_j\tau_i^z\tau_j^z \tag{2.4}$$

where, $\chi_{\sigma\tau}$ is the spin-isospin interaction strength; and $\vec{\sigma}_i$ and τ_i are the Pauli matrices that represent the spin and the isospin, respectively. All relations, that are used and not explained in this paper are similar to those in Ref. [11]. In the QRPA method, the collective 1^+ states are considered as one-phonon excitations described by

$$|t\rangle = Q_t^+ |\Psi_0\rangle = \frac{1}{\sqrt{2}}\sum_{ss',\tau}\left[\psi^t_{ss'}(\tau)C^t_{ss'}(\tau) - \phi^t_{ss'}(\tau)C_{ss'}(\tau)\right]|0\rangle \tag{2.5}$$

where Q_t^+ is the phonon creation operator, $|0\rangle$ is the phonon vacuum which corresponds to the ground state of the even-even nucleus and $C^+_{ss'}(C_{ss'})$ is a two-quasiparticle creation (annihilation) operator. Further s(s') denotes the single-quasiparticle states of the nucleons and the isospin index τ takes the values n(p) for neutrons(protons). Our system has a discrete spectrum and the wave functions $|t\rangle$ form complete set satisfying $\sum\limits_t |t\rangle\langle t| = 1$. The two quasiparticle amplitudes $\psi^t_{ss'}(\tau)$ and $\phi^t_{ss'}(\tau)$, corresponding to the operator $C_{ss'}$ and $C^+_{ss'}$ are normalized as follows:

$$\sum_{ss'\tau} [\psi^i_{ss'}(\tau)\psi^k_{ss'}(\tau) - \phi^i_{ss'}(\tau)\phi^k_{ss'}(\tau)] = \delta_{i,k}. \tag{2.6}$$

Following the well-known procedure of the RPA method, one can find the eigenfunctions and the eigenvalues of the Hamiltonian. Employing the conventional procedure of the QRPA with the equation of motion:

$$\left[H_{sqp} + V_{\sigma\tau}, Q^+_t\right] = \omega_t Q^+_t \tag{2.7}$$

and omitting the details of the solution of (7), we obtain [11] the secular equation for the excitation energy $\omega_{\mathrm{t}} = \mathrm{E_t} - \mathrm{E_0}$ of the 1^+-states

$$D(\omega_t) = 1 + \chi_{\sigma\tau}\left(F_n(\omega_t) + F_p(\omega_t)\right) = 0 \tag{2.8}$$

where

$$F_\tau(\omega_t) = 2\sum_\mu{}^{(\tau)} \frac{\varepsilon_\mu \sigma^2_\mu L^2_\mu}{\varepsilon^2_\mu - \omega^2_t},\ \ \tau = n, p. \tag{2.9}$$

Here, $\varepsilon_{ss'} = \varepsilon_s + \varepsilon_{s'}$ and s are the energies of the deformed single-quasiparticle states $|s>$. The single-particle matrix elements for spin operator σ_{+1} are denoted by $\sigma_{ss'}$. The expression $L_{ss'} = u_s v_{s'} - u_{s'} v_s$ is defined in the usual Bogolyubov notation. Hereafter in order to simplify the notation we use a single index μ instead of the pair index (ss'). The sum $\sum^{(\tau)}$ denotes the summation over the neutron or the proton states. Finally, the neutron-neutron and proton-proton two-quasiparticle amplitudes are given by:

$$\psi^t_\mu = \frac{1}{\sqrt{Y(\omega_t)}}.\frac{\sigma_\mu L_\mu}{\varepsilon_\mu - \omega_t}, \tag{2.10}$$

$$\phi^t_\mu = \frac{1}{\sqrt{Y(\omega_t)}}.\frac{\sigma_\mu L_\mu}{\varepsilon_\mu + \omega_t} \tag{2.11}$$

where

$$Y(\omega_t) = 4\omega_k \sum_t \frac{\varepsilon_\mu \sigma^2_\mu L^2_\mu}{\left(\varepsilon^2_\mu - \omega^2_t\right)^2}. \tag{2.12}$$

