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NONEXISTENCE OF GLOBAL SOLUTIONS FOR A KIRCHHOFF-TYPE VISCOELASTIC EQUATION WITH DISTRIBUTED DELAY

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Abstract. In this paper, we consider a Kirchhoff-type viscoelastic equation with distributed delay and source terms. We obtain the nonexistence of global solutions under suitable conditions.

1. Introduction

In this paper, we consider the following Kirchhoff-type viscoelastic equation with distributed delay and source terms

$$(1.1)\quad \begin{cases} u_{tt} - M\left(\|\nabla u\|^2\right)\Delta u + \int_0^t g(t-s)\,\Delta u(s)\,ds & \\ +\mu_1 u_t + \int_{\tau_1}^{\tau_2} |\mu_2(q)|\, u_t(x,t-q)\,dq & \\ = b\,|u|^{p-2}\,u, & (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0, & x \in \partial\Omega, \\ u_t(x,-t) = f_0(x,t), & (x,t) \in \Omega \times (0,\tau_2), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & x \in \Omega, \end{cases}$$

where b, $\mu_1 > 0$, $p > 2$ and τ_1, τ_2 are the time delay with $0 \leq \tau_1 < \tau_2$, μ_2 is an L^∞ function, and g is a differentiable function under the assumptions (A1), (A2), and (A3). $M(s)$ is a nonnegative function of C^1 for $s \geq 0$ satisfy, $M(s) = m_0 + \alpha s^\gamma$, $m_0 > 0$, $\alpha \geq 0$ and $\gamma \geq 0$, specially we take $M(s) = 1 + s^\gamma$ where $m_0 = 1$, $\alpha = 1$.

Problems about the mathematical behavior of solutions for PDEs with time delay effects have become interesting for many authors mainly because time delays often appear in many practical problems such as thermal, economic phenomena, biological, chemical, physical, electrical engineering systems, mechanical applications and medicine. Moreover, it is well known that delay effects may destroy the stabilizing properties of a well-behaved system. In the literature, there are several examples that illustrate how time delays destabilize some internal or boundary control system [5, 6]. Viscous materials are the opposite of elastic materials that posses the ability to dissipate and store the mechanical energy. The mechanical

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properties of these viscous substances are of great importance when they seem in many natural sciences applications [2]. The problem (1.1) is a general form of a model introduced by Kirchhoff [7]. To be more precise, Kirchhoff recommended a model denoted by the equation for $f = g = 0$,

$$\rho h \frac{\partial^2 u}{\partial t^2} + \delta \frac{\partial u}{\partial t} + g\left(\frac{\partial u}{\partial t}\right) = \left\{\rho_0 + \frac{Eh}{2L}\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\right\} \frac{\partial^2 u}{\partial x^2} + f(u), \tag{1.2}$$

for $0 < x < L$, $t \geq 0$, where $u(x,t)$ is the lateral displacement, E is the Young modulus, ρ is the mass density, h is the cross-section area, L is the lenght, ρ_0 is the initial axial tension, δ is the resistance modulus, and f and g are the external forces. Furthermore, (1.2) is called a degenerate equation when $\rho_0 = 0$ and nondegenerate one when $\rho_0 > 0$.

In 1986, Datko et al. [4] indicated that delay is a source of instability. In [9], Nicaise and Pignotti considered the following wave equation with a linear damping and delay term

$$u_{tt} - \Delta u + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) = 0. \tag{1.3}$$

They obtained some stability results in the case $0 < \mu_2 < \mu_1$. In the absence of delay, Zuazua [23] looked into exponentially stability for the equation (1.3).

Wu and Tsai [24], considered the following Kirchhoff-type equation

$$u_{tt} - M\left(\|\nabla u\|_2^2\right)\Delta u + |u_t|^{r-2} u_t = |u|^{p-2} u, \tag{1.4}$$

with the positive upper bounded initial energy and they obtained the blow-up of solutions for the equation (1.4). In 2013, Ye [22], considered the global existence results by constructing a stable set in $H_0^1(\Omega)$ and showed the decay by using a lemma of Komornik for the nonlinear Kirchhoff-type equation (1.4) with dissipative term.

