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PREDICTION MODEL FOR BASE SHEAR INCREASE DUE TO VERTICAL GROUND MOTION IN FRICTION PENDULUM ISOLATED STRUCTURES

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Abstract: Seismic isolation is one of the most effective seismic hazard mitigation techniques, which has been implemented in many structures. Friction pendulum isolators are one of the most popular isolation devices to achieve energy dissipation and shear resistance that depends on the effective radius and instantaneous vertical force on them. So, the horizontal response of this type of isolators is coupled with the axial load on them, which could change due to vertical acceleration during ground shaking. Thus, it makes consideration of the vertical excitation in the design and analysis phase inevitable, especially for structures in those regions where vertical ground accelerations are more pronounced. In this study a model for the prediction of increase in base shear due to vertical motion for friction pendulum isolated structures is proposed. A simple single friction pendulum isolator system is utilized with the assumption of rigid superstructure on top and subjected to a hundred earthquake ground motions with different characteristics. A multi-layer perceptron model with three layers is utilized, and a simple computer program, which is open-source, is prepared to construct probability curves. With the help of the program, within the range of the considered structural parameters in the study, probability curves related to increase in base shear, maximum isolator displacement and residual isolator displacement can be constructed, and designers can have a rough estimation on modification of these parameters due to vertical ground motion.

Keywords: Seismic isolation, Friction pendulum system, Vertical ground motion, Base shear increase

Düşey yer hareketinden dolayı sürtünmeli sarkaç sistemleri ile izole edilmiş yapılarda taban kesme kuvveti artışı tahmin modeli

Öz: Birçok yapıya uygulanan sismik izolasyon, sismik tehlikenin azaltılması için en etkin yollardan biridir. Sürtünmeli sarkaç izolatörleri de enerji sönümü ve etkili yarı çapa ve üzerindeki anlık yüke bağlı olarak kayma direnci oluşturan en popüler sismik izolatörlerdendir. Bu yüzden, bu tür izolatörlerin yatay tepkisi üzerinde bulunan eksenel yüke bağlıdır ve bu yük de düşey yer ivmesine göre sarsıntı boyunca değişebilir. Böylece düşey yer sarsıntısının etkili olduğu bölgelerde bu doğrultudaki ivmeyi tasarım ve analiz aşamasında göz önünde bulundurmayı kaçınılmaz kılar. Bu nedenle, bu çalışmada taban kesme kuvvetinde düşey hareketten dolayı meydana gelen artımın tahmini için bir yöntem sunulmaktadır. Rijit yapı kabulüne dayalı basit bir model oluşturulmuş ve farklı karakterlere sahip yüz deprem hareketine maruz bırakılmıştır. Çok katmanlı algılayıcı modeli kullanılmış ve kırılganlık eğrilerinin oluşturulması için açık kaynak kodlu basit bir bilgisayar programı hazırlanmıştır. Program yardımıyla çalışmada verilen yapısal parametrelerin aralığında taban kesme kuvvetinde, maksimum ve kalıcı izolatör deplasmanında düşey yer ivmesinden dolayı meydana gelecek artışa dair olasılık eğrileri elde edilebilir ve tasarımcılar düşey yer ivmesinden dolayı bu parametrelerde yapılacak olan modifikasyonlar için yaklaşık bir tahmin yapabilir.

Anahtar Kelimeler: Sismik izolasyon, Sürtünmeli sarkaç sistemi, Düşey yer hareketi, Taban kesme kuvveti artışı

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1. INTRODUCTION

Friction pendulum isolators, which have different types depending on the number of sliding surface that yields different regimes in force displacement relation, provide stiffness based on the axial load on them and the radius of the curvature on which they are sliding (Zayas et al., 1987; Mokha et al., 1991; Fenz and Constantinou, 2006, 2008; Kumar et al., 2015a). Also, friction between articulated slider and curved surface forms a hysteretic force-displacement relation and dissipates energy under horizontal displacement. Both stiffness and frictional force of the friction pendulum bearings are dependent on the axial load on them, and determine horizontal response. So, base shear increase due to vertical ground motions can be observed in friction pendulum isolated structures, especially for those sites that observe relatively higher accelerations in vertical direction. Hence, vertical ground motion can have significant effect on the response of friction isolators, in both structural and non-structural component levels (Ryan and Dao, 2016; Guzman Pujols and Ryan 2017). There are number of factors affecting severity of change of structural response of friction pendulum isolated structures.

Intensity of ground motion, interaction with the soil, and its parameters, and type of ground motions are key parameters that determine base shear increase due to vertical ground motion. Dao and Ryan (2020) shows effect of soil-structure interaction and vertical ground motion on the response of friction isolated structures. Once structures are subjected to three-dimensional shaking, inter-story drift ratio is affected due to directional coupling of the isolators. Cilsalar and Constantinou (2017) analyzed twenty-four simplified models that were subjected to both Near-Fault and Far-Field ground motions given in FEMA P-695 (FEMA, 2009). Probability curves and some other statistical parameters are given in the study, and maximum value in base shear increases are given as 1.22 and 1.48 under Far-Field and Near-Fault motions, respectively.