The sum $\sum$ runs over all neutron and proton states. On the other hand, since energies of the magnetic dipole 1^+ states are the solutions of the function $D(\omega_t)$, after simple manipulation for $Y(\omega_t)$ the following formula is obtained

$$Y\left(\omega_t\right) = \frac{1}{\chi} D'\left(\omega_t\right) \tag{2.13}$$

where

$$D' = \frac{dD(z)}{dz}.$$

Due to the symmetry between the used spin-spin forces and magnetic dipole operator, the most characteristic quantity of the 1^+ states is transition matrix elements M1 from ground state to all excited states in nucleus:

$$\vec{M}_t = \langle t|\, \vec{M}\, |0\rangle \tag{2.14}$$

where the magnetic-dipole operator is

$$\vec{M} = \sqrt{\frac{3}{4\pi}} \sum_{m,\tau} [(g_s^\tau - g_l^\tau)\frac{1}{2}\vec{\sigma}_m^\tau - g_e^\tau \vec{j}_m^\tau]. \tag{2.15}$$

Here g_s^τ and g_l^τ are the spin and the orbital gyromagnetic ratios of nucleons, respectively. By using the wave function (5) and by means of (10) and (11), transition matrix elements of 1^+ states from ground state to excited states $|t\rangle$ can be expressed as

$$M_t = \sqrt{\frac{3}{4\pi}} \frac{\sum_\tau [\frac{1}{2}(g_s^\tau - g_l^\tau)F_\tau(\omega_t) - g_\ell^\tau J_\tau(\omega_t)]}{\sqrt{Y(\omega_t)}} \tag{2.16}$$

where

$$J_\tau(\omega_t) = 2\sum_\mu {}^{(\tau)} \frac{\varepsilon_\mu j_\mu^2 L_\mu^2}{\varepsilon_\mu^2 - \omega_t^2}. \tag{2.17}$$

Here, j_μ denotes the single-particle matrix elements of the angular momentum operator.

The energy-weighted sum rule (2) for the M1 transitions (15) calculated using Hartree-Fock-Bogolyubov (HFB) ground state is given as follows [13]

$$S(\delta_0) = \sum_{t>0} \omega_t B_t(M1) = \frac{3}{4\pi} \sum_{\mu,\tau} \varepsilon_\mu^\tau [(g_s^\tau - g_l^\tau)\, s_\mu^\tau - g_e^\tau j_\mu^\tau]^2. \tag{2.18}$$

Here, $B_t(M1) =< t|\, M|0 >^2$ is the M1 transition probability of the excitation from the ground state. As seen the right-hand side of eq. (18) does not depend on the spin-spin interaction strength parameter and represents the quasiparticle estimate of the sum rule. Let us now generalize the sum rule in eq. (2) for transitions between the ground and excited states which have different form. Let us suppose that shape of the excited states $|k\rangle$ have different deformation parameter from the ground state one. After this, the quantities corresponding to excited states $|k\rangle$ which have different form from the ground state are denoted by (tilda) over themselves. Also, by taking the fact that the excited state wave functions $|i\rangle = Q_i^+ |0\rangle$ in the ground state bases form a complete set into consideration, in QRPA we obtain the generalized expression for the left-hand side of the sum rule (1) for the transition matrix elements between different shapes as follows:

$$S\left(\delta_0, \delta_{ex.}\right) = \sum_{k>0} \tilde{\omega}_k \left| \sum_{i>0} M_i \langle k \mid i \rangle \right|^2 = \frac{1}{4} \sum_k \tilde{\omega}_k \left| \sum_i M_i \langle k \mid i \rangle \right|^2. \tag{2.19}$$

