Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 11(4): 3086-3097, 2021
Journal of the Institute of Science and Technology, 11(4): 3086-3097, 2021
ISSN: 2146-0574, eISSN: 2536-4618
Matematik / *Mathematics* **DOI: 10.21597/jist.958229**
Araştırma Makalesi / *Research Article*
Geliş tarihi / Received: 27.06.2021 **Kabul tarihi / Accepted: 17.09.2021**

To Cite: Kırcı Ö, Aktürk T, Bulut H, 2021. Simulation of Wave Solutions of a Mathematical Model Representing Communication Signals, Journal of the Institute of Science and Technology, 11(4): 3086-3097.

Simulation of Wave Solutions of a Mathematical Model Representing Communication Signals

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ABSTRACT: In this study, the Lonngren-wave equation is considered to be analyzed for its wave solutions. To implement this purpose the modified exponential function method is used and ultimately new hyperbolic, trigonometric and rational forms of the exact solutions are obtained. Furthermore, it was tested whether these forms satisfy the Lonngren-wave equation or not and it was seen that they verify the equation. Besides this, the two and three dimensional graphics together with the contour and density plots are presented.

Keywords: Lonngren-wave equation, the modified exponential function method, exact wave solutions, nonlinear partial differential equation, the hyperbolic, trigonometric and rational functions

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INTRODUCTION

The nonlinear partial differential equations (NLPDEs) interest a wide range of applied scientists due to their potential of being extensively arisen in all the fields of engineering and science. Thus various powerful and effective methods are implemented for the exact solutions of the evolution equations owing to their importance in nonlinear science and their broad usage area. The solutions obtained as a result of applying these techniques allow commenting on the behavior of mathematical models. Some of them are the (G'/G)-expansion technique and its modifications (Wang et al., 2008; Naher, 2012; Naher and Abdullah, 2013; Akbar et al., 2016; Duran, 2020; Duran, 2021), the (1/G')-expansion method (Duran, 2021), sine-Gordon expansion method and $(m+G'/G)$-expansion method (Ismael et al., 2020), the improved Bernoulli sub-equation function method (Bulut et al., 2016; Duran et al., 2021), the Riccati-Bernoulli sub-ODE method (Yang et al., 2015), the $exp(-\phi(\xi))$-expansion method and its improved forms (Misirli and Gurefe, 2011; Arshed et al., 2019; Chen et al., 2019; Yel et al., 2019; Baskonus, 2021; Duran, 2021), the generalized Kudryashov method (Demiray et al., 2015; Mahmud et al., 2017; Rahman et al., 2019), the new function method (Aktürk et al., 2017), the Hirota's bilinear transformation (Hietarinta, 2005), the Backlund transformation method (Hirota and Satsuma, 1977; Lu et al., 2006), rational sine-cosine method (Marwan et al. 2011; Qawasmeh and Alquran, 2014) the tanh method and its various extension (Fan, 2000; Elwakil et al., 2005; Yang and Hon, 2006), the tanh-coth expansion method (Wazwaz, 2007a, 2007b; Parkes, 2010), the homotopy perturbation method (He, 2006a, 2006b, 2008; Biazar et al., 2009), the simplified Hirota's method (Wazwaz, 2016), the extended sinh-Gordon equation expansion method (Kumar et al., 2018; Gao et al., 2019), Lie transformation method and singular manifold method (Saleh et al., 2021), the power index method (Shrauner, 2019), ϕ^6-model expansion method (Seadawy et al., 2021), the truncated Painleve expansion (Radha et al., 2007), the Jacobi elliptic-function method (Parkes et al., 2002), etc.

In this study the Lonngren wave Equation (1), which is one of the NLPDEs, is considered and the new forms of the exact solutions are obtained by modified exponential function method (MEFM).

$$(u_{xx}-\alpha u+\beta u^2)_{tt}+u_{xx}=0. \quad (1)$$

The Lonngren wave equation is used in the field of telecommunication and network engineering. For this equation, Akcagil and Aydemir have presented (G′/G)–expansion method, the modified extended tanh method and the unified method to reveal the new exact solutions (Akcagil and Aydemir, 2016; Akcagil and Aydemir, 2018). Kayum et al. have investigated the soliton solutions through the modified simple equation method (Kayum et al., 2020). Then it comes out that the Lonngren-wave equation is not taken into consideration by the modified exponential function method. In the light of those papers, the aim of this study is to present the new form of solutions to this equation. The flow of the manuscript is as follows: The materials and methods section includes the description of the MEFM and the application of the method for the Lonngren-wave equation, in results and discussion section the graphical results are given, finally it is ended with the conclusion part.

MATERIALS AND METHODS

Methodology

In this section, the application of the modified exponential function method to a NLPDE will be described. According to the method, the general form of a nonlinear evolution equation is written as follows;

$$P(U,U_x,U_t,U_{xx},U_{xt},U_{tt},U_{xxtt},\dots)=0, \quad (2)$$

where P is a function of $U=U(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved.