Eltahawy et al. (2018) investigated parameters that had effect on the dynamic response of three-dimensional seismic isolation systems, and showed that the rigid block assumption was good enough to capture fundamental behavior of the system, although, it did not capture variation in stiffness change of superstructure and second order effects. Also, system parameters are more influential than the ratio between spectral acceleration values in horizontal and vertical directions. Although, inclusion of vertical ground motion in the time history analyses of friction pendulum isolators has impact on base shear and floor accelerations in superstructure, and its impact on maximum isolator displacement is insignificant (Sarlis et al., 2013; Morgan 2007; Oikonomou et al., 2016). Other studies also show importance of the vertical earthquake motions on the response of different base isolated structures with different sliding isolators (Shakib and Fuladgar 2003; Panchal et al., 2010).

Dao et al. (2019) studied the ability of simplified models for capturing the response of tested five-story steel moment resisting frame with triple friction isolator. Results reveal that response of a simplified stick model cannot capture floor acceleration well; however, with the mechanical properties of stick model computed from a fixed-base structure is almost identical with the isolated system. Also, displacement results for both are almost identical in the study.

Since base shear is increased with the inclusion of the vertical acceleration, and simplified models reveal behavior of seismically isolated structure to some extent, this study presents a simple Python program to anticipate base shear increase in simple single friction pendulum isolated structures depending on a number of parameters due to vertical components of ground motions. Moreover, effect of vertical ground motion on maximum isolator displacement and residual isolator displacement are also investigated. Although, this study focuses on the single friction pendulum isolator, result can give some insight about the behavior of double concave isolators if the both surfaces have the same frictional coefficient. Three thousand different rigid mass systems are analyzed under a hundred ground motions with and without vertical ground motion, and base shear, maximum isolator displacement and residual isolator displacement value of those analyses with vertical ground motions are normalized with the result of analyses without vertical ground motion. Those isolator parameters such as effective period, damping coefficients etc. were selected in a way that mean values of all parameters were around the quantities mostly used in the literature (Cilsalar and Constantinou 2017; Kumar et al., 2015b; McVitty et al., 2015; Kitayama and Constantinou 2018).

A neural network model is prepared for estimation of any structural systems' probability curves. Then, with the help of this model a Python program written, which has simple graphical user interface, so that users can obtain estimation of probability curves for either base shear ratio, maximum isolator displacement ratio or maximum residual isolator displacement ratio. Also, data for the construction of those probability curves can also be obtained using the program written within the scope of this study. Loss and accuracy value of the neural network model are given, and estimation of probability curves by the program is compared with the real data, and results show that prediction of the given program is very close to those of real data. This program can be used to obtain cumulative distribution function of base shear increase of friction pendulum isolated structures to consider increased loads in design phase without performing large number of nonlinear response history analyses.

2. SYSTEMS AND SIMULATION OF GROUND MOTION

2.1. Analyzed models

Structural systems used in this study consist of simple rigid block assumption, a mass on top of a single friction pendulum isolator, attached to a damper to provide damping to the system. Following Figure 1 illustrates representation of single friction pendulum and mass system. This representation is also valid in bi-directional ground shaking as well, and dashpot and friction isolator work in two dimensions.

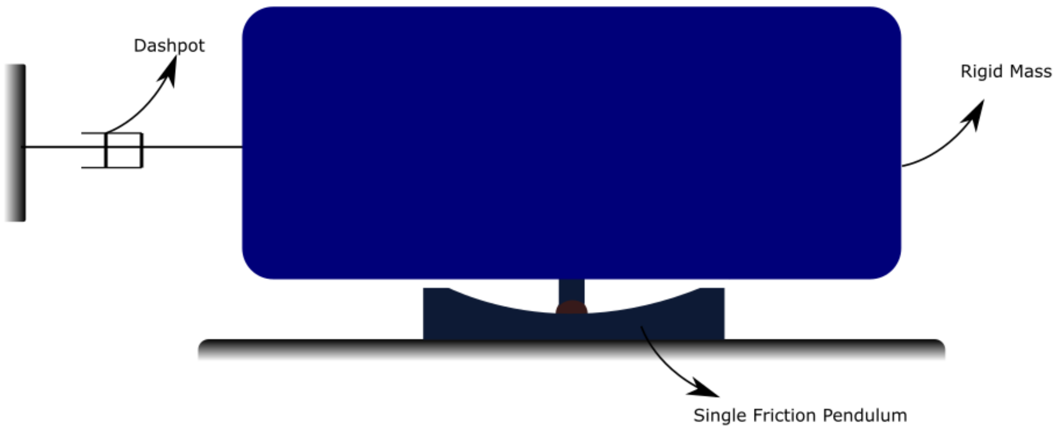
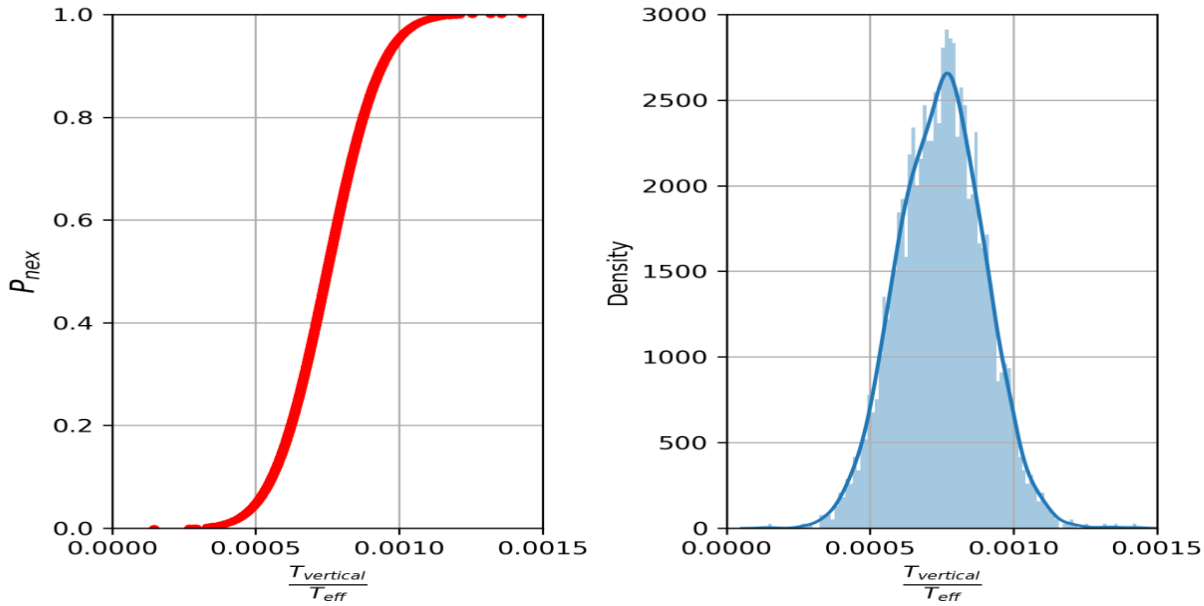