The overlap of the wave functions $|k\rangle$ and $|i\rangle$ has the following form:

$$\langle k \mid i \rangle = \frac{1}{2} \sum_{\mu\tau} [g_\mu^i(\tau) w_\mu^k(\tau) + w_\mu^i(\tau) g_\mu^k(\tau)] \tag{2.20}$$

where

$$g_q^t(\tau) = \psi_q^t(\tau) + \phi_q^t(\tau). \tag{2.21}$$

Here $\vec{M}_i = \langle i|\, \vec{M}\, |0\rangle$, $\delta_{0.}$ and $\delta_{ex.}$ are quadrupole deformation parameters of the ground (core) and excited states, respectively. Further, k and i runs over all the negative and positive solutions of the $D(\omega_t) = 0$. If we use (20), we find that general expression for $S(\delta_0, \delta_{ex.})$ given by (19), assumes the form

$$S(\delta_{0.}, \delta_{ex.}) = \frac{1}{4} \sum_{\mu\nu} .\tilde{\Omega}_{\mu\nu} d_\mu d_\nu. \tag{2.22}$$

where

$$\tilde{\Omega}_{\mu\nu} = \sum_k \tilde{\omega}_k g_\mu^k g_\nu^k,\ d_q = \sum_i M_i w_q^i,\ q = \mu, \nu. \tag{2.23}$$

As a matter of convenience, let as calculate sum rule for spin part of the $S(\delta_0, \delta_{ex.})$. In this case by putting $M = \sigma$ in (22), and by exploiting the equations (8)-(13), we obtain:

$$S_\sigma(\delta_0, \delta_{ex.}) = \frac{1}{4} \sum_{\mu\nu} .\tilde{\Omega}_{\mu\nu} d_\mu d_\nu. \tag{2.24}$$

where

$$d_q = 2\chi_{\sigma\tau} \sum_i \frac{\omega_i F(\omega_i)}{D'(\omega_i)} \frac{\sigma_q L_q}{\left(\varepsilon_q^2 - \omega_i^2\right)}. \tag{2.25}$$

$$\tilde{\Omega}_{\mu\nu} = 4\chi_{\sigma\tau} \sum_k \frac{\tilde{\omega}_k}{D'(\tilde{\omega}_k)} \frac{\tilde{\varepsilon}_\mu \tilde{\sigma}_\mu \tilde{L}_\mu \tilde{\varepsilon}_\nu \tilde{\sigma}_\nu \tilde{L}_\nu}{(\tilde{\varepsilon}_\mu^2 - \tilde{\omega}_k^2)(\tilde{\varepsilon}_\nu^2 - \tilde{\omega}_k^2)}. \tag{2.26}$$

Let us calculate right-hand side of (24) using analytical properties expressions (25) and (26) for d_q and $\tilde{\Omega}_{\mu\nu}$, respectively. The mathematical formalism of the model and method of calculation sum rules are discussed in details in [3,4]. Since ω_i and ω_k are the zeros of the function $D(\omega_t)$ of (8), the basic theorem of the theory of residues [14] now allow us to write the expression for d_q and $\tilde{\Omega}_q$ in the form of the contour integral:

$$d_q = \chi_{\sigma\tau} \frac{1}{\pi i} \sum_i \sigma_q L_q \oint_{L_i} \frac{z_i F(z_i)}{\left(\varepsilon_q^2 - z_i^2\right) D(z_i)} dz_i, \tag{2.27}$$

$$\tilde{\Omega}_{\mu\nu} = \frac{2\chi_{\sigma\tau}}{\pi i} \oint_{L_i} \sum_k \frac{z_k}{D(z_k)} \frac{\tilde{\varepsilon}_\mu \tilde{\sigma}_\mu \tilde{L}_\mu \tilde{\varepsilon}_\nu \tilde{\sigma}_\nu \tilde{L}_\nu}{(\tilde{\varepsilon}_\mu^2 - \tilde{\omega}_k^2)(\tilde{\varepsilon}_\nu^2 - \tilde{\omega}_k^2)} dz_k. \tag{2.28}$$

The contour of the integration is given in Fig.1. Analysis shows that integral (27) contains first-order singularities of the integrand at $z_i = \pm\varepsilon_q$ (see Fig.1). The same integral extended over the contour L_∞ is proportional to ${}^1\!/_{z^3}$ vanishes for large z, and therefore

$$d_q = -\oint_{L_q} - \oint_{L_{-q}} = 2\sigma_q L_q. \tag{2.29}$$

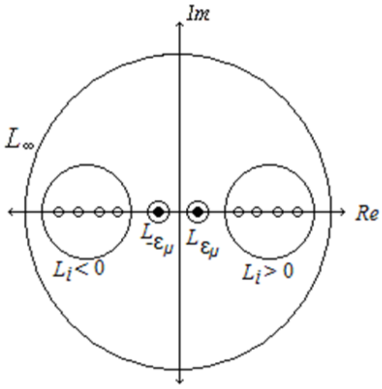

Figure 1. The contour of integration in the complex plane for (24).