Step1: In order to obtain the solution functions of a NLPDE, the equation must be reduced into a nonlinear ordinary differential equation (NLODE) module. Therefore according to the method the wave transformation given below can be used for equation (1)

$$U(x,t) = U(\xi), \xi = k(x-ct), \tag{3}$$

where k represents the wave height and c represents the wave frequency. The derivative terms required in equation (1) are obtained using the wave transformation (3) above. These terms are then substituted in equation (1) and as a result (1) reduced to a NLODE whose general form can be given as in the following,

$$N(U, U^2, U', U'', \dots) = 0. \tag{4}$$

Step2: According to the method the solution of Equation (1) is

$$U(\xi) = \frac{\sum_{i=0}^{n} A_i[exp(-\Omega(\xi))]^i}{\sum_{j=0}^{m} B_j[exp(-\Omega(\xi))]^j} = \frac{A_0 + A_1 exp(-\Omega(\xi)) + \cdots + A_n exp(-n\Omega(\xi))}{B_0 + B_1 exp(-\Omega(\xi)) + \cdots + B_m exp(-m\Omega(\xi))}, \tag{5}$$

where $A_i, B_j, (0 \le i \le n, 0 \le j \le m)$ are constants to be determined.

In order to state equation (5) clearly, it is necessary to determine the upper limits of the summation symbols, the omega function and the coefficients, respectively. The balancing principle is used in the process of determining the upper limits, namely m and n. For this, a relation is obtained between m and n by balancing the term containing the highest order derivative and the highest order nonlinear term in equation (4). Then, the upper limits of the summation symbols are determined by giving values to the parameters so that they can provide the correlation. The expansion of the sums in (5) are provided up to the upper limit values after determination of m and n. After explicitly expressing $U(\xi)$ in (5), the derivative terms required in Equation (4) are obtained from here. Substituting (5) together with the required derivatives into (4) creates the need of the omega function and its first order derivative. Therefore it is utilized from the following ordinary differential equation whose solution is $\Omega(\xi)$.

$$\Omega'(\xi) = exp(-\Omega(\xi)) + \mu exp(\Omega(\xi)) + \lambda. \tag{6}$$

Step3: The substitution of (5) into (4), taking into consideration (6), results in a system of algebraic equation. From this system the coefficients are determined using the package program. By writing the obtained coefficients in equation (4), the solution functions are investigated according to the following family states (Bulut and Baskonus, 2016).

Family1: When $\mu \neq 0, \lambda^2 - 4\mu > 0$,

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right) - \frac{\lambda}{2\mu}\right). \tag{7}$$

Family2: When $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$\Omega(\xi) = \ln\left(\frac{\sqrt{-\lambda^2+4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}(\xi+E)\right) - \frac{\lambda}{2\mu}\right). \tag{8}$$

Family3: When $\mu = 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu > 0$,

$$\Omega(\xi) = -\ln\left(\frac{\lambda}{exp(\lambda(\xi+E)) - 1}\right). \tag{9}$$

Family4: When $\mu \neq 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu = 0$,

$$\Omega(\xi) = \ln\left(-\frac{2\lambda(\xi+E)+4}{\lambda^2(\xi+E)}\right). \tag{10}$$

Family5: When $\mu = 0,\ \lambda = 0$ and $\lambda^2 - 4\mu = 0$,

$$\Omega(\xi) = \ln(\xi + E), \tag{11}$$

where $A_0, A_1, \dots, A_n, B_0, B_1, \dots, B_m, E, \lambda, \mu$ are constants.

Application of MEFM to Lonngren-wave equation

If the wave transformation (3) is applied to the equation (1), the following NLODE model is obtained,

$$c^2k^2U'' + (1-\alpha c^2)U + \beta c^2U^2 + R = 0, \tag{12}$$

where R is the integral constant. Balancing the terms including the highest order derivative U'' and the highest power nonlinear term U^2 in (12) results in the following equation

$$n = m + 2. \tag{13}$$

According to this equation for the values $m=1,\ n=3$, the solution function and the derivatives sought for equation (12) are as follows;

$$U(\xi) = \frac{\psi}{\varphi} = \frac{A_0+A_1e^{-\Omega(\xi)}+A_2e^{-2\Omega(\xi)}+A_3e^{-3\Omega(\xi)}}{B_0+B_1e^{-\Omega(\xi)}},\ U'(\xi) = \frac{\psi'\varphi-\psi\varphi'}{\varphi^2},$$

$$U''(\xi) = \frac{\psi''\varphi^3-\varphi^2\psi'\varphi'-(\psi\varphi''+\psi'\varphi')\varphi^2+2(\psi')^2\psi\varphi}{\varphi^4}. \tag{14}$$

If the derivative concepts obtained above are written into equation (4), the following coefficient states and the solution functions depending on them are obtained.

Case1:

$$A_0 = \frac{\left(1+c^2\left(-\alpha+k^2(\lambda^2+8\mu)\right)\right)A_2B_0}{12c^2k^2(B_0+\lambda B_1)},A_1 = \frac{A_2\left(B_1+c^2\left(12k^2\lambda B_0+\left(-\alpha+k^2(\lambda^2+8\mu)\right)B_1\right)\right)}{12c^2k^2(B_0+\lambda B_1)},$$

$$A_3 = \frac{A_2B_1}{B_0+\lambda B_1},\beta = -\frac{6k^2(B_0+\lambda B_1)}{A_2},R = \frac{\left(-1+2c^2\alpha+c^4\left(-\alpha^2+k^4(\lambda^2-4\mu)^2\right)\right)A_2}{24c^2k^2(B_0+\lambda B_1)}. \tag{15}$$

The coefficients obtained by solving the algebraic equation system, are written in (5) and the solution function of equation (1) is analyzed according to the families as mentioned in step3. Graphical results that belong to the solution forms of (1) according to case1 are presented.

