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A New Unit Root Test Against LSTAR Nonlinearity without Threshold

Atilla Hepkorucu[1]

Eşiksiz LSTAR Doğrusal-dışılığına Karşı Yeni Bir Birim Kök Testi

Öz

Çalışmada, eşik içermeyen durağan LSTAR doğrusal-dışı yapısı alternatifine karşı basit bir birim kök testi önerilmiştir. Monte Carlo simülasyonları ile kritik değerler, boyut ve güç özellikleri incelenmiştir. Geliştirilen testin gücü, doğrusal Dickey ve Fuller (DF) (1979) ve doğrusal olmayan Kapetanios, Shin ve Snell (KSS) (2003) birim kök testleri ile karşılaştırılmıştır. Eşik etkisi olmadığı varsayılarak geliştirilen test $\left(F_{LSTAR,c=0}\right)$, karşılaştırılanlara göre daha uygundur. Testin ampirik uygulaması OECD ülkeleri ve Avrupa 1961(i)-1986(iv) endüstriyel üretim verileri için yapılmıştır. Uygulama kısmında kullanılan veriler, LSTAR model yapısına uygun olduğu için seçilmiştir. Çalışmanın literatüre katkısı, eşiksiz LSTAR model yapısına sahip zaman serilerinin birim kök yapısını açıklayan alternatif bir test mekanizması elde etmektir. Ampirik uygulama sonuçları göstermektedir ki, testin kullanımı ilgili model yapısı altında uygundur.

Anahtar Kelimeler: LSTAR Model, Birim Kök Testi, Doğrusal-dışılık, Endüstriyel Üretim

JEL Kodları: C1, C15, C22

A New Unit Root Test Against LSTAR Nonlinearity without Threshold

Abstract

In this paper, a simple unit root test was proposed against the alternative of stationary LSTAR nonlinearity without a threshold effect. The critical values, size and power properties were examined with Monte Carlo simulations. The power of the developed test was compared with linear Dickey and Fuller (DF) (1979) and nonlinear Kapetanios, Shin and Snell (KSS) (2003) unit root tests. The developed test $\left(F_{LSTAR,c=0}\right)$ assumed that no-threshold effect is more suitable than the comparable ones. The empirical application of the test was carried out for industrial production data from OECD countries and Europe 1961(i) - 1986(iv). The data used in the application part has been chosen, because it is suitable for the LSTAR model structure. The contribution of the study to the literature is to obtain an alternative test mechanism that explains the unit root structure of time series LSTAR model structure without a threshold. Empirical application results show that the use of the test is appropriate under the relevant model structure.

Keywords: LSTAR Model, Unit Root Test, Nonlinearity, Industrial Production

JEL Codes: C1, C15, C22

Araştırma ve Yayın Etiği Beyanı	Yazar, makalede etik kurul raporuna gerek olmadığını belirtmektedir.
Yazarların Makaleye Olan Katkıları	Yazar, makalenin tamamının kendisi tarafından hazırlandığını beyan etmiştir.
Çıkar Beyanı	Yazar kendisi ve üçüncü kişiler için herhangi bir çıkar çatışması olmadığını beyan etmiştir.

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1. Introduction

The introduction of smooth regime switching is aimed at the STAR (smooth threshold autoregressive) model which is constructed by Teräsvirta and Anderson (1992). Using a nonlinear process is an alternative description of smooth adjustment, because the adjustment is continuous over time. Recently, it has been strongly accepted that economic time series can be modeled nonlinearly. This nonlinearity can be caused by regime changes, business cycle, structural breaks or the data generation process. The differentiation of the data movements in the nonlinear form can be explained by STAR instead of discrete change. It should also be considered that the transition between regimes may continue smoothly for several periods.