The integral $\tilde{\Omega}_{\mu\nu}$ over the contour L_∞ is equal to zero because as $z \to \infty$, the integrand tends to zero also as $1/_{z^3}$. We note that in the case $\mu \neq \nu$, $z_k = \pm\varepsilon_\mu$ and $z_k = \pm\varepsilon_\nu$ are removable singularities of the integrand (28), after certain manipulations we obtain $\Omega_{\mu\neq\nu} = 0$. On other hand diagonal part of $\Omega_{\mu\nu}\delta_{\mu\nu}$ contains first-order singularities of the integrand at $z_i = \pm\varepsilon_\mu$ (see Fig.1). Omitting the intermediate computations after laborious calculations, one can obtain the expression for the $\Omega_{\mu\nu}$ in the form

$$\Omega_{\mu\nu} = 2\tilde{\varepsilon}_\mu \delta_{\mu\nu}. \tag{2.30}$$

Substituting (29) and (30) into the (24) we obtain

$$S_\sigma\left(\delta_0, \delta_{ex.}\right) = \sum_\mu \tilde{\varepsilon}_\mu \sigma_\mu^2 L_\mu^2. \tag{2.31}$$

Applying analogous procedure of calculations for $M = j$ we obtain:

$$S_j\left(\delta_0, \delta_{ex.}\right) = \sum_\mu \tilde{\varepsilon}_\mu j_\mu^2 L_\mu^2. \tag{2.32}$$

Finally, in general case for left-hand side of EWSR (2) in QRPA by exploiting (31) and (32) for M1 transitions accompanied by change of nuclear form we derived very simple formula

$$S(\delta_0, \delta_{ex.}) = \frac{3}{4\pi} \sum_{\mu,\tau} \tilde{\varepsilon}_\mu^\tau L_\mu^2 [(g_s^\tau - g_l^\tau)\, s_\mu^\tau - g_e^\tau j_\mu^\tau]^2. \tag{2.33}$$

This is quite interesting result in our approach. As can bee seen from (33) the deformation dependence of the EWSR is caused by the interplay of the ground and excited states structure. Thus, for M1 transitions followed by shape changing the two-quasiparticle energies calculate for the shape changed base of the excited states, meanwhile two-quasiparticle matrix elements calculate on the ground state base. As a natural consequence, when $\delta_{ex.} = \delta_0$ the formula (33) is transformed into the known sum rule expression (18) for magnetic dipole transitions:

$$S(\delta_0) = \frac{3}{4\pi} \sum_{\mu,\tau} \varepsilon_\mu^\tau \mathrm{L}_\mu^2 [(g_s^\tau - g_l^\tau)\, s_\mu^\tau - g_e^\tau j_\mu^\tau]^2. \tag{2.34}$$

As seen in the QRPA the EWSR (2) for M1 transitions is satisfied exactly, i.e, the value (34) of left hand side of the EWSR (2) calculated with the QRPA is equal the value of the (18) of right hand side of (2) calculated using the HFB ground state wave function (18). Thus, by using analytical properties of the nuclear matrix elements with help the theory of residue and contour integrals, the known energy weighted sum rule for the magnetic dipole transitions is generalized for transitions to levels which have different forms from the ground state. As can bee seen for transitions followed by shape changing the single-quasiparticle matrix elements calculated for ground states with δ_0 deformation where two-quasiparticle matrix elements calculated for the shape changed excited states.