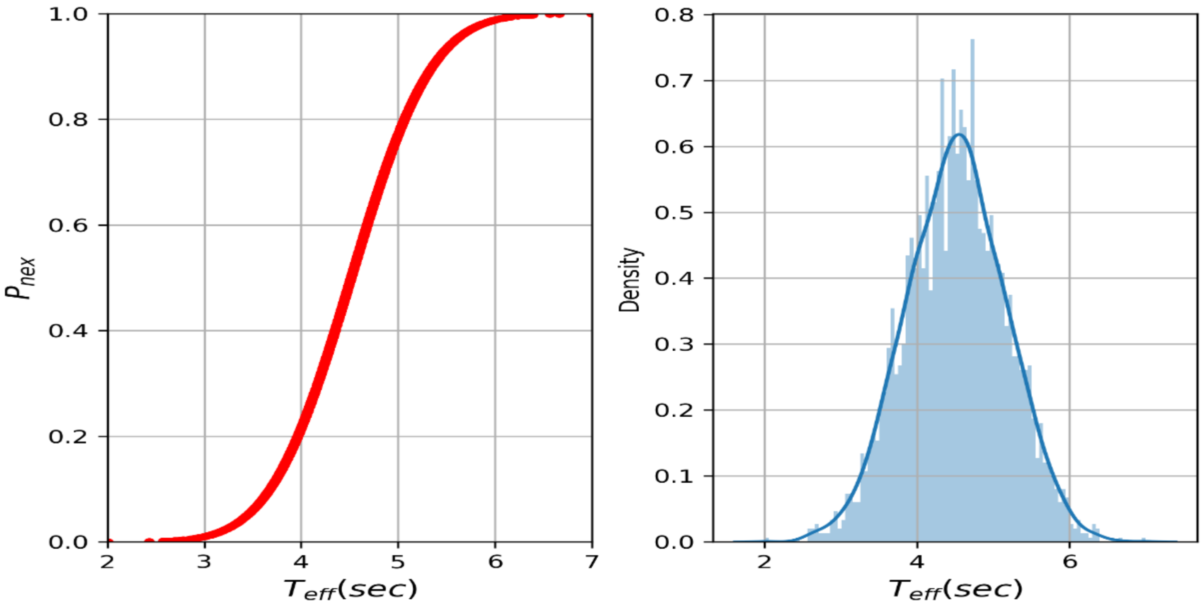
Figure 1:
Rigid mass block and single friction isolator representation

OpenSeesPy (Zhu et al., 2018), which is python library for OpenSees (McKenna, 1997), is used for modeling and analyses. Friction pendulum isolator is modeled with the *singleFPBearing*, while *ViscousDamper* is used for dampers. Three thousand systems are modeled with different properties in terms of effective period, friction coefficient at high velocity, damping ratio, and ratio of vertical structural period to effective period that is calculated based on the post-elastic stiffness of the isolators under constant axial load. Damping ratio is also calculated based on the post-elastic stiffness of the isolators. Radius of the curvatures of isolators are calculated based on the corresponding effective period value. Friction coefficient at low velocity is assumed to be two-third of the coefficient at high velocity (Cilsalar and Constantinou 2017) to ensure velocity dependent friction model for the isolation system (Constantinou et al., 1990). These four parameters are randomly produced based on the normal distribution to have wide range of systems that cover combination of different isolator properties. Three thousand values for each parameter are produced, and cumulative distribution functions and density plots are given in Figures 1-4. Effective period values are determined so that mean value is around 4.5 sec and standard deviation is 0.65 sec. Damping ratios are chosen with mean value of 0.4 and standard deviation of 0.1. Friction coefficients are determined considering mean value of 0.075 and standard deviation of 0.0175. Ratio of vertical period to effective period is chosen with mean value of 0.00075 and standard deviation of 0.00015. Cumulative distribution function and density plot of these four parameters are given in Figures 2-5. In these figures, P_{nex} shows probability of non-exceedance of a value selected from the dataset of ratio between vertical and horizontal period ($T_{vertical}/T_{effective}$), effective period of the isolator (μ_{Fast}), friction coefficients at high velocities, and damping ratio (ξ). Density plots depict the gathering of data around a value for each quantity.