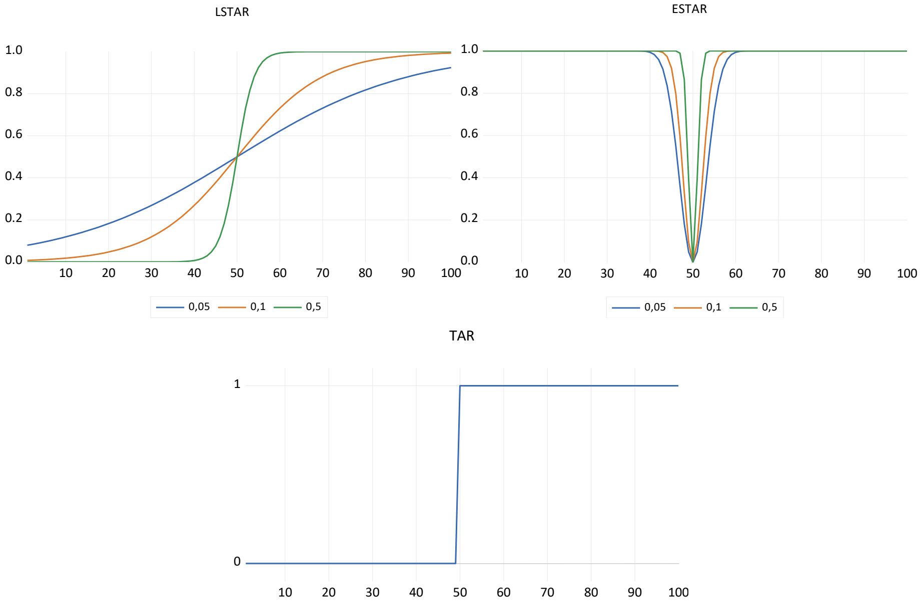
The TAR (threshold autoregressive) model is a predecessor of the STAR model that enables transition across regimes only. However, the transition between regimes does not occur suddenly and the transition mechanism needs to be explained. But it cannot explain the transitional phase. In the TAR model, the transition between regimes is sharply gradual. The basic difference between the STAR and TAR models is that the adjustment approaches smoother rather than the discrete adjustment to STAR model. The smooth transition is accepted as more adequate for economic cycles (Henry and Shields, 2004, p. 486). In the STAR model structure, it is allowed the change of the autoregressive coefficient. This change is determined by a stimulus variable; It is determined by the deviation function, which is determined by the distance from the threshold value. If the variation of the series is smooth within itself, two basic situations are taken into account while modeling the structure in applications. If the determined function is exponential, the ESTAR structure, which takes into account the quadratic distance, will be considered. If it is similar in structure to the logistic function, taking into account the Euclidean distance, The STAR structure is used. Despite the smooth transition between the regimes of the STAR models, if the distance to the threshold value gets larger, it gains a segmented linear appearance and may appear suitable for TAR models. This is where the most important difference between ESTAR and LSTAR structures lies. So, to understand the difference of the models, mathematical structures can be examined in the following equations. The transition functions of these structures with different θ mean reversion speeds as $0.05, 0.1, 0.5$ are visualized in Figure 1.

TAR model: $y_t = \begin{cases} \beta_1 y_{t-1} + \varepsilon_t & if \;\; y_{t-1} \leq c \\ \beta_2 y_{t-1} + \varepsilon_t & if \;\; y_{t-1} > c \end{cases}$

Logistic STAR (LSTAR) model: $y_t = \beta y_{t-1} + \gamma y_{t-1}(1 + \exp(-\theta(y_{t-d} - c)))^{-1} + \varepsilon_t$

Exponential STAR (ESTAR) model: $y_t = \beta y_{t-1} + \gamma y_{t-1}(1 - \exp(-\theta(y_{t-d} - c)^2)) + \varepsilon_t$

Figure 1: TAR, ESTAR and TAR model transition functions with different mean reversions