Family1:

$$U_{1,1}(\xi) = \frac{\mathrm{Sech}[\vartheta]^2(2(1-c^2(\alpha+5k^2(\lambda^2-4\mu)))\mu+(1-c^2(\alpha-k^2(\lambda^2-4\mu)))((\lambda^2-2\mu)\mathrm{Cosh}[2\vartheta] +\lambda\sqrt{\lambda^2-4\mu}\mathrm{Sinh}[2\vartheta]))A_2}{12c^2k^2(B_0+\lambda B_1)\left(\lambda+\sqrt{\lambda^2-4\mu}\mathrm{Tanh}[\vartheta]\right)^2},$$

where $\vartheta = \frac{1}{2}\sqrt{\lambda^2-4\mu}(\mathrm{E}+\xi)$, (Figure1).

Family2:

$$U_{1,2}(\xi) = \frac{\mathrm{Sec}[\psi]^2(2(1-c^2(\alpha+5k^2(\lambda^2-4\mu)))\mu+(1-c^2(\alpha-k^2(\lambda^2-4\mu)))((\lambda^2-2\mu)\mathrm{Cosh}[\psi] -\lambda\sqrt{-\lambda^2+4\mu}\mathrm{Sinh}[2\psi]))A_2}{12c^2k^2(B_0+\lambda B_1)\left(\lambda-\sqrt{-\lambda^2+4\mu}\mathrm{Tan}[\psi]\right)^2},$$

where $\psi = \frac{1}{2}(E+\xi)\sqrt{-\lambda^2+4\mu}$, (Figure2).

Family3:

$$U_{1,3}(\xi) = \frac{e^{(\mathrm{E}+\xi)\lambda}(-1+c^2(\alpha+5k^2\lambda^2)+(1+c^2(-\alpha+k^2\lambda^2))\mathrm{Cosh}[(\mathrm{E}+\xi)\lambda])A_2}{6c^2(-1+e^{(\mathrm{E}+\xi)\lambda})^2k^2(B_0+\lambda B_1)}.$$

(Figure3).

Family 4:

$$U_{1,4}(\xi) = \frac{-(((-\theta^2+c^2(\alpha\theta^2+2k^2(\lambda^2(-2+\phi(\theta+2))-4\theta^2\mu)))A_2)}{12c^2k^2\theta^2(B_0+\lambda B_1)},$$

where $\phi = \mathrm{E}\lambda + \xi\lambda, \theta = 2 + \phi$, (Figure4).

Family5:

$$U_{1,5}(\xi) = \frac{\left(\frac{12}{(\mathrm{E}+\xi)^2} + \frac{\frac{1}{c^2} - \alpha}{k^2}\right) A_2}{12B_0}.$$

(Figure5).

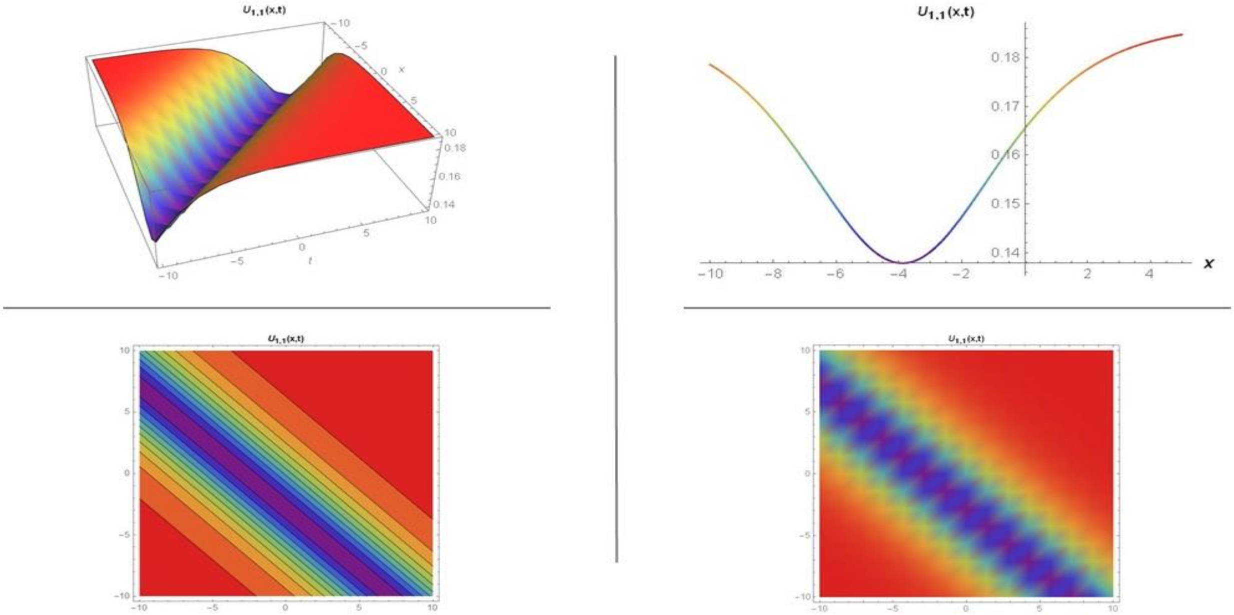

Figure 1. The three dimensional graph, contour graph, density graph of solution $U_{1,1}(\xi)$ for the values $c = -1, k = 0.5, B_1 = 2, \lambda = 3, \mu = 2, A_2 = 1.25, B_0 = 0.35, \alpha = -1.6, A_0 = 0.203248, A_1 = 1.36811, A_3 = 0.393701, \beta = -7.62, R = -0.219734, E = 0.75$ and two-dimensional graph for $t = 1$