When $M(s) = 1$, the equation (1.1) becomes the following form

$$\begin{aligned} & u_{tt} - \Delta u - \omega \Delta u_t + \textstyle\int_0^t g(t-s)\Delta u(s)\,ds \\ & + \mu_1 u_t + \textstyle\int_{\tau_1}^{\tau_2} |\mu_2(\rho)|\, u_t(x,t-\rho)\,d\rho \\ & = b|u|^{p-2} u. \end{aligned} \tag{1.5}$$

In [2], Choucha et al. obtained the blow-up of solutions under appropriate conditions of the equation (1.5). In [3], the authors showed the exponential growth of solution for the equation (1.5). In recent years, some other authors investigate hyperbolic type equations (see [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]).

In this paper, we consider the Kirchhoff-type ($M\left(\|\nabla u\|^2\right)$) viscoelastic equation (1.1) with distributed delay ($\int_{\tau_1}^{\tau_2} |\mu_2(q)|\, u_t(x,t-q)\,dq$) and source ($b|u|^{p-2}u$) terms. Our aim is to obtain the nonexistence of global solutions for the equation (1.1).

The paper is organized as follows: In section 2, we give some materials that will be used later. In section 3, we state and prove our main result.

2. Preliminaries

In this part, we give materials for the proof of our result. As usual, the notation $\|.\|_p$ denotes L^p norm, and $(.,.)$ is the L^2 inner product. In particular, we write $\|.\|$ instead of $\|.\|_2$.

Now, we denote some assumptions used in this paper:

(A1) $g : R_+ \to R_+$ is a decreasing and differentiable function, that

$$g(t) \geq 0,\ 1 - \int_0^\infty g(s)\,ds = l > 0. \tag{2.1}$$

(A2) There exists a constant $\xi > 0$, that

$$g'(t) \leq -\xi g(t),\ t \geq 0. \tag{2.2}$$

(A3) $\mu_2 : [\tau_1, \tau_2] \to R$ is an L^∞ function, that

$$\left(\frac{2\delta - 1}{2}\right) \int_{\tau_1}^{\tau_2} |\mu_2(\rho)|\, d\rho \leq \mu_1,\ \delta > \frac{1}{2}. \tag{2.3}$$

Let $B_p > 0$ be the constant satisfies [1]

$$\|v\|_p \leq B_p \|\nabla v\|_p\,,\ \text{for } v \in H_0^1(\Omega). \tag{2.4}$$

It holds

(2.5)

$$\int_0^t g(t-s)(\nabla u(s), \nabla u_t(t))\,ds = -\tfrac{1}{2} g(t)\|\nabla u(t)\|^2 + \tfrac{1}{2}(g' o \nabla u)(t)$$
$$-\tfrac{1}{2}\tfrac{d}{dt}\left[(go\nabla u)(t) - \left(\int_0^t g(s)\,ds\right)\|\nabla u(t)\|^2\right],$$

where

$$(go\nabla u)(t) = \int_\Omega \int_0^t g(t-s)\,|\nabla u(t) - \nabla u(s)|^2\,ds. \tag{2.6}$$

Firstly, as in [8], we introduce the new variable

$$y(x, \rho, q, t) = u_t(x, t - q\rho),$$

thus, we get

$$\begin{cases} q y_t(x, \rho, q, t) + y_\rho(x, \rho, q, t) = 0, \\ y(x, 0, q, t) = u_t(x, t). \end{cases} \tag{2.7}$$