In addition, STAR model can reflect the arbitrage behavior, depict economic variables explained by the rates, and illustrate the complicated and chaotic dynamic behavior of economic variables. This topic was taken into account by Gregoriou and Kontonikas (2009), Guidolin et al. (2009), Yoon (2010) and Pavlidis et al. (2013). The different transition functions in the STAR model reveal different types of regime-switching models. If a logistic function is used as a transition function, the LSTAR model is created (Dijk et al., 2002, p. 2-4). The LSTAR model implies that the time series could have different regimes, which may have different dynamics, but the transition mechanism links them smoothly. In addition, the ESTAR model suggests that regimes with similar dynamics can contain different transition dynamics in the data generation process (Sarantis, 1999, p. 33). These two models are frequently used in various applications lately to explain nonlinear behavior of economic variables. These potentially adaptable models were examined in the studies of Teräsvirta (1994), Escribano and Jordá (2001), Chen and Kuan (2002) and Chen (2003). Applications of LSTAR models are allowed for regime-switching behavior and to describe modeling asymmetric cycles such as expansions and recessions appropriate. Previous reviews of the smooth transition model include Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993), Leybourne et al. (1998), Potter (1999) and Hall et al. (2001).

There are two components in the STAR model. One of these allows switching regimes in a smooth manner and also can be described as a monotone function. The other part is related to the behavior of two different regimes continuum, associated with the extreme values of the transition function. The exhibition of the two regimes is concretized depending on whether it is above or below the equilibrium level. In addition, the adjustment speed varies with the deviation from the equilibrium permanently (Puspaningrum, Lin and Gulati, 2013, p. 558). The logistic transition function characterizes the nonlinear part of the LSTAR model and it is identified by probability values. It also implies the asymmetric behavior of the time series. For defining and modeling nonlinear time series should take a nonlinear-specific approach.

Otherwise, describing a nonlinear time series with conventional linear approaches will cause modeling error. This error also implies that the behavior of the transition function is ignored.

Zhang (2013) applied Monte Carlo simulation to show the linear spurious regression phenomenon between two independent partial units for TAR, LSTAR and ESTAR data generation processes (DGP) based on the model. Under these conditions, the re-estimation of regressions by adding an auto-regressive parameter removes this phenomenon. This study also points out that these nonlinear model structures can be identified with overestimated linear restrictions by adding a lag. Adding higher order lags can help to explain the time series sufficiently. However, even assuming that there is no modeling error, the explanation of the nature of changing regimes will be neglected. Therefore, if a nonlinear model can be determined by using linear conditions, its unit root structure can be examined by using standard linear tests under the null hypothesis by accepting power loss. Leybourne et al. (1998), Hamori and Tokihisa (1997) and Nelson et al. (2001) show that the standard linear tests are affected by size distortions if there is a structural break or regime switching in the time series. Many unit root tests have also been developed under nonlinear restrictions. Pippenger and Goering (1993), Balke and Fomby (1997), Enders and Granger (1998), Berben and van Dijk (1999), Caner and Hansen (2001) and Lo and Zivot (2001) indicate that standard linear unit root tests are performed poorly in nonlinear situations. However, there are studies that show the opposite. For example, Zhang (2016) declares that the ADF and KSS have better power characteristics than the PP, M-TAR, and inf-t tests for nonlinear DGP based on the model. Although the ADF test is chosen as the most robust test among the linear and nonlinear stationary tests, the power properties of the near-unit root process is known to be low. In this case, the need to examine the power properties of the ADF unit root test together with other tests for structures with nonlinear DGP draws attention.

2. Unit Root Structure under LSTAR Nonlinearity

The LSTAR model is assumed as follows. Let the model structure be redefined by considering the first differences of the time series. It is a logistic function that gives nonlinearity to the structure, and the transition variable is determined as y_{t-d}. For the transition variable, it was assumed that d = 1 to clarify the empirical practices.

$$y_t = \beta y_{t-1} + \gamma y_{t-1}(1 + \exp(-\theta(y_{t-d} - c)))^{-1} + \varepsilon_t \quad (1)$$

$$\triangle y_t = \phi y_{t-1} + \gamma y_{t-1}(1 + \exp(-\theta(y_{t-d} - c)))^{-1} + \varepsilon_t \quad (2)$$

The first-order autoregressive term becomes as $\phi = \beta - 1$. In the LSTAR model θ refers to the measure of reversion speed. The threshold parameter c is another term of the ESTAR model that can be explained as an exhibition of time series behavior such as a repressed or broken appearance when the time-series approach it. This can be interpreted as a change in the regime when the threshold approached. When moving away from threshold c, the instability of the time series increases. If the stationary of the return series is being questioned, it can be assumed that this assumption is correct. As such, these parameters in the LSTAR model (the reversion speed and threshold value) must be identified. In this way, the unit root structure of time series can be accurately defined. Under the assumption of $\theta \geq 0$ in the model structure, the realization of $\gamma < 0$ and $\gamma + \phi < 0$ indicates global stationary. In the following part, developed unit root tests for LSTAR model structure in recent years are examined.