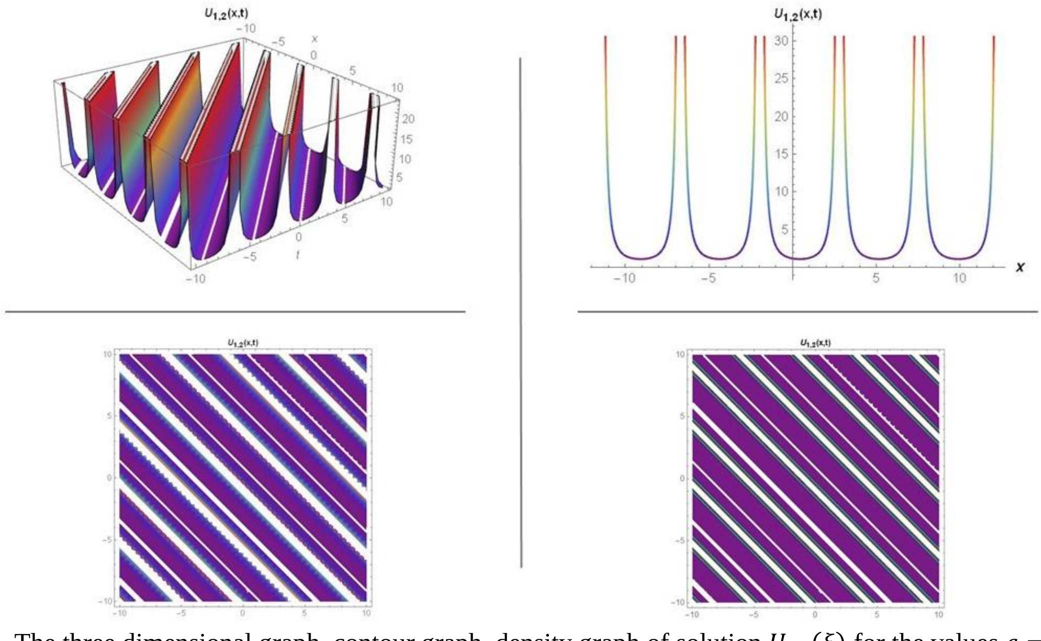

Figure 2. The three dimensional graph, contour graph, density graph of solution $U_{1,2}(\xi)$ for the values $c = -1, k = 0.5, B_1 = 2, \lambda = 1, \mu = 2, A_2 = 1.25, B_0 = 0.35, \alpha = -1.6, A_0 = 0.42089, A_1 = 2.61525, A_3 = 1.06383, \beta = -2.82, R = -0.327793, E = 0.75$ and two-dimensional graph for $t = 1$

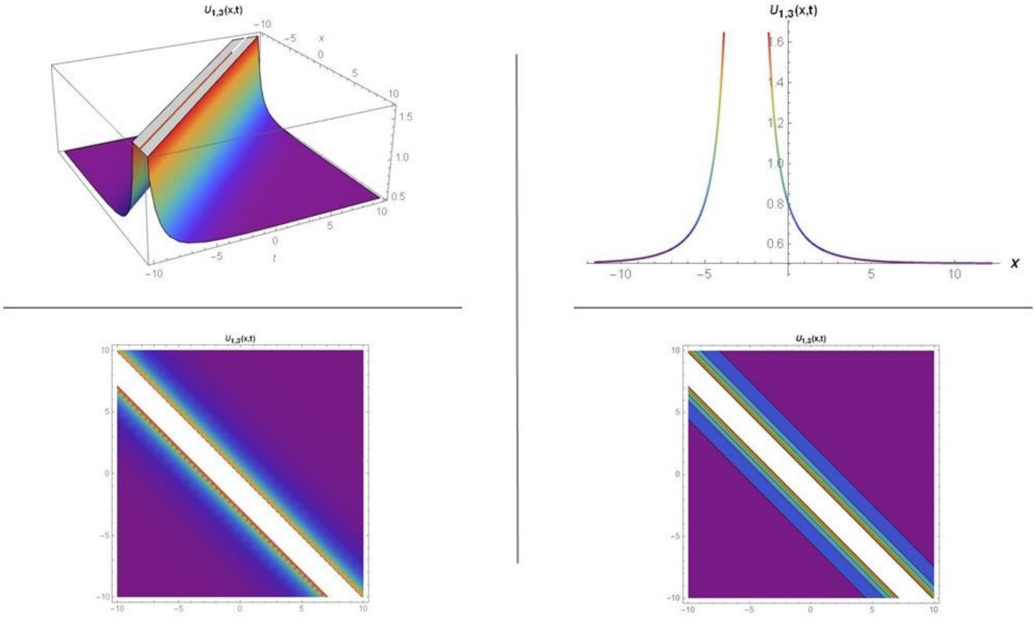

Figure 3. The three dimensional graph, contour graph, density graph of solution $U_{1,3}(\xi)$ for the values$c = -1, k = 0.5, B_1 = 2, \lambda = 1, \mu = 0, A_2 = 1.25, B_0 = 0.35, \alpha = -1.6, A_0 = 0.176862, A_1 = 1.19681, A_3 = 1.06383, \beta = -2.82, R = -0.59375, E = 0.75$ and two-dimensional graph for $t = 1$