Hence, problem (1.1) is equivalent to:

$$\begin{cases} u_{tt} - M\left(\|\nabla u\|^2\right)\Delta u + \int_0^t g(t-s)\Delta u(s)\,ds \\ +\mu_1 u_t + \int_{\tau_1}^{\tau_2} |\mu_2(q)|\,|y(x,1,q,t)|\,dq \\ = b\,|u|^{p-2}\,u, \qquad x \in \Omega,\ t > 0, \\ q y_t(x, \rho, q, t) + y_\rho(x, \rho, q, t) = 0, \end{cases} \tag{2.8}$$

with initial and boundary conditions

$$\begin{cases} u(x,t) = 0, \qquad x \in \partial\Omega, \\ y(x, \rho, q, 0) = f_0(x, q\rho), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), \end{cases} \tag{2.9}$$

where

$$(x, \rho, q, t) \in \Omega \times (0,1) \times (\tau_1, \tau_2) \times (0, \infty).$$

Theorem 2.1. *Suppose that (2.1), (2.2) and (2.3) hold. Let*

$$\begin{cases} p \geq 2, & n = 1, 2, \\ 2 < p < \frac{2n-2}{n-2}, & n \geq 3. \end{cases} \tag{2.10}$$

Thus, for any initial data

$$(u_0, u_1, f_0) \in H_0^1(\Omega) \times H_0^1(\Omega) \times L^2(\Omega \times (0,1) \times (\tau_1, \tau_2)),$$

the problem (2.8)-(2.9) has a unique solution

$$u \in C\left([0,T]; H_0^1(\Omega)\right),$$

for some $T > 0$.

Now, we define the energy functional as follows:

Lemma 2.2. *Suppose that (2.1), (2.2), (2.3) and (2.10) hold. Let u be a solution of (2.8). Then, $E(t)$ is nonincreasing, such that*

$$\begin{aligned} E(t) &= \tfrac{1}{2}\|u_t\|^2 + \tfrac{1}{2}\left(1 - \int_0^t g(s)\,ds\right)\|\nabla u\|^2 \\ &+ \tfrac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} + \tfrac{1}{2}(go\nabla u)(t) \\ &+ \tfrac{1}{2}\int_\Omega \int_0^1 \int_{\tau_1}^{\tau_2} q\,|\mu_2(q)|\left|y^2(x,\rho,q,t)\right| dq\,d\rho\,dx - \tfrac{b}{p}\|u\|_p^p, \end{aligned} \tag{2.11}$$

which satisfies

$$E'(t) \leq -c_1\left(\|u_t\|^2 + \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)|\left|y^2(x,1,q,t)\right| dq\,dx\right). \tag{2.12}$$

Proof. By multiplying the first equation of (2.8) by u_t and integrating over Ω, we obtain

$$\begin{aligned} &\tfrac{d}{dt}\left\{\tfrac{1}{2}\|u_t\|^2 + \tfrac{1}{2}\left(1 - \int_0^t g(s)\,ds\right)\|\nabla u\|^2 \right. \\ &\left. + \tfrac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} + \tfrac{1}{2}(go\nabla u)(t) - \tfrac{b}{p}\|u\|_p^p\right\} \\ &= -\mu_1\|u_t\|^2 - \int_\Omega u_t \int_{\tau_1}^{\tau_2} |\mu_2(q)|\,|y(x,1,q,t)|\,dq\,dx \\ &+ \tfrac{1}{2}(g' o\nabla u)(t) - \tfrac{1}{2}g(t)\|\nabla u\|^2, \end{aligned} \tag{2.13}$$

and

$$\begin{aligned} &\tfrac{d}{dt}\tfrac{1}{2}\int_\Omega \int_0^1 \int_{\tau_1}^{\tau_2} q\,|\mu_2(q)|\left|y^2(x,\rho,q,t)\right| dq\,d\rho\,dx \\ &= -\tfrac{1}{2}\int_\Omega \int_0^1 \int_{\tau_1}^{\tau_2} 2\,|\mu_2(q)|\,y y_\rho\,dq\,d\rho\,dx \\ &= \tfrac{1}{2}\int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)|\left|y^2(x,0,q,t)\right| dq\,dx \\ &- \tfrac{1}{2}\int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)|\left|y^2(x,1,q,t)\right| dq\,dx \\ &= \tfrac{1}{2}\left(\int_{\tau_1}^{\tau_2} |\mu_2(q)|\,dq\right)\|u_t\|^2 \\ &- \tfrac{1}{2}\int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)|\left|y^2(x,1,q,t)\right| dq\,dx. \end{aligned} \tag{2.14}$$