Leybourne et al. (1998) examined stationarity around a linear trend with an abrupt break by describing the situation as a smooth transition one trend to another. They developed a Dickey-Fuller type unit root test against LSTAR nonlinearity by claiming under a stationary time series, including various forms of structural change in the deterministic structure. The nature of structural change can be described as a smooth transition that considers aggregate behavior to grade change of deterministic components rather than instantaneous ones. It can be seen in many economic time series. By using the nonlinear least squares (NLS) algorithm, estimation of deterministic component parameters is completed, and the residuals are calculated. The lag degrees of the obtained residuals are determined using the ordinary least squares (OLS) approach to apply the Dickey-Fuller type test mechanism. The power of the developed test compared with its linear-type origin of ADF and becomes more successful. This approach is suitable for determining model parameters of a nonlinear LSTAR structure; however, the aim is missed. The efficiency of approach is dependent on determining correctly the deterministic parameters of LSTAR. If the parameters obtained by NLS are biased, the model determination and unit root test approach will be misleading. The advantage of this approach is seen that it is quite practical to use.

Sollis (2004) developed a unit root test that combines Enders and Granger (1998) and Leybourne et al. (1998). Their methodology uses the Dickey-Fuller tests generalize re-defining nonlinearity as an alternative. Enders and Granger (1998) modeled asymmetric adjustment toward a long-run attractor as a threshold. The value of combining these similar tests is shown with an empirical application. The results were influenced by rejection of the unit-root hypothesis for the chosen data. However, the combination of tests reveals statistically significant evidence against the unit root hypothesis for all time series. The methodology is similar to that Leybourne et al. (1998), but the difference is a dummy variable that takes value according to the threshold value of obtained residuals. The power characteristics of combined test are compared with its origins, as well as the ADF. Briefly, the most important result of the study is that ignoring large breaks in deterministic parameters leads to a significantly greater loss in power than ignoring asymmetric adjustment when testing for a unit root. Another remarkable point is that the ADF unit root test yields results close to those of the comparative tests when the break is small. As summarized previously, ignoring breaks is much more effective in linear unit root tests because of the power loss. It was concluded that ADF are a good alternative when testing the power of unit root tests against LSTAR nonlinearity when structural breaks are not taken into account.

Eklund (2003) aimed to determine a unit root test against the alternative of logistic smooth transition autoregressive nonlinearity. The novelty of this approach is that nonlinearity is explained under the Taylor expansion. The obtained auxiliary regression is redefined under the Taylor expansion of the difference equation to eliminate the definition problem defined by Luukonen et al. (1988). The use of the Taylor expansion, especially the ESTAR model structure in unit root tests is an application that is included in many nonlinear unit root tests. This can also be seen in the tests listed by KSS (2003), Sollis (2009) and Kruise (2011). Thus, the problem of determining the coefficients describing the nonlinear structure is eliminated by redefining the difference equation. The developed test structure considers determining unit root structure under nonlinearity. It is seen as the obtained auxiliary regression similarity with the ADF test with constant, but also including one lag of $\triangle y_{t-1}$. However, when Taylor expansion is applied under the ESTAR structure, the auxiliary regressions turn into two or higher-order polynomial forms. Therefore, according to the model, the nonlinearity of the ESTAR model is more effective than that of the LSTAR model. The auxiliary regression obtained in this study is linear. It is thought that it would be incomplete to explain the nonlinear form in a linear structure. Therefore, our study was developed from this point. Unit root applications, which take into account the recently developed nonlinearity of LSTAR, have been examined, and an alternative test application has been implemented.