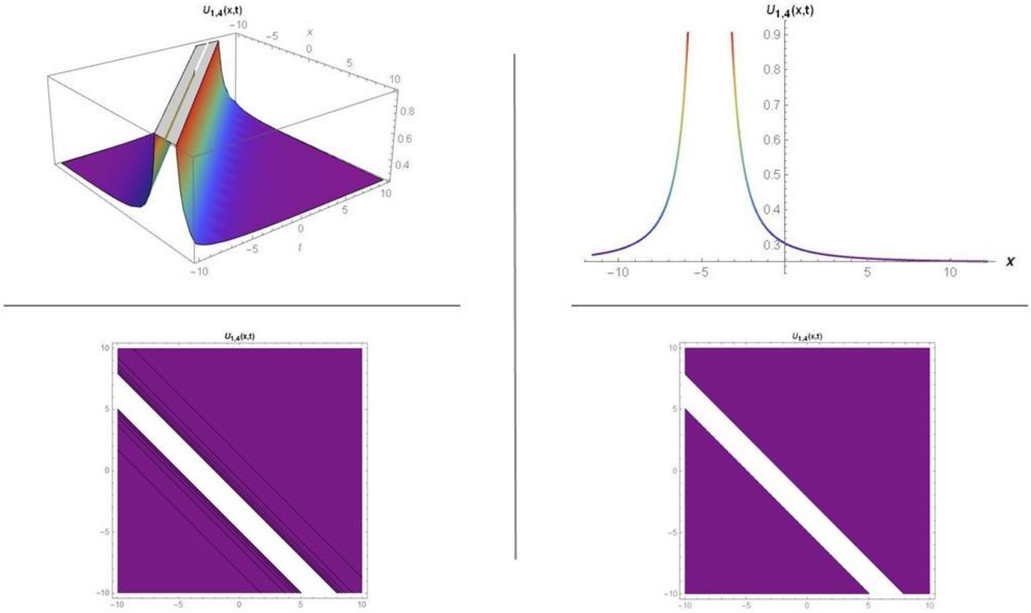

Figure 4. The three dimensional graph, contour graph, density graph of solution $U_{1,4}(\xi)$ for the values$c = -1, k = 0.5, B_1 = 2, \lambda = 2, \mu = 1, A_2 = 1.25, B_0 = 0.35, \alpha = -1.6, A_0 = 0.187739, A_1 = 1.27395, A_3 = 0.574713, \beta = -5.22, R = -0.323755, E = 0.75$ and two-dimensional graph for $t = 1$

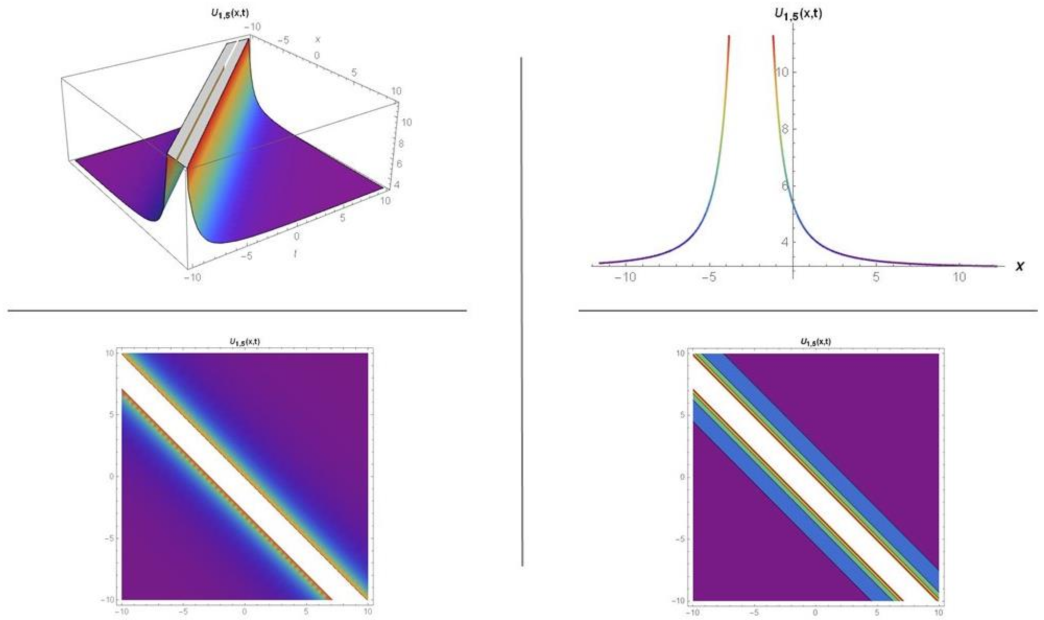

Figure 5. The three dimensional graph, contour graph, density graph of solution $U_{1,5}(\xi)$ for the values $c=-1, k=0.5, B_1=2, \lambda=0, \mu=0, A_2=1.25, B_0=0.35, \alpha=-1.6, A_0=1.08333, A_1=6.19048, A_3=7.14286, \beta=-0.42, R=-4.02381, E=0.75$ and two-dimensional graph for $t=1$

Another group of coefficient called as case2 is presented below.