Therefore,

$$\begin{aligned}\frac{d}{dt}E(t) &= -\mu_1 \|u_t\|^2 - \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| \, |u_t y(x,1,q,t)| \, dqdx + \frac{1}{2}(g' o \nabla u)(t) \\ &\quad -\frac{1}{2} g(t) \|\nabla u\|^2 + \frac{1}{2}\left(\int_{\tau_1}^{\tau_2} |\mu_2(q)| \, dq\right) \|u_t\|^2 \\ &\quad -\frac{1}{2}\int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| \left|y^2(x,1,q,t)\right| dqdx. \qquad (2.15)\end{aligned}$$

By using (2.13) and (2.14), we obtain (2.11). Utilizing Young's inequality, (2.1), (2.2), (2.3) and (2.15), we get (2.12). Hence, we complete the proof. □

Lemma 2.3. [2] *There exists $c > 0$, depending on Ω only, such that*

$$\left(\int_\Omega |u|^p \, dx\right)^{s/p} \leq c\left[\|\nabla u\|^2 + \|u\|_p^p\right], \qquad (2.16)$$

for all $u \in L^{p+1}(\Omega)$ and $2 \leq s \leq p$.

Using the fact that $\|u\|_2^2 \leq c\|u\|_p^2 \leq c\left(\|u\|_p^p\right)^{2/p}$, we have the corollary as follows:

Corollary 2.3.1. There exists $C > 0$, depending on Ω only, that

$$\|u\|_2^2 \leq c\left[\|\nabla u\|_2^{4/p} + \left(\|u\|_p^p\right)^{2/p}\right]. \qquad (2.17)$$

Lemma 2.4. [2] *There exists $C > 0$, depending on Ω only, such that*

$$\|u\|_p^s \leq C\left[\|\nabla u\|^2 + \|u\|_p^p\right], \qquad (2.18)$$

for all $u \in L^{p+1}(\Omega)$ and $2 \leq s \leq p$.

Now, we define the functional as follows:

$$\begin{aligned}H(t) &= -E(t) \\ &= \frac{b}{p}\|u\|_p^p - \frac{1}{2}\|u_t\|^2 \\ &\quad -\frac{1}{2}\left(1 - \int_0^t g(s) \, ds\right)\|\nabla u\|^2 \\ &\quad -\frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} - \frac{1}{2}(g o \nabla u)(t) \\ &\quad -\frac{1}{2}\int_\Omega \int_0^1 \int_{\tau_1}^{\tau_2} q \, |\mu_2(q)| \left|y^2(x,\rho,q,t)\right| dqd\rho dx. \qquad (2.19)\end{aligned}$$

3. Nonexistence of solutions

In this part, we obtain the nonexistence of global solutions for the problem (2.8)-(2.9).

Theorem 3.1. *Suppose that (2.1)-(2.3) and (2.10) hold. Suppose further that $E(0) < 0$ holds. Then, the solution of the problem (2.8)-(2.9) blows up in finite time.*

Proof. By (2.11), we get

$$E(t) \leq E(0) \leq 0. \tag{3.1}$$

Hence

$$\begin{aligned} H'(t) &= -E'(t) \\ &\geq c_1 \left(\|u_t\|^2 + \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| \left| y^2(x,1,q,t) \right| dq dx \right) \\ &\geq c_1 \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| \left| y^2(x,1,q,t) \right| dq dx \geq 0, \end{aligned} \tag{3.2}$$