2.1. Unit Root Tests against LSTAR Nonlinearity under the Assumptions of Symmetric-Asymmetric Reversions and Threshold Effect.

A logistic smooth transition autoregressive model without threshold $(c = 0)$ was identified by inspiration of the redefinition of the first differences of time series by Taylor expansion recently used. The aim of this redefinition extinguish of the parameter estimation problem was introduced by Davies (1987), KSS (2003) and others expressed in nonlinear models. This is because the values of the nonlinear parameters are not known. The reason of assuming threshold value is assumed to be zero $c = 0$ to avoid the same problem. Our aim is not to estimate the LSTAR model parameters correctly, but to determine its unit root structure in a correct and simple way. In many applications, the transition function parameter y_{t-d} is accepted as y_{t-1} for ease of solution. This is accepted in the continuation of the study.

$$y_t = \beta y_{t-1} + \gamma y_{t-1}(1 + \exp(-\theta(y_{t-d})))^{-1} + \varepsilon_t \qquad (1)$$

This new approach is inspired by Eklund (2003) by Taylor expansion to better explain nonlinearity than standard linear models. Thus, the explanation of nonlinearity reaches a more reasonable auxiliary model. Similarly, we assumed a threshold value of zero in a nonlinear unit root structure. Under this assumption, the first difference in the time series is.

$$\triangle y_t = \phi y_{t-1} + \gamma y_{t-1}(1 + \exp(-\theta(y_{t-d})))^{-1} + \varepsilon_t \qquad (2)$$

For $\beta = 1$ or $\phi = 0$ shows that the unit root structure. If we assume that there is a unit root in the time series, the first difference is obtained as follows:

$$\triangle y_t = \gamma y_{t-1}(1 + \exp(-\theta(y_{t-d})))^{-1} + \varepsilon_t$$
(2.1)

The first-order Taylor approximation around $y = 0$ is applied to the auxiliary regression of the LSTAR structure. The auxiliary regression parameters were simplied as $\beta_1 = \frac{1}{2}\theta$ and $\beta_2 = \frac{1}{4}\gamma\theta$, so the s-shaped transition mechanism can mimic the transition mechanism. Therefore, auxiliary regression occurs in with two parts, one of which is linear by expression of y_{t-1} and the other is nonlinear by expression of${y_{t-1}}^2$. β_1, β_2 terms contain θ the speed of reversions. β_2 term also contains γ parameter. In short, the following situations where$\beta_1 > 0$, $\beta_2 < 0$ are pointed out on the coefficient values of the auxiliary regression obtained.

$$\triangle y_t = \frac{1}{2}\theta y_{t-1} + \frac{1}{4}\gamma\theta {y_{t-1}}^2 + \varepsilon_t \qquad (2.2)$$

$$\triangle y_t = \beta_1 y_{t-1} + \beta_2 {y_{t-1}}^2 + \varepsilon_t \qquad (2.2.1)$$

Therefore, the null hypothesis is $H_0\colon \beta_1 = \beta_2 = 0$ against the alternative $H_0\colon \beta_1 \neq \beta_2 \neq 0$. The standard Wald test is appropriate for deriving the critical values. This is because standard critical values of the F-test cannot be used because the null hypothesis contains a unit root structure. Let denote the test statistics for the null hypothesis for zero mean, non-zero mean and deterministic trend cases. y_t is replaced with $\hat{y}_t = y_t - \mu$ for non-zero case. The model constant μ is determined as the mean of time series y_t. y_t is replaced with $\hat{\hat{y}}_t = y_t - \hat{\alpha}_0 - \hat{\alpha}_1 t$ for the deterministic trend case. The model constant and trend term parameters $\hat{\alpha}_0, \hat{\alpha}_1$ for the developed $F_{LSTAR,c=0}$ tests are chosen with least squares from estimation of y_t. Then, a finite sample of critical values was obtained under the data generation process. 10000 times simulated series were used at each sample size $t = 50, 100, 200, 500, 1000\ ,10000$. The critical values of the developed test are tabulated as follows for %1, %5 and %10 statistical significances.