Case2:

$$A_0=\frac{(\sqrt{4R\beta+c^2k^4(\lambda^2-4\mu)^2}-ck^2(\lambda^2+8\mu))B_0}{2c\beta}, A_1=\frac{(-\sqrt{4R\beta+c^2k^4(\lambda^2-4\mu)^2}+ck^2(\lambda^2+8\mu))A_3}{12ck^2}-\frac{6k^2\lambda B_0}{\beta}$$

$$A_2=\lambda A_3-\frac{6k^2B_0}{\beta}, B_1=-\frac{\beta A_3}{6k^2}, \alpha=\frac{1+c\sqrt{4R\beta+c^2k^4(\lambda^2-4\mu)^2}}{c^2}.$$

Now the wave solutions of equation (1) are investigated according to case2 which is a result of the algebraic equation system emerged from MEFM.

Family1:

$$U_{2,1}(\xi)=\frac{\text{Sech}[\vartheta]^2(2(\eta+5ck^2(\lambda^2-4\mu))\mu+(\eta-ck^2(\lambda^2-4\mu))((\lambda^2-2\mu)\text{Cosh}[2\vartheta]+\lambda\sqrt{\lambda^2-4\mu}\text{Sinh}[2\vartheta]))}{2c\beta(\lambda+\sqrt{\lambda^2-4\mu}\text{Tanh}[\vartheta])^2},$$

where $\vartheta=\frac{1}{2}\sqrt{\lambda^2-4\mu}(\text{E}+\xi), \eta=\sqrt{4R\beta+c^2k^4(\lambda^2-4\mu)^2}$, (Figure6).

Family2:

$$U_{2,2}(\xi)=\frac{\text{Sec}[\psi]^2(2(\eta-5ck^2\sigma^2)\mu+(\eta+ck^2\sigma^2)((\lambda^2-2\mu)\text{Cos}[2\vartheta]-\lambda\sigma\text{Sin}[2\psi]))}{2c\beta(\lambda-\sigma\text{Tan}[\psi])^2},$$

where $\psi=\frac{1}{2}\sqrt{-\lambda^2+4\mu}(\text{E}+\xi), \eta=\sqrt{4R\beta+c^2k^4(\lambda^2-4\mu)^2}, \sigma=\sqrt{-\lambda^2+4\mu}$, (Figure7).

Family3:

$$U_{2,3}(\xi)=\frac{\frac{\sqrt{4R\beta+c^2k^4\lambda^4}}{c}-k^2\lambda^2(1+3\text{Csch}[\omega]^2)}{2\beta},$$

where $\omega=\frac{1}{2}\lambda(\text{EE}+\xi)$, (Figure8).

Family4:

$$U_{2,4}(\xi) = \frac{\frac{\eta}{c} + 2k^2(\lambda^2(1 - \frac{6}{\phi^2}) - 4\mu)}{2\beta},$$

where $\eta = \sqrt{4R\beta + c^2k^4(\lambda^2 - 4\mu)^2}, \phi = \mathrm{E}\lambda + \xi\lambda, \theta = 2 + \phi$, (Figure9).

Family 5:

$$U_{2,5}(\xi) = \frac{-\frac{6k^2}{(\mathrm{E} + \xi)^2} + \frac{\sqrt{R\beta}}{c}}{\beta}.$$

(Figure10).

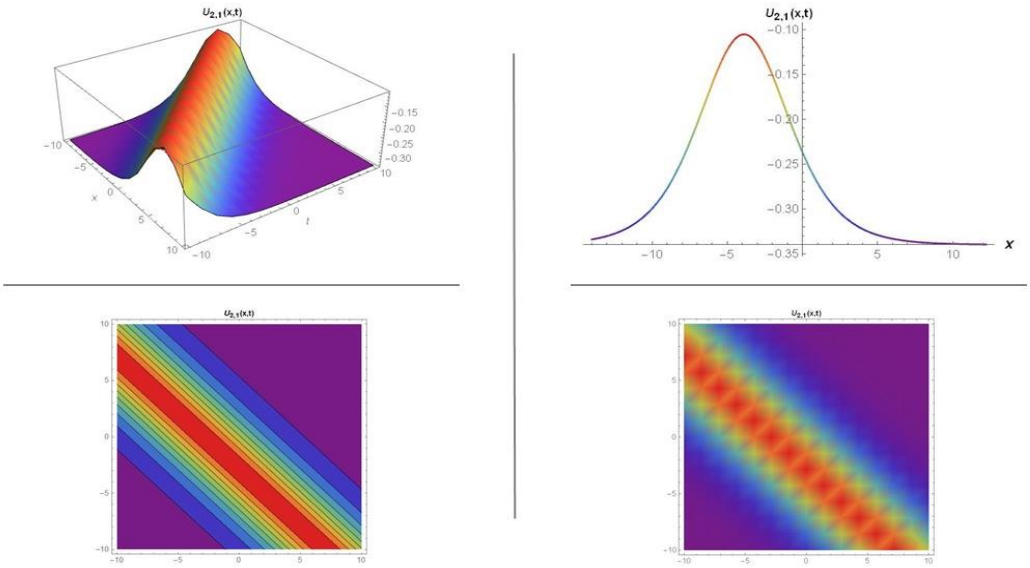

Figure 6. The three dimensional graph, contour graph, density graphs of solution $U_{2,1}(\xi)$ for the values $c = -1, k = 0.5, B_1 - 1.33333, \lambda = 3, \mu = 2, A_2 = 3.42188, B_0 = 0.35, \alpha = 0.161847, A_0 = -0.775267, A_1 = 1.96902, A_3 = 1.25, \beta = 1.6, R = 0.1, E = 0.75$ and two-dimensional graph for $t = 1$