and

$$0 \leq H(0) \leq H(t) \leq \frac{b}{p} \|u\|_p^p. \tag{3.3}$$

Set

$$\mathcal{K}(t) = H^{1-\alpha}(t) + \varepsilon \int_\Omega u u_t dx + \frac{\varepsilon \mu_1}{2} \int_\Omega u^2 dx, \tag{3.4}$$

here $\varepsilon > 0$ to be specified later and

$$\frac{2(p-2)}{p^2} < \alpha < \frac{p-2}{2p} < 1. \tag{3.5}$$

We multiply the first equation of (2.8) by u and with a derivative of (3.4), to obtain

$$\begin{aligned} \mathcal{K}'(t) &= (1-\alpha) H^{-\alpha}(t) H'(t) \\ &\quad + \varepsilon \|u_t\|^2 + \varepsilon \int_\Omega \nabla u \int_0^t g(t-s) \nabla u(s) ds dx \\ &\quad - \varepsilon \|\nabla u\|^2 - \varepsilon \|\nabla u\|^{2(\gamma+1)} + \varepsilon b \int_\Omega |u|^p dx \\ &\quad - \varepsilon \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| |u y(x,1,q,t)| dq dx. \end{aligned} \tag{3.6}$$

By using

$$\begin{aligned} & \varepsilon \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| |u y(x,1,q,t)| dq dx \\ \leq\ & \varepsilon \left\{ \delta_1 \left(\int_{\tau_1}^{\tau_2} |\mu_2(q)| dq \right) \|u\|^2 \right. \\ & \left. + \frac{1}{4\delta_1} \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| \left| y^2(x,1,q,t) \right| dq dx \right\}, \end{aligned} \tag{3.7}$$

and

$$\begin{aligned}
& \varepsilon \int_0^t g(t-s)\, ds \int_\Omega \nabla u \nabla u(s)\, dx ds \\
= & \ \varepsilon \int_0^t g(t-s)\, ds \int_\Omega \nabla u (\nabla u(s) - \nabla u(t))\, dx ds \\
& + \varepsilon \int_0^t g(s)\, ds \|\nabla u\|^2 \\
\geq & \ \frac{\varepsilon}{2} \int_0^t g(s)\, ds \|\nabla u\|^2 - \frac{\varepsilon}{2} (go\nabla u)(t) .
\end{aligned} \tag{3.8}$$

By (3.6), we get

$$\begin{aligned}
\mathcal{K}'(t) \geq & \ (1-\alpha) H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \left(1 - \frac{1}{2}\int_0^t g(s)\, ds\right) \|\nabla u\|^2 \\
& - \varepsilon \|\nabla u\|^{2(\gamma+1)} + \varepsilon b \|u\|_p^p - \varepsilon \delta_1 \left(\int_{\tau_1}^{\tau_2} |\mu_2(q)|\, dq\right) \|u\|^2 \\
& - \frac{\varepsilon}{4\delta_1} \int_\Omega \int_{\tau_1}^{\tau_2} |\mu_2(q)| \left|y^2(x,1,q,t)\right| dq dx + \frac{\varepsilon}{2} (go\nabla u)(t) .
\end{aligned} \tag{3.9}$$

By using (3.2) and setting δ_1 such that, $\frac{1}{4\delta_1 c_1} = \kappa H^{-\alpha}(t)$, we obtain

$$\begin{aligned}
\mathcal{K}'(t) \geq & \ [(1-\alpha) - \varepsilon\kappa] H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^2 \\
& - \varepsilon \left[\left(1 - \frac{1}{2}\int_0^t g(s)\, ds\right)\right] \|\nabla u\|^2 - \varepsilon \|\nabla u\|^{2(\gamma+1)} \\
& + \varepsilon b \|u\|_p^p - \varepsilon \frac{H^\alpha(t)}{4c_1\kappa} \left(\int_{\tau_1}^{\tau_2} |\mu_2(q)|\, dq\right) \|u\|^2 + \frac{\varepsilon}{2} (go\nabla u)(t) .
\end{aligned} \tag{3.10}$$