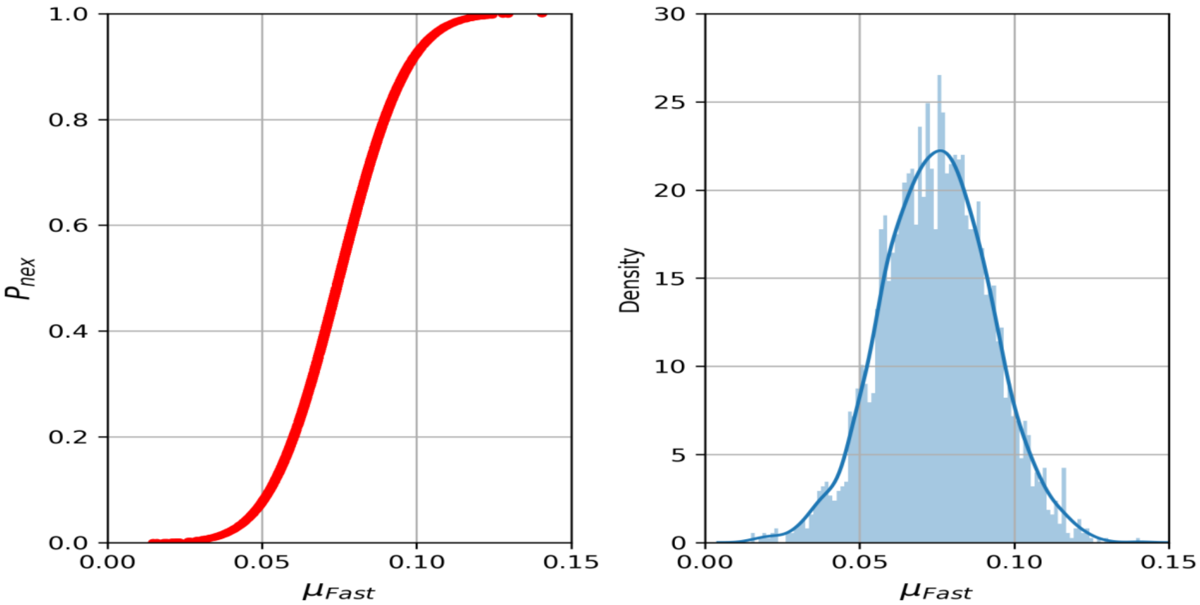
a. *b.*

Figure 2:
Cumulative distribution and density plot of ratio of vertical period to effective period
a. *Cumulative distribution plot* **b.** *Density plot*



b. *b.*

Figure 3:
Distribution of effective period in the data set
a. *Cumulative distribution* ***b.*** *Density distribution*



a. *b.*

Figure 4:
Distribution of friction coefficients at high velocities the data set
a. *Cumulative distribution* ***b.*** *Density distribution*

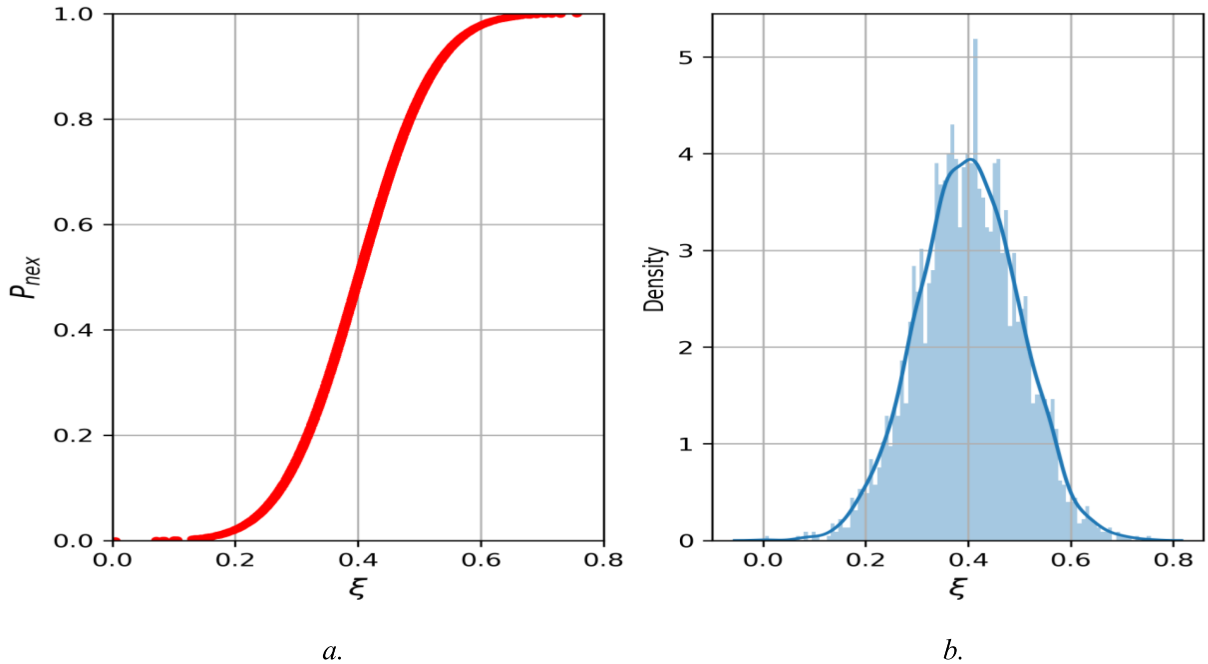


Figure 5:
Distribution of damping ratio in the data set
***a.** Cumulative distribution **b.** Density distribution*