3. Numerical Calculations and Discussion

It is well-known that many transitional and deformed nuclei demonstrate softness against δ deformations. As stressed in [2] the large variety of the equilibrium shapes of nuclei in deformation region $50 < \mathrm{Z.N} < 126$ is related to the considerable differences between the δ_2 values of excited states ($\delta_{ex.}$) and ground state values (δ_0). Experimental data confirming the existing different deformation form of nuclei is discussed in [6,7].

Aim of the present calculations is study the states with different shapes and to demonstrate importance the shape transformation in describing the quenching phenomenon of the transition matrix elements in deformed nuclei. This can be obtained by investigating the deformation dependence of the $S(\delta_0, \delta_{ex.})$ value and comparison of its results with the results of the QRPA calculated on the ground state base with δ_0. Numerical calculations are performed in a wide interval of the deformation parameter $\delta_{ex.}$ for ^{140}Ce, ^{150}Ce, ^{154}Sm, ^{156}Gd and ^{196}Pt in the deformed Woods-Saxon potential [15]. The calculations are performed by using the sum rule (33) for the M1 excitations. The ground state mean field deformation parameters δ_0 are calculated according to [16] using deformation parameters β_2 defined from the experimental quadrupole moments [17]. The pairing-interaction constants chosen according to [2] are based on the single-particle states corresponding to the nucleus in question. The calculated values of the pairing quantities Δ and λ corresponding to the $\mathrm{G_N}$ and $\mathrm{G_Z}$ and the mean-field deformation parameters δ_0 are shown in Table 1. The isovector spin-spin interaction strength was chosen as $\chi_{\sigma\tau} = {}^{40}\!/_{\mathrm{A}}$ MeV [11]. This value allows a satisfactory description of the scissors mode fragmentation in well-deformed rare earth nuclei [12]. In calculating the B(M1) value, we have used bear spin and orbital gyromagnetic factors for nucleons.

Here we want to study the effect of shape transformation of excited states to the EWSR of the M1 excitation matrix elements for $\mathrm{K}^\pi = 1^+$ states. The importance of deformation dependence of sum rule can be demonstrated by the comparison of the QRPA results with the experimental data. Dependence of the calculated S in transitional ^{140}Ce and ^{196}Pt and well deformed the ^{154}Sm, ^{156}Gd isotopes with respect to deformation parameter $\delta_{ex.}$ are represented in Fig.2 and Fig.3, respectively.

The results were compared with the experimentally observed M1 dipole excitations data from refs. [5,18,19] in the experimentally investigated energy region of 6.0-9.0 MeV. The Ce and Pt isotopes are an important link in the transition region

TABLE 1. Pairing correlation parameters (in MeV) and δ_2 values.

Nuclei	Δ_n	λ_n	$G_N.A$	Δ_p	λ_p	$G_Z.A$	δ_0
^{140}Ce	1.190	-7.581	23.0	1.542	-7.118	24.1	0.090
^{154}Sm	1.183	-6.822	19.2	0.783	-8.589	22.3	0.271
^{156}Gd	1.07	-7.486	19.3	1.175	-7.408	25.3	0.271
^{196}Pt	0.815	-6.961	19.2	1.073	-7.087	25.4	0.115

from spherical to deformed shape and from deformed to spherical nuclei, respectively. In Fig. 2, we compare the $\delta_{ex.}$-dependence of the calculated in the QRPA (solid line) $S(\delta_0, \delta_{ex.})$ value for the 1^+ states with the single-quasiparticle model values (dashed line) and the experimental data for ^{140}Ce[19] and ^{196}Pt.

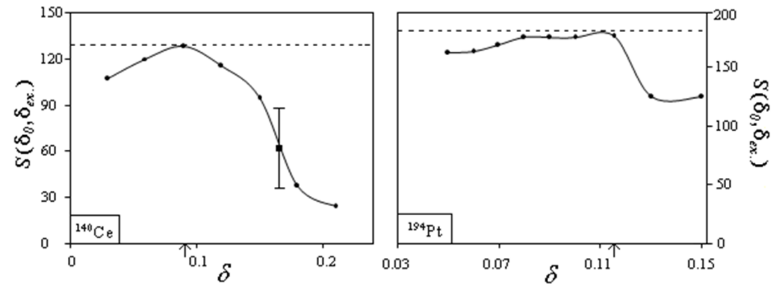

Figure 2. Deformation dependence of the $S(\delta_0, \delta_{ex.})$ values (in units of MeV μ_N^2) for the M1 transitions with $K^\pi = 1^+$ in transitional ^{140}Ce[19] and ^{196}Pt isotopes. The right-hand side of the sum rule (34) is shown by dashed line. The solid line corresponds to the function S calculated by (33). Symbol $\top$ denotes the experimental data for EWSR [19]. The value of the ground state deformation parameter δ_0 is marked with the arrows.