Table 1: The critical values of extended developed $F_{LSTAR,c=0}$ tests

Number of observations	Zero mean model			Non-zero mean model			Deterministic trend model		
	%1	%5	%10	%1	%5	%10	%1	%5	%10
50	6.88	4.73	3.81	6.91	4.66	3.76	6.77	4.74	3.85
100	6.72	4.71	3.86	6.65	4.58	3.67	6.40	4.46	3.63
200	6.60	4.58	3.72	6.24	4.48	3.63	6.66	4.56	3.68
500	6.46	4.37	3.56	6.34	4.34	3.51	5.79	3.96	3.21
1000	6.27	4.36	3.52	6.24	4.42	3.58	5.23	3.62	2.90
10000	6.17	4.24	3.46	5.01	3.44	2.77	5.84	4.10	3.38

We investigated the size properties of the test with the null data generation process. The data generation process was as follows:

$y_t = y_{t-1} + \varepsilon_t$ for $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$ under assumption of $u_t \sim iid(0,1)$.

Where $\rho = \{-0.5, 0, 0.5\}$ for the sample size $t = 50, 100, 200, 500, 1000\ ,10000$, 5000 times were simulated. Test statistics are computed from the relevant model with one lag of Δy_t for $\rho = \{-0.5, 0.5\}$ and the nominal size was set to 5%. It is accepted that models are specified correctly, neither over nor less by in terms of lag. When the results of the size properties are examined, it is seen that the obtained results are examined for all three model structures, and the size distortion is mostly in the model with a deterministic trend. In general, the size distortions increased as the sample size increased.

Table 2: The size properties of extended developed $F_{LSTAR,c=0}$ tests

Number of Observation	Zero mean model			Non-zero mean model			Deterministic trend model		
	0.5	0	-0.5	0.5	0	-0.5	0.5	0	-0.5
50	0.0506	0.0502	0.0600	0.0562	0.0542	0.0474	0.0480	0.0424	0.0506
100	0.0420	0.0534	0.0548	0.0548	0.0410	0.0514	0.0400	0.0460	0.0528
200	0.0550	0.0488	0.0380	0.0506	0.0388	0.0426	0.0364	0.0422	0.0342
500	0.0474	0.0580	0.0596	0.0324	0.0414	0.0468	0.0432	0.0556	0.0502
1000	0.0442	0.0552	0.0386	0.0494	0.0324	0.0292	0.0640	0.0444	0.0558
10000	0.0404	0.0396	0.0582	0.0676	0.0768	0.0826	0.0662	0.0748	0.0838

The finite sample power calculation results were computed for the sample size at $t = 50, 200, 1000$ nominal sizes at 5% and 10000 times were repeated. The results are examined without adding a lag. The combination of $\gamma = \{-1.50, -1.00, -0.50, -0.01\}$ and $\theta = \{0.01, 0.05, 0.10, 0.50, 1.00\}$ values are employed.

In the case of non-zero model, y_t is replaced with $y_t^* = y_t - \hat{\mu}$ to deal with a non-zero mean. $\hat{\mu}$ is mean of y_t. In addition, the deterministic trend model y_t is replaced with $y_t^* = y_t - \hat{\alpha}_1 - \hat{\alpha}_2 t$ to deal with non-zero mean and deterministic trend. $\hat{\alpha}_1$ and $\hat{\alpha}_2$ population parameters were estimated by least squares. KSS (2003), Dickey-Fuller (1979) and developed tests $F_{LSTAR,c=0}$ were compared to determine the power characteristics. The results are tabulated for each model in Table 3, 4 and 5. The specific lag lengths were chosen by using information criteria for all experimentation.