2.2. Ground motions

Each system is analyzed using a hundred ground motions. Simulations are performed with three-dimensional and bi-directional application of ground motions. All records are selected based on random distribution of twenty earthquake events with different magnitudes, source-to-site distance and shear wave velocity in the upper 30 m part of the soil. Each set of events contains five ground motions, hence, total number of analyses is a hundred for each system. A script given by Baker Research Group is used for the selection and scaling of the motions (Baker, 2021). With the given properties of earthquake events, the script returns ground motions record-sequence-number and the scaling factors depending on a target spectrum (Baker and Jayaram 2008; Jayaram and Baker 2010; Jayaram et al., 2011).

Ground motions are obtained from PEER (2021), and scaled according to the factor given by the script. Each components, two horizontal and vertical, are scaled using the same factors so that the ratio of accelerations in horizontal and vertical directions stays as recorded. Distribution of magnitude, source-to-site distance and shear wave velocity of the earthquakes to obtain ground motions is shown in Figure 6. Distribution of these parameters is determined based on the expected seismic hazard in Oakland, California, (Baker et al., 2011) with the event that has 2475-year of return period, 2% probability of exceedance in 50 years. Structural period range considered is the key parameter for the selection of magnitude, and distance values, which consists the base for the seismic hazard evaluation. Mean magnitude value is 7.5 with a standard deviation of 0.1, while mean value of source-to-site distance is chosen as 14.5 km with the standard deviation of 1 km. Mean value of shear wave velocity in the upper 30m part of the soil is around 360 m/s which is classified as C/D boundary based on NEHRP. Also, standard deviation of the shear wave velocity is around 75 m/s. 5%-damped response spectra of the selected and scaled ground motions in two horizontal and vertical directions are given in Figure 7.

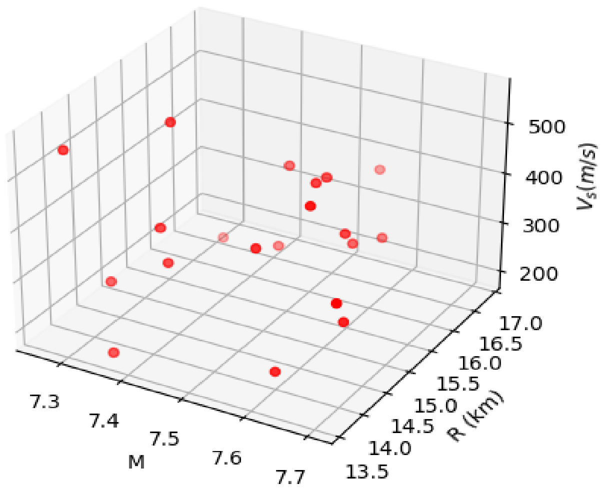
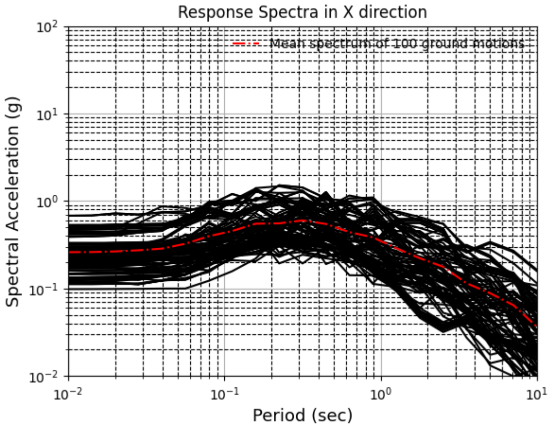
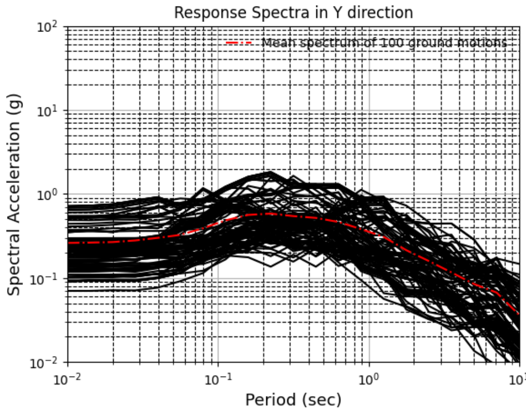


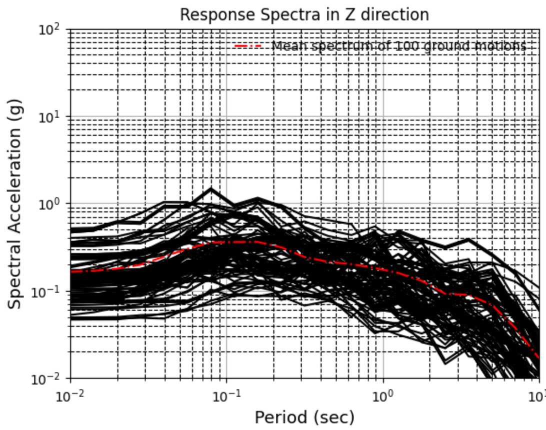
Figure 6:
Magnitude, distance and shear wave velocity distribution of the earthquake events considered to produce ground motions



a.



b.



c.