As seen from figures the single quasiparticle model exceeds the experimental values almost two times in ^{140}Ce. One can observe strong deformation dependence of the calculated sum rule $S(\delta_0, \delta_{ex.})$. The calculation results of the $S(\delta_0, \delta_{ex.})$ show that deviations from the QRPA results below the δ_0 are small. In contrast in the case $\delta_{ex.} > \delta_0$ the sum rules $S(\delta_0, \delta_{ex.})$ for both nuclei change steeply with increasing $\delta_{ex.}$ which leads to the conclusion that in heavy deformed and transition nuclei a quenching M1 strength does occur mainly for $\delta_{ex.} > \delta_0$. This is also the case for the ^{154}Sm, ^{156}Gd isotopes (see Fig.3). Therefore, it is not surprise that, in fitting the data by adjusting the parameter $\delta_{ex.}$ we have found $\delta_{ex.} > \delta_0$ for nuclei investigated. Analysis shows that the strong deformation dependence is caused by the interplay of the two-quasiparticle energy $\tilde{\varepsilon}_{ss'}$ and $\delta_{ex.}$. Thus, the results confirm the importance of the shape transformation in the quenching. The quenching phenomenon as described above for transitional nuclei can be more clearly seen for nuclei with a larger deformation. As an example, the comparison of the present results obtained in the deformation range of 0.2-0.3 for well deformed ^{154}Sm, ^{156}Gd isotopes to the sum rule resulting from the single-quasiparticle model values (dashed line) and the experimental [5,18] data is given in Fig. 3.

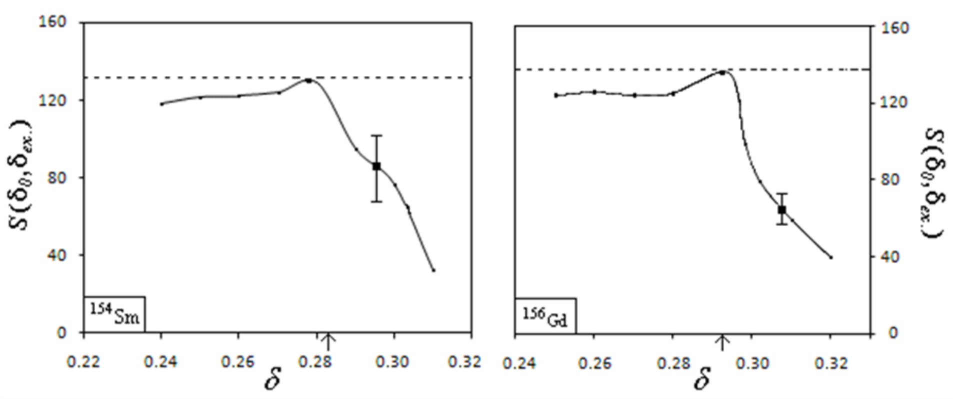

Fig. 3. Deformation dependence of the $S\,(\delta_0, \delta_{ex.})$ values (in units of MeV μ_N^2) for the M1 transitions with $\mathrm{K}^\pi = 1^+$ in deformed ^{154}Sm, ^{156}Gd isotopes. The right-hand side of the sum rule (34) is shown by dashed line. The solid line corresponds to the function $S\,(\delta_0, \delta_{ex.})$ calculated by (33). Symbol $\top$ denotes the experimental data for ^{154}Sm and ^{156}Gd [18]. The value of the ground state deformation parameter δ_0 is marked with the arrows.