Table 3: The comparison of power properties of KSS (2003), DF (1979) and developed test $F_{LSTAR,c=0}$ (LSTAR) for zero mean models

Zero mean model		**DF**	**KSS**	$F_{LSTAR,c=0}$	**DF**	**KSS**	$F_{LSTAR,c=0}$	**DF**	**KSS**	$F_{LSTAR,c=0}$
		T=50			T=200			T=1000		
$\gamma = -1.50$	$\theta = 0.01$	1.000	0.803	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma = -1.50$	$\theta = 0.05$	1.000	0.799	0.999	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma = -1.50$	$\theta = 0.10$	1.000	0.780	0.999	1.000	0.999	1.000	1.000	1.000	1.000
$\gamma = -1.50$	$\theta = 0.50$	0.991	0.462	0.982	0.997	0.855	0.997	0.996	0.960	0.997
$\gamma = -1.50$	$\theta = 1.00$	0.220	0.035	0.230	0.238	0.024	0.262	0.246	0.005	0.278
$\gamma = -1.00$	$\theta = 0.01$	1.000	0.431	0.966	1.000	0.998	1.000	1.000	1.000	1.000
$\gamma = -1.00$	$\theta = 0.05$	0.999	0.424	0.962	1.000	0.998	1.000	1.000	1.000	1.000
$\gamma = -1.00$	$\theta = 0.10$	0.999	0.419	0.956	1.000	0.996	1.000	1.000	1.000	1.000
$\gamma = -1.00$	$\theta = 0.50$	0.893	0.231	0.787	0.960	0.542	0.966	0.959	0.726	0.966
$\gamma = -1.00$	$\theta = 1.00$	0.142	0.018	0.147	0.157	0.009	0.169	0.154	0.002	0.166
$\gamma = -0.50$	$\theta = 0.01$	0.930	0.085	0.359	1.000	0.886	1.000	1.000	1.000	1.000
$\gamma = -0.50$	$\theta = 0.05$	0.927	0.083	0.358	1.000	0.869	1.000	1.000	1.000	1.000
$\gamma = -0.50$	$\theta = 0.10$	0.916	0.084	0.357	1.000	0.838	1.000	1.000	0.999	1.000
$\gamma = -0.50$	$\theta = 0.50$	0.426	0.037	0.205	0.561	0.122	0.549	0.596	0.132	0.629
$\gamma = -0.50$	$\theta = 1.00$	0.083	0.005	0.074	0.096	0.004	0.097	0.100	0.01	0.097
$\gamma = -0.01$	$\theta = 0.01$	0.072	0.001	0.038	0.104	0.002	0.030	0.361	0.009	0.052
$\gamma = -0.01$	$\theta = 0.05$	0.071	0.001	0.040	0.098	0.001	0.024	0.338	0.008	0.056
$\gamma = -0.01$	$\theta = 0.10$	0.057	0.001	0.042	0.078	0.001	0.038	0.243	0.006	0.058
$\gamma = -0.01$	$\theta = 0.50$	0.041	0.001	0.050	0.055	0.001	0.041	0.113	0.003	0.057
$\gamma = -0.01$	$\theta = 1.00$	0.046	0.000	0.053	0.056	0.001	0.044	0.116	0.002	0.060

Note: In the ADF test, the critical values were used -1.947 for 50 observations, -1.942 for 200 observations and -1.941 for 1000 observations. The critical value was used -2.22 in the KSS test. In the developed test, the critical values were used 4.73 for 50 observations, 4.58 for 200 observations and 4.36 for 1000 observations.

Table 4: The comparison of power properties of KSS (2003), DF (1979) and developed test $F_{LSTAR,c=0}$ (LSTAR) for non-zero mean models