Figure 7:
Response spectra of selected and scaled ground motions
***a.** X direction **b.** Y direction **c.** Z direction*

3. RESPONSE OF FRICTION ISOLATOR SYSTEM AND THE PREDICTION MODEL

Base shear and isolator displacement are calculated considering quantities in two horizontal directions, and squared root of sum of the squares of the force and displacement values are monitored during the analysis, and maximum value is kept tracking. Each ground motion records are extended ten more seconds with zero value so that all motions are damped out and remaining displacement can be recorded as the residual displacement. Base shear, maximum isolator displacement and residual displacement values of analyses results including vertical ground motions are normalized with those results without vertical motion.

Data of time history analyses are processed and those results with uplift and divergence are removed. Statistical distribution parameters of models are obtained with remaining data after first processing. Mean and standard deviation of log data are, then, obtained. Also, outliers are eliminated from these data for the robust estimation of the parameters for the neural network model. Figure 8 shows correlation between the input and output parameters. In this figure BR, UR and URr are values of base shear, maximum isolator displacement and residual displacement obtained in analyses with vertical ground motions normalized without the vertical component analyses. μ and σ denote standard deviation and mean value of log data to be used for construction of probability curves. α is the ratio of vertical period of friction isolator system to the effective period. The correlation figure shows that mean residual displacement has strong correlation with the coefficient of friction and effective period, while base shear is more related to the effective period of vibration.

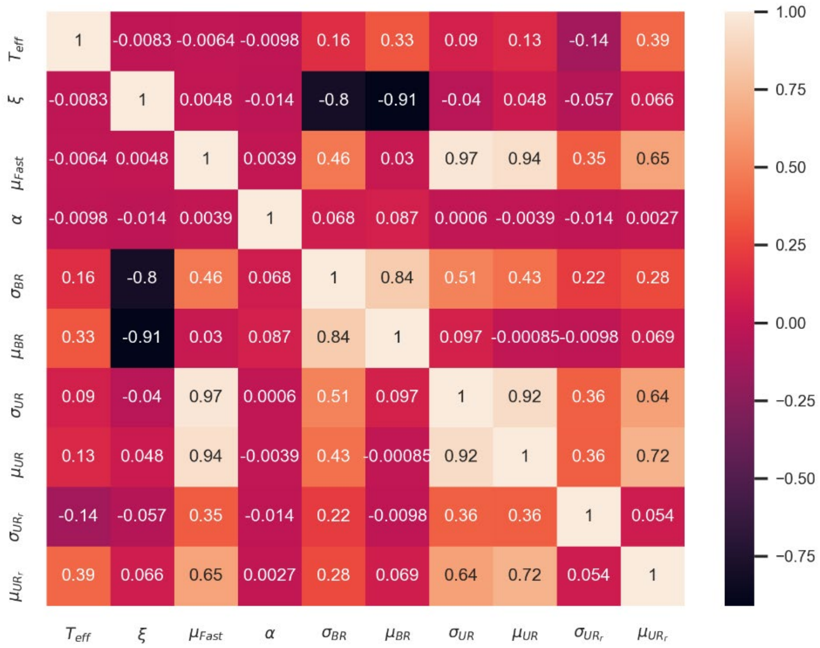


Figure 8:
Correlation relation of the paramaters considered in the study

A multi-layer perceptron model with three layers is utilized in Python using TensorFlow library (Abadi et al., 2015), and this model is trained and saved so that prediction for those systems can be made without complex calculation. Four structural parameters are the inputs of the neural network model, and the output values are standard deviation and mean values for construction of probability curves depending on log-normal distribution. There are three hidden layers within the model. First, second and the third layers have 16, 32, 32 units. The last layer, which is output layer, has six units which are those values to be estimated. 30% of the data is split for testing, and 15% of the training data is also considered as validation set. Loss and accuracy value of the trained model are around 0.0018 and 0.86, respectively. Mean error values of each parameter are given in Table 1, and accuracy and loss plot of the neural network model are given in Figures 9 and 10. Error values related to displacement quantities are observed to be relatively higher compared to base shear ratio.

Table 1. Mean error values for the parameters considered in the study

Parameter	Mean error (%)
σ_{BR}	1.79
μ_{BR}	3.0
σ_{UR}	2.88
μ_{UR}	2.95
σ_{UR_r}	13.57
μ_{UR_r}	9.49