For $0 < a < 1$, by (2.19)

$$\begin{aligned}
\varepsilon b \|u\|_p^p = & \ \varepsilon p (1-a) H(t) + \frac{\varepsilon p (1-a)}{2} \|u_t\|^2 + \varepsilon b a \|u\|_p^p \\
& + \frac{\varepsilon p (1-a)}{2} \left(1 - \int_0^t g(s)\, ds\right) \|\nabla u\|^2 \\
& + \frac{\varepsilon p (1-a)}{2(\gamma+1)} \|\nabla u\|^{2(\gamma+1)} + \frac{\varepsilon}{2} p (1-a) (go\nabla u)(t) \\
& + \frac{\varepsilon p (1-a)}{2} \int_\Omega \int_0^1 \int_{\tau_1}^{\tau_2} q |\mu_2(q)| \left|y^2(x,\rho,q,t)\right| dq d\rho dx,
\end{aligned} \tag{3.11}$$

with (3.10), it gives

$$\begin{aligned}\mathcal{K}'(t) &\geq [(1-\alpha)-\varepsilon\kappa] H^{-\alpha}(t) H'(t) + \varepsilon\left[\frac{p(1-a)}{2}+1\right]\|u_t\|^2 \\ &+\varepsilon\left[\left(\frac{p(1-a)}{2}\right)\left(1-\int_0^t g(s)\,ds\right)-\left(1-\frac{1}{2}\int_0^t g(s)\,ds\right)\right]\|\nabla u\|^2 \\ &+\varepsilon\left(\frac{p(1-a)}{2(\gamma+1)}-1\right)\|\nabla u\|^{2(\gamma+1)}-\varepsilon\frac{H^\alpha(t)}{4c_1\kappa}\left(\int_{\tau_1}^{\tau_2}|\mu_2(q)|\,dq\right)\|u\|^2 \\ &+\frac{\varepsilon p(1-a)}{2}\int_\Omega\int_0^1\int_{\tau_1}^{\tau_2} q|\mu_2(q)|\left|y^2(x,\rho,q,t)\right| dq d\rho dx \\ &+\varepsilon p(1-a) H(t)+\varepsilon ba\|u\|_p^p+\frac{\varepsilon}{2}(p(1-a)+1)(go\nabla u)(t). \end{aligned} \tag{3.12}$$

By using (2.17), (3.3) and Young's inequality, we obtain

$$\begin{aligned} H^\alpha(t)\|u\|_2^2 &\leq \left(b\int_\Omega |u|^p\,dx\right)^\alpha \|u\|_2^2 \\ &\leq c\left\{\left(\int_\Omega |u|^p\,dx\right)^{\alpha+2/p}+\left(\int_\Omega |u|^p\,dx\right)^\alpha \|\nabla u\|_2^{4/p}\right\} \\ &\leq c\left\{\left(\int_\Omega |u|^p\,dx\right)^{(p\alpha+2)/p}+\|\nabla u\|_2^2+\left(\int_\Omega |u|^p\,dx\right)^{p\alpha/(p-2)}\right\}. \end{aligned} \tag{3.13}$$

By exploiting (3.5), we obtain

$$2<\alpha p+2\leq p \text{ and } 2<\frac{\alpha p^2}{p-2}\leq p.$$

As a result, by Lemma 2.2, such that

$$H^\alpha(t)\|u\|_2^2\leq c\left(\|u\|_p^p+\|\nabla u\|_2^2\right). \tag{3.14}$$