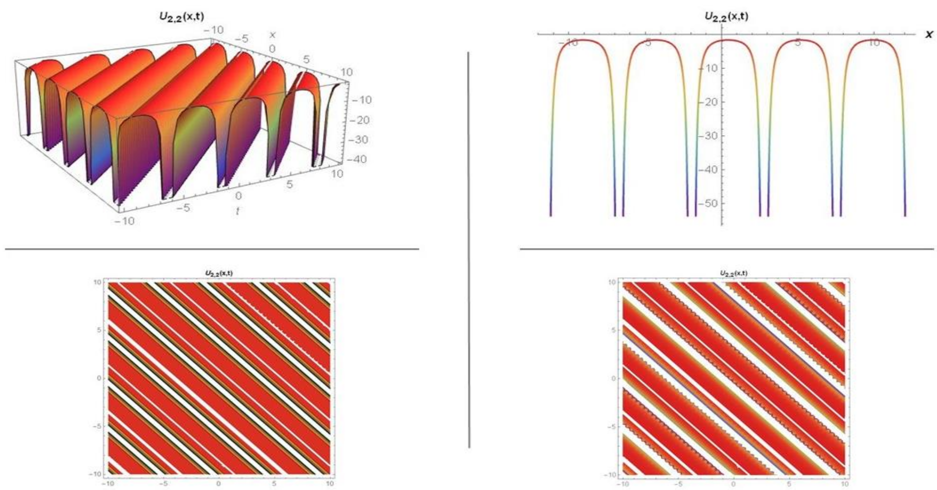

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Figure 7. The three dimensional graph, contour graph, density graph of solution $U_{2,2}(\xi)$ for the values $c = -1, k = 0.5, B_1 - 1.33333, \lambda = 1, \mu = 2, A_2 = 0.921875, B_0 = 0.35, \alpha = -0.924188, A_0 = -0.675302, A_1 = 2.24445, A_3 = 1.25, \beta = 1.6, R = 0.1, E = 0.75$ and two-dimensional graph for $t = 1$

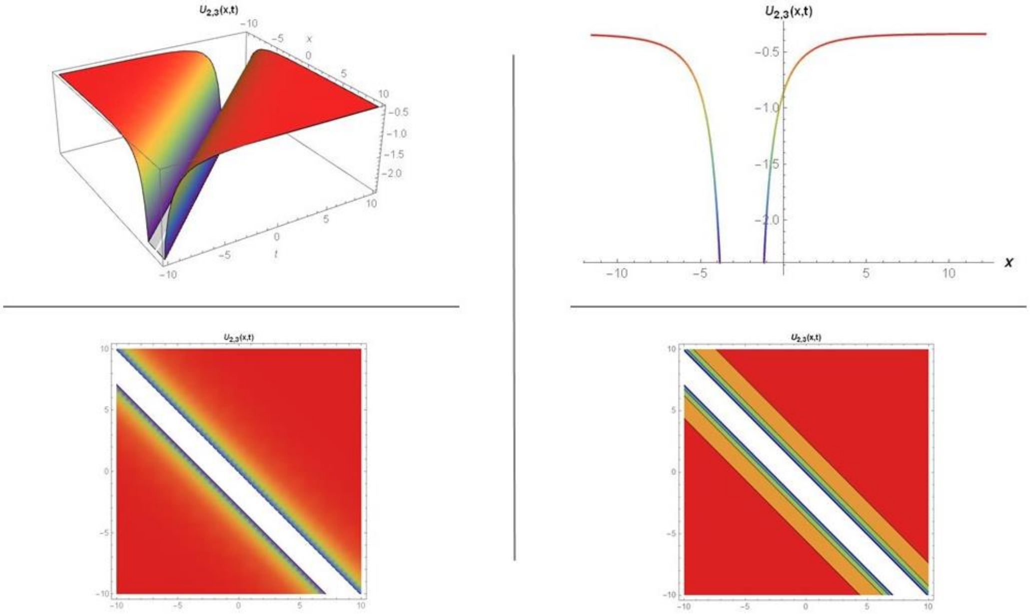

Figure 8. The three dimensional graph, contour graph, density graph of solution $U_{2,3}(\xi)$ for the values $c = -1, k = 0.5, B_1 - 1.33333, \lambda = 1, \mu = 0, A_2 = 0.921875, B_0 = 0.35, \alpha = 0.161847, A_0 = -0.119017, A_1 = 0.125272, A_3 = 1.25, \beta = 1.6, R = 0.1, E = 0.75$ and two-dimensional graph for $t = 1$