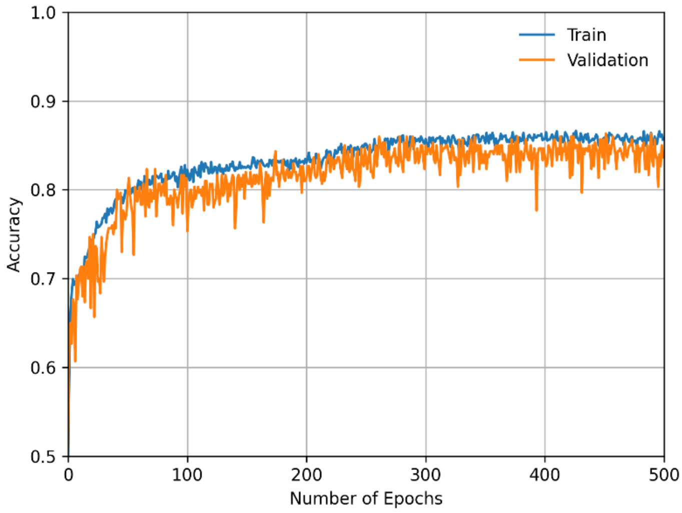


Figure 9:
Train and validation accuracy graph of the model

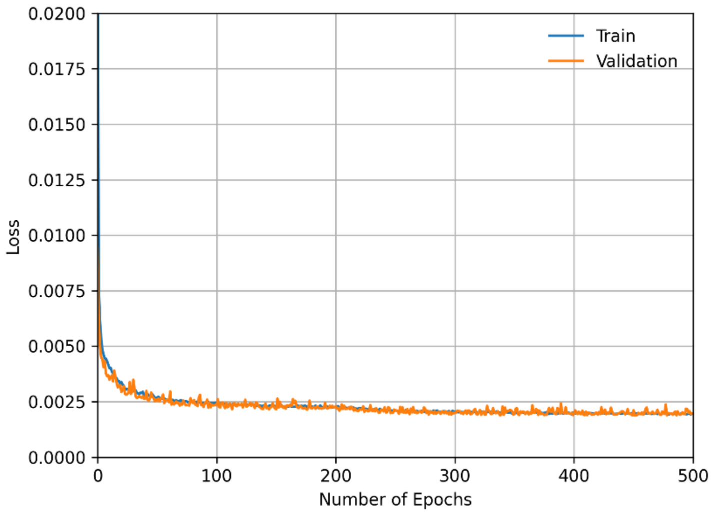


Figure 10:
Train and validation loss graph of the model

Results of the network model is compared with the real data, and Figures 11 and 12 show standard devitaions and mean values normalized with the maximum value in the set for the base shear ratio. There is strong correleation between the observed and predicted results in both standart deviation and mean values.

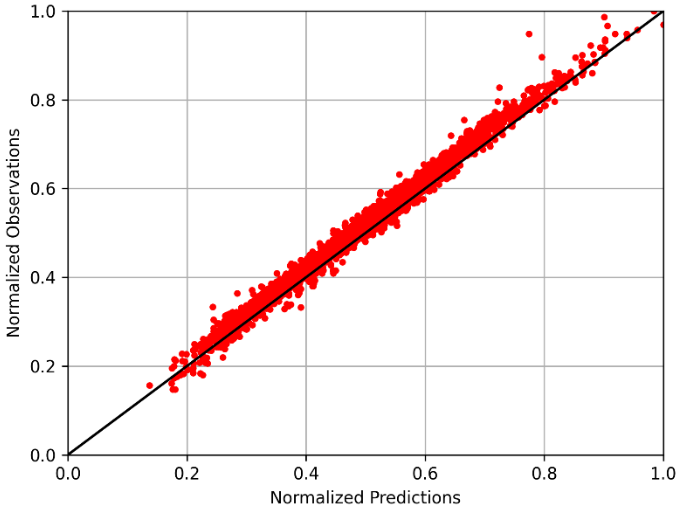


Figure 11:
Normalized observations with respect to normalized predictions of mean values

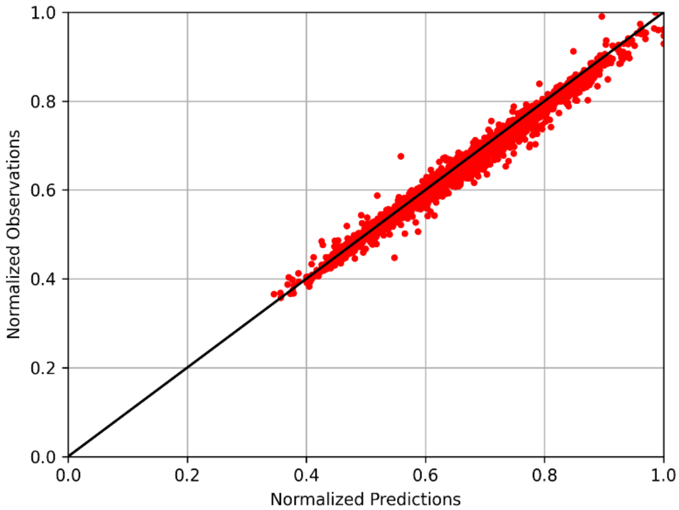


Figure 12:
Normalized observations with respect to normalized predictions of standart deviations

A Python code with the graphical user interface, as illustrated in Figure 13, predicts probability curves for a given system with four parameters. These curves can be constructed for ratios of base shear (BR), maximum isolator displacement (UR), and maximum residual isolator displacement (URr). Output layers have six elements that are standard deviation and mean value for the lognormal distribution of base shear, maximum isolator displacement and residual displacement ratio. Following Figure 14 shows four representative probability curves of base shear ratios predicted by neural network results and observed from the time history analyses.