By combining (3.12) and (3.14), we have

$$\begin{aligned}\mathcal{K}'(t) &\geq [(1-\alpha)-\varepsilon\kappa] H^{-\alpha}(t) H'(t) \\ &+\varepsilon\left[\frac{p(1-a)}{2}+1\right]\|u_t\|^2+\frac{\varepsilon}{2}(p(1-a)+1)(go\nabla u)(t) \\ &+\varepsilon\left\{\left(\frac{p(1-a)}{2}-1\right)-\int_0^t g(s)\,ds\left(\frac{p(1-a)-1}{2}\right)\right. \\ &\left.-\frac{c}{4c_1\kappa}\left(\int_{\tau_1}^{\tau_2}|\mu_2(q)|\,dq\right)\right\}\|\nabla u\|^2 \\ &+\varepsilon\left[ab-\frac{c}{4c_1\kappa}\left(\int_{\tau_1}^{\tau_2}|\mu_2(q)|\,dq\right)\right]\|u\|_p^p \\ &+\varepsilon\left(\frac{p(1-a)}{2(\gamma+1)}-1\right)\|\nabla u\|^{2(\gamma+1)}+\varepsilon p(1-a)H(t) \\ &+\frac{\varepsilon p(1-a)}{2}\int_\Omega\int_0^1\int_{\tau_1}^{\tau_2} q|\mu_2(q)|\left|y^2(x,\rho,q,t)\right| dq d\rho dx. \end{aligned} \tag{3.15}$$

Taking $a > 0$ small enough, that

$$\alpha_1 = \frac{p(1-a)}{2} - 1 > 0$$

and suppose

$$\int_0^\infty g(s)\,ds < \frac{\frac{p(1-a)}{2} - 1}{\left(\frac{p(1-a)}{2} - \frac{1}{2}\right)} = \frac{2\alpha_1}{2\alpha_1 + 1}. \tag{3.16}$$

Choosing κ such that,

$$\begin{aligned}\alpha_2 &= \left(\frac{p(1-a)}{2} - 1\right) - \int_0^t g(s)\,ds \left(\frac{p(1-a)-1}{2}\right) \\ &\quad - \frac{c}{4c_1\kappa}\left(\int_{\tau_1}^{\tau_2} |\mu_2(q)|\,dq\right) \\ &> 0\end{aligned}$$

and

$$\alpha_3 = ab - \frac{c}{4c_1\kappa}\left(\int_{\tau_1}^{\tau_2} |\mu_2(q)|\,dq\right) > 0 \text{ and } \frac{p(1-a)}{2(\gamma+1)} - 1 > 0.$$

Fixing κ and a, we have ε small enough,

$$\alpha_4 = (1-\alpha) - \varepsilon\kappa > 0.$$

Hence, for some $\beta > 0$, (3.15) becomes

$$\begin{aligned}\mathcal{K}'(t) &\geq \beta\Big\{H(t) + \|u_t\|^2 + \|\nabla u\|^2 + \|\nabla u\|^{2(\gamma+1)} + (go\nabla u)(t) \\ &\quad + \|u\|_p^p + \int_\Omega \int_0^1 \int_{\tau_1}^{\tau_2} q\,|\mu_2(q)|\,\left|y^2(x,\rho,q,t)\right| dq\,d\rho\,dx\Big\}.\end{aligned} \tag{3.17}$$

Therefore,

$$\mathcal{K}(t) \geq \mathcal{K}(0) > 0,\ t > 0. \tag{3.18}$$

Now, utilizing Holder's and Young's inequalities, we obtain

$$\begin{aligned}\|u\|_2 &= \left(\int_\Omega u^2 dx\right)^{\frac{1}{2}} \\ &\leq \left[\left(\int_\Omega \left(|u|^2\right)^{p/2} dx\right)^{\frac{2}{p}} \left(\int_\Omega 1dx\right)^{1-\frac{2}{p}}\right]^{\frac{1}{2}} \\ &\leq C\,\|u\|_p\end{aligned} \tag{3.19}$$

and

$$\left|\int_\Omega u u_t dx\right| \leq \|u_t\|_2 \|u\|_2 \leq c\,\|u_t\|_2 \|u\|_p.$$