Fig. 3. Deformation dependence of the $S\,(\delta_0, \delta_{ex.})$ values (in units of MeV μ_N^2) for the M1 transitions with $\mathrm{K}^\pi = 1^+$ in deformed ^{154}Sm, ^{156}Gd isotopes. The right-hand side of the sum rule (34) is shown by dashed line. The solid line corresponds to the function $S\,(\delta_0, \delta_{ex.})$ calculated by (33). Symbol $\top$ denotes the experimental data for ^{154}Sm and ^{156}Gd [18]. The value of the ground state deformation parameter δ_0 is marked with the arrows.

As seen in Fig. 3, in the case of using $\delta_{ex.} = \delta_0$, the sum rules are well above the experimental data for both nuclei. Namely, the sum rule values have an $S(\delta_0, \delta_{ex.}) =$ 122.3 MeV μ_N^2 for ^{154}Sm and $S(\delta_0, \delta_{ex.}) = 129$ MeV μ_N^2 for ^{156}Gd. Analysis shows that for a considerable consequence of the use of the different deformation shape for the ground and excited states is the strong decreasing of the $S(\delta_0, \delta_{ex.})$ with increasing $\delta_{ex.}$ between $\delta_0 = 0.27$ and $\delta_0 = 0.3$. For example, for ^{154}Sm in case of using $\delta_{ex.} = \delta_0$ (dashed lines in the figures), the sum $S(\delta_0, \delta_{ex.}) = 122.3$ MeV μ_N^2, while for the $\delta_{ex.} = 1.1.\delta_i$ the sum $S(\delta_0, \delta_{ex.}) = 50$ MeV μ_N^2. This is also the case for the all nuclei under investigation.

The theory predicts a giant spin-flip Ml resonance at an energy of 8.0- 9.0 MeV while M1 strength exhibit strong fragmentation in a wide region energy between 2.0 MeV and 12 MeV. Naturally the calculation predicts substantially more dipole strength than is indicated by experiment [5,18,19] in the energy range of 6.0-9.0 MeV. This discrepancy indicates some additional strength out of the experimental energy interval of 6.0-9.0 MeV. One of possible reason for this discrepancy could be scissors mode excitations observed in ^{154}Sm and ^{156}Gd. For example, experimental data [20] indicate scissors mode 1^+-excitations with an energy centroid $\overline{\omega} = 3.26$ MeV for ^{154}Sm and $\overline{\omega} = 3.06$ MeV for ^{156}Gd with summed $\sum \omega.\mathrm{B(M1)} = 7.95$ MeV μ_N^2 and 8.35 MeV μ_N^2 , respectively. Another possible reason for this discrepancy could be that the M1 strength in these nuclei is so fragmented that a part of it might have just escaped detection [5,18,19]. It seems, therefore, that the above-mentioned experimental data of the sum rule $S(\delta_0, \delta_{ex.})$ of the M1 mode (see Fig.3) in ^{154}Sm and ^{156}Gd in the energy region of 6.0-8.5 MeV should be regarded rather as a lower limit.

To summarize, we have reported on results of the investigation of the deformation dependence of the energy-weight sum rule for M1 dipole excitation strength in the γ-soft ^{140}Ce and ^{196}Pt and well deformed ^{154}Sm and ^{156}Gd isotopes. The calculated sum rules for M1 transitions demonstrate strong deformation dependence. It is shown, that an essential decrease of the M1transitions rates may be due to the change of nuclear shape by the excitations of the nuclear excited states.

It would be desirable to investigate the features of the M1-resonance in the well deformed nuclei more deeply. Furthermore, the scarcity of the data on EWSR does not allow a systematic analysis of the properties of the mode. The available experimental data of some deformed nuclei (see Richter [5,18]) are as yet not sufficient to reach a decisive conclusion regarding the shape transformation in the M1 transitions. Consequently, it would be important to extend the experimental studies with improved sensitivity in order to investigate the M1 mode in a wide energy range of the excitations, though several experimental results on the spin-mode strength in rare-earth and actinide nuclei have been published. Therefore additional experimental evidence is needed to resolve the issue.

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