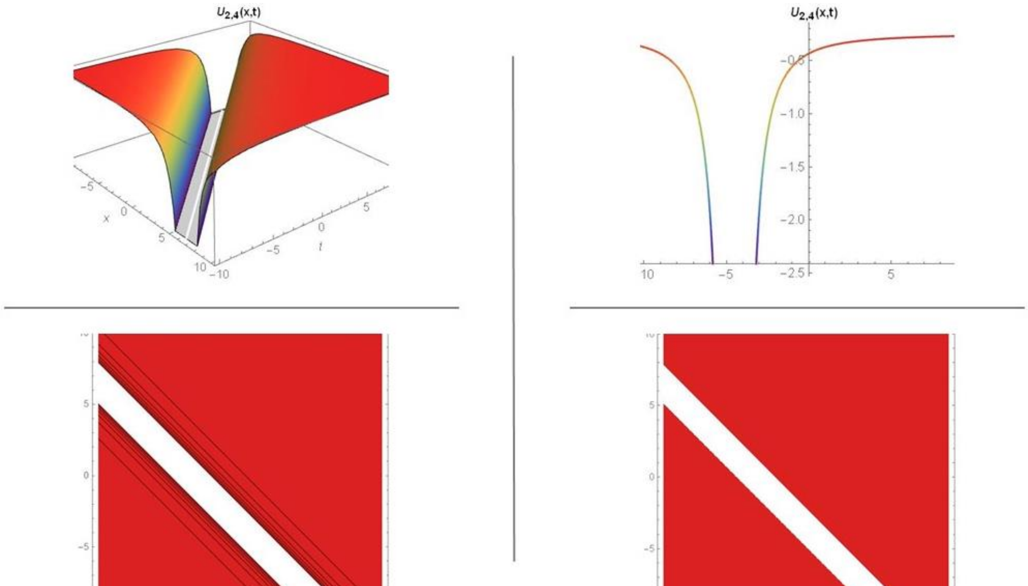

Figure 9. The three dimensional graph, contour graph, density graph of solution $U_{2,4}(\xi)$ for the values $c = -1, k = 0.5, B_1 - 1.33333, \lambda = 2, \mu = 1, A_2 = 2.17188, B_0 = 0.35, \alpha = 0.2, A_0 = -0.415625, A_1 = 0.927083, A_3 = 1.25, \beta = 1.6, R = 0.1, E = 0.75$ and two-dimensional graph for $t = 1$

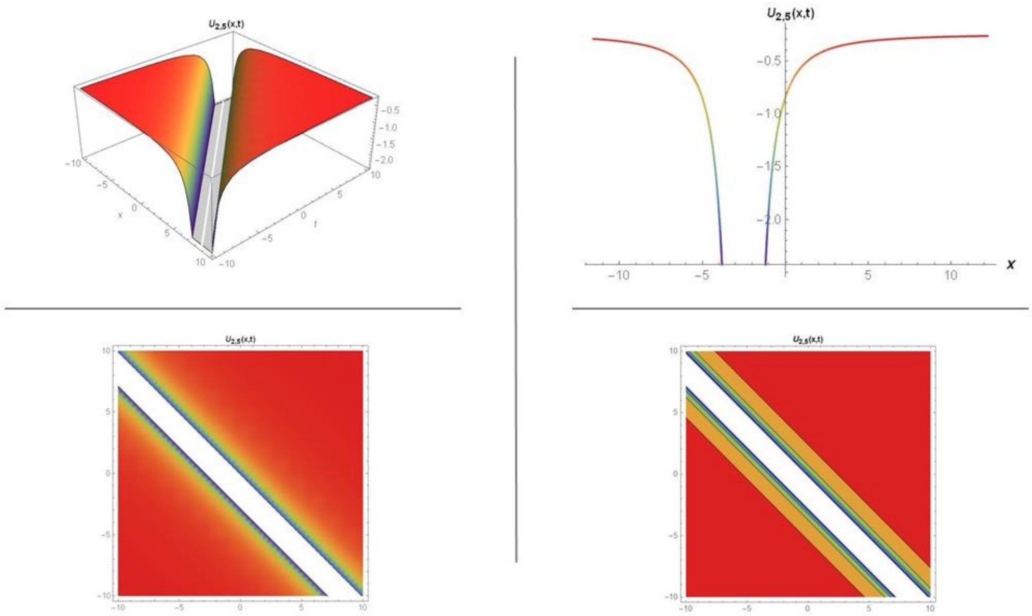

Figure 10. The three dimensional graph, contour graph, density graph of solution $U_{2,5}(\xi)$ for the values $c = -1, k = 0.5, B_1 - 1.33333, \lambda = 0, \mu = 0, A_2 = -0.328125, B_0 = 0.35, \alpha = 0.2, A_0 = -0.0875, A_1 = 0.33333, A_3 = 1.25, \beta = 1.6, R = 0.1, E = 0.75$ and two-dimensional graph for $t = 1$

RESULTS AND DISCUSSION

The graphs representing the behavior of the mathematical model are obtained by determining the appropriate parameters, for the solution forms. The 3D, 2D graphs together with the contour and the density plots of $U_{1,1}, U_{1,2}, U_{1,3}, U_{1,4} U_{1,5}, U_{2,1}, U_{2,2}, U_{2,3}, U_{2,4}, U_{2,5}$ are illustrated in Figures 1-10. Additionally, these forms are tested whether they are the exact solution of (1) or not with the help of a package program and verification is acquired.

CONCLUSION

We have determined the new exact solution forms of the Lonngren-wave equation as hyperbolic, trigonometric and rational functions via the modified exponential function method which is an effective and functioning method. It is observed that the MEFM is not applied for this equation before. The process of plotting the graphs and the computations are overcome with the aid of a package program. The Lonngren wave equation is used in the field of telecommunication and network engineering. Therefore the newly obtained wave solutions may be useful for analyzing and understanding the information as signals for transmission. This shows that the method is a very effective technique for the NLPDEs.

Conflict of Interest

The article authors declare that there is no conflict of interest between them.

Author's Contributions

The authors declare that they have contributed equally to the article.

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