Hence,

$$\begin{aligned}\left|\int_\Omega u u_t dx\right|^{\frac{1}{1-\alpha}} &\leq c\,\|u_t\|_2^{\frac{1}{1-\alpha}} \|u\|_p^{\frac{1}{1-\alpha}} \\ &\leq c\left[\|u_t\|_2^{\frac{\theta}{1-\alpha}} + \|u\|_p^{\frac{\mu}{1-\alpha}}\right],\end{aligned} \tag{3.20}$$

here $\frac{1}{\mu}+\frac{1}{\theta}=1$. Taking $\theta=2\left(1-\alpha\right)$, we have

$$\frac{\mu}{1-\alpha}=\frac{2}{1-2\alpha}\leq p.$$

For $s=\frac{2}{(1-2\alpha)}$, we get

$$\left|\int_{\Omega}uu_{t}dx\right|^{\frac{1}{1-\alpha}}\leq c\left(\|u_{t}\|_{2}^{2}+\|u\|_{p}^{s}\right).$$

Hence, Lemma 2.3 gives

$$\begin{aligned}\left|\int_{\Omega}uu_{t}dx\right|^{\frac{1}{1-\alpha}} &\leq c\left[\|u_{t}\|_{2}^{2}+\|u\|_{p}^{p}+\|\nabla u\|_{2}^{2}\right]\\ &\leq c\left[\|u_{t}\|_{2}^{2}+\|u\|_{p}^{p}+\|\nabla u\|_{2}^{2}+\|\nabla u\|^{2(\gamma+1)}+\left(go\nabla u\right)(t)\right].\end{aligned}\tag{3.21}$$

Therefore,

$$\begin{aligned}\mathcal{K}^{\frac{1}{1-\alpha}}\left(t\right) &= \left(H\left(t\right)^{1-\alpha}+\varepsilon\int_{\Omega}uu_{t}dx+\frac{\varepsilon\mu_{1}}{2}\int_{\Omega}u^{2}dx\right)^{\frac{1}{1-\alpha}}\\ &\leq c\left[H\left(t\right)+\left|\int_{\Omega}uu_{t}dx\right|^{\frac{1}{1-\alpha}}+\|u\|_{2}^{\frac{2}{1-\alpha}}+\|\nabla u\|_{2}^{\frac{2}{1-\alpha}}\right]\\ &\leq c\left[H\left(t\right)+\|u_{t}\|^{2}+\|u\|_{p}^{p}+\|\nabla u\|^{2}+\|\nabla u\|^{2(\gamma+1)}+\left(go\nabla u\right)(t)\right].\end{aligned}\tag{3.22}$$

By (3.17) and (3.22), we obtain

$$\mathcal{K}'\left(t\right)\geq\lambda\mathcal{K}^{\frac{1}{1-\alpha}}\left(t\right),\tag{3.23}$$

here $\lambda>0$, which depends on β and c. An integration of (3.23), we get

$$\mathcal{K}^{\frac{\alpha}{1-\alpha}}\left(t\right)\geq\frac{1}{\mathcal{K}^{\frac{-\alpha}{1-\alpha}}\left(0\right)-\lambda\frac{\alpha}{(1-\alpha)}t}.$$

Therefore, $\mathcal{K}\left(t\right)$ blows up in time

$$T\leq T^{*}=\frac{1-\alpha}{\lambda\alpha\mathcal{K}^{\alpha/(1-\alpha)}\left(0\right)}.$$

Then, the proof is completed. □

4. Conclusion

In recent years, there has been published much work concerning the wave equations (Kirchhoff, Petrovsky, Bessel,... etc.) with different state of delay time (constant delay, time-varying delay,... etc.). However, to the best of our knowledge, there were no nonexistence of global results for the Kirchhoff-type viscoleastic equation with distributed delay and source terms. We have been obtained the nonexistence of global solutions under suitable conditions.

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The Declaration of Research and Publication Ethics

The author(s) declared that they comply with the scientific, ethical, and citation rules of Journal of Universal Mathematics in all processes of the study and that they do not make any falsification on the data collected. Besides, the author(s) declared that Journal of Universal Mathematics and its editorial board have no responsibility for any ethical violations that may be encountered and this study has not been evaluated in any academic publication environment other than Journal of Universal Mathematics.

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