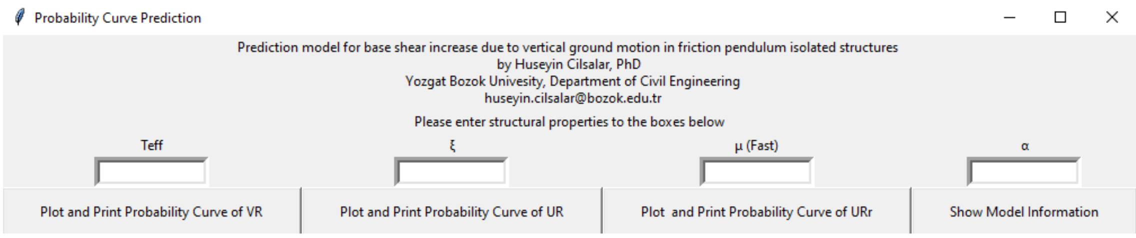


Figure 13:
Graphical user interface of the program for the prediciton of probability curves

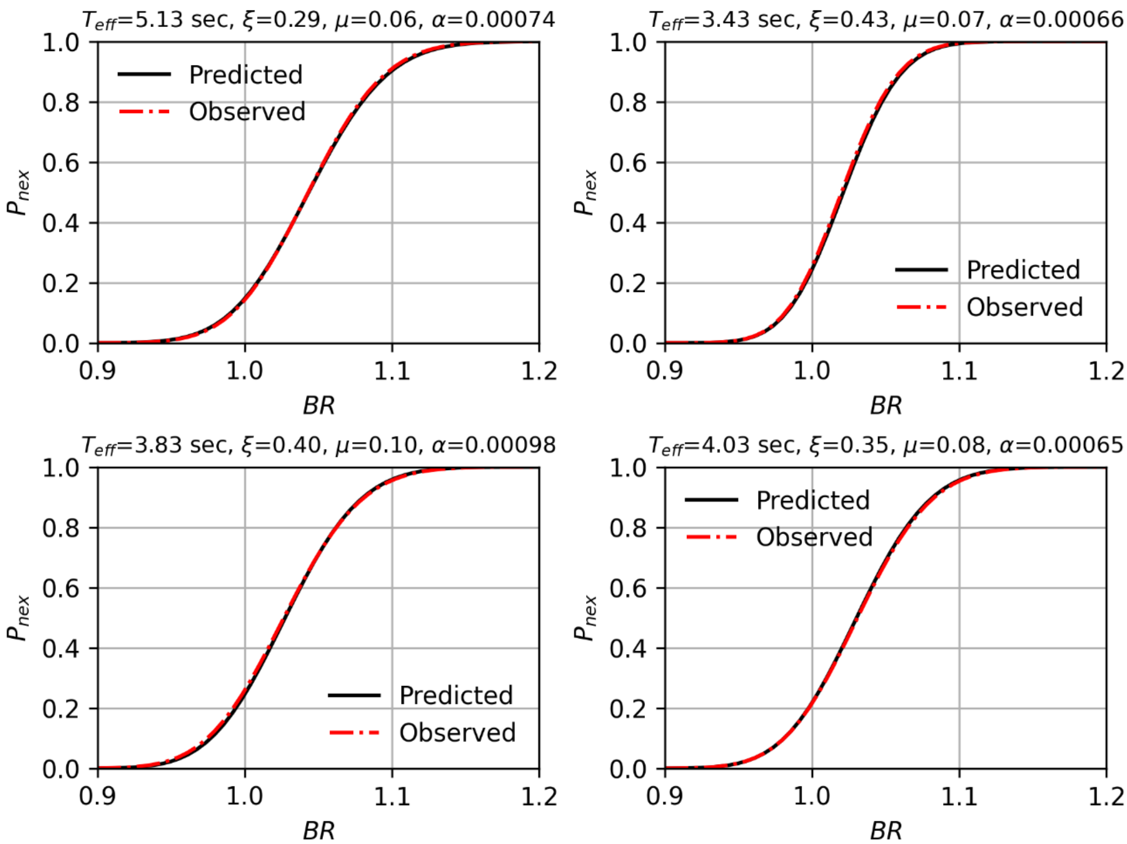


Figure 14:
Comparison of prediction and obervation of probability curves for base shear ratio

4. CONCLUSION

In this paper, three thousand simple rigid block systems are subjected to a hundred ground motions selected based on the seismic hazard in Oakland, California. Analyses were performed with and without the inclusion of the vertical components. Then, base shear, maximum isolator displacement and residual displacement results obtained from three-dimensional analysis were normalized with the bi-directional analyses. Each set of structures' response subjected to selected ground motions are evaluated and standard deviation and mean values are obtained, which are used in obtaining of probability curves based on log-normal distribution. Then, a neural network model is constructed in Python using TensorFlow library and trained. This model is saved and with the help of a script that utilizes a simple program with graphical user interface, probability curves of base shear, maximum isolator displacement and residual displacement ratios are obtained. Users can use those probability curves to decide in base shear increase with its probability to include additional loads on isolator in design phase without computational expense of nonlinear response history analyses on structural frames. Since, model is simple rigid mass, it does not account for overturning moment, second order effect, and it is necessary to analyze three- or two-dimensional structures after designing and modeling of the real model. Error in prediction of isolator maximum and residual displacement is higher compared to base shear, however, model is capable of estimation of base shear with relatively low error. Double concave isolators with same friction coefficients in each surface behave in the same manner with the single friction pendulum isolator. So, those findings given in the study can also be extended for those kind of isolators.

CONFLICT OF INTEREST

Author declares that there is no any conflict of interest.

AUTHOR CONTRIBUTION

Huseyin Cilsalar, as the sole author, conducted all the works in the paper.

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Oguzhan Ayyildiz from Abdullah Gul University commented on training and results of multi-layer perceptron model of the study. This help is gratefully acknowledged.

DATA AVAILABLTY

Data to train deep neural network model with the model's itself, and Python program written for the construction of probability curves are available at: https://github.com/hclslr/FrictionPendulum_Prediction

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