Non-zero mean model		DF	KSS	$F_{LSTAR,c=0}$	DF	KSS	$F_{LSTAR,c=0}$	DF	KSS	$F_{LSTAR,c=0}$
		T=50			T=200			T=1000		
$\gamma=-1.50$	$\theta=0.01$	1.000	0.940	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma=-1.50$	$\theta=0.05$	1.000	0.941	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma=-1.50$	$\theta=0.10$	1.000	0.936	0.999	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma=-1.50$	$\theta=0.50$	0.989	0.812	0.984	0.999	0.954	0.999	0.999	0.977	0.999
$\gamma=-1.50$	$\theta=1.00$	0.045	0.005	0.098	0.043	0.005	0.194	0.057	0.030	0.036
$\gamma=-1.00$	$\theta=0.01$	1.000	0.716	0.969	1.000	0.999	1.000	1.000	1.000	1.000
$\gamma=-1.00$	$\theta=0.05$	1.000	0.712	0.970	1.000	0.999	1.000	1.000	1.000	1.000
$\gamma=-1.00$	$\theta=0.10$	0.999	0.698	0.966	1.000	0.998	1.000	1.000	1.000	1.000
$\gamma=-1.00$	$\theta=0.50$	0.904	0.554	0.829	0.985	0.808	0.989	0.986	0.846	0.989
$\gamma=-1.00$	$\theta=1.00$	0.039	0.005	0.210	0.053	0.005	0.134	0.066	0.005	0.172
$\gamma=-0.50$	$\theta=0.01$	0.925	0.258	0.369	1.000	0.972	1.000	1.000	1.000	1.000
$\gamma=-0.50$	$\theta=0.05$	0.925	0.254	0.365	1.000	0.970	1.000	1.000	1.000	1.000
$\gamma=-0.50$	$\theta=0.10$	0.922	0.250	0.375	1.000	0.956	1.000	1.000	1.000	1.000
$\gamma=-0.50$	$\theta=0.50$	0.046	0.006	0.093	0.047	0.006	0.173	0.783	0.651	0.780
$\gamma=-0.50$	$\theta=1.00$	0.050	0.005	0.057	0.046	0.004	0.087	0.091	0.066	0.092
$\gamma=-0.01$	$\theta=0.01$	0.075	0.008	0.043	0.096	0.012	0.029	0.383	0.055	0.049
$\gamma=-0.01$	$\theta=0.05$	0.070	0.008	0.042	0.102	0.011	0.032	0.357	0.055	0.052
$\gamma=-0.01$	$\theta=0.10$	0.045	0.004	0.056	0.088	0.010	0.027	0.295	0.044	0.045
$\gamma=-0.01$	$\theta=0.50$	0.043	0.004	0.057	0.061	0.005	0.044	0.081	0.006	0.051
$\gamma=-0.01$	$\theta=1.00$	0.051	0.006	0.053	0.062	0.005	0.043	0.080	0.005	0.051

Note: In the ADF test, the critical values were used as -2.921 for 50 observations, -2.875 for 200 observations and -2.864 for 1000 observations. The critical value was used -2.93 in the KSS test. In the developed test, the critical values were used 4.66 for 50 observations, 4.48 for 200 observations and 4.42 for 1000 observations.

Table 5: The comparison of power properties of KSS (2003), DF (1979) and developed test $F_{LSTAR,c=0}$ (LSTAR) for deterministic trend models

Deterministic trend model		DF	KSS	$F_{LSTAR,c=0}$	DF	KSS	$F_{LSTAR,c=0}$	DF	KSS	$F_{LSTAR,c=0}$
		T=50			T=200			T=1000		
$\gamma=-1.50$	$\theta=0.01$	0.998	0.825	0.999	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma=-1.50$	$\theta=0.05$	0.999	0.823	0.999	1.000	1.000	1.000	1.000	1.000	1.000
$\gamma=-1.50$	$\theta=0.10$	0.999	0.820	1.000	1.000	0.999	1.000	1.000	1.000	1.000
$\gamma=-1.50$	$\theta=0.50$	0.945	0.678	0.990	0.999	0.922	1.000	0.999	0.988	0.999
$\gamma=-1.50$	$\theta=1.00$	0.091	0.001	0.096	0.097	0.001	0.106	0.344	0.094	0.416
$\gamma=-1.00$	$\theta=0.01$	0.898	0.476	0.963	1.000	0.999	1.000	1.000	1.000	1.000
$\gamma=-1.00$	$\theta=0.05$	0.901	0.472	0.962	1.000	0.999	1.000	1.000	1.000	1.000
$\gamma=-1.00$	$\theta=0.10$	0.887	0.470	0.963	1.000	0.997	1.000	1.000	1.000	1.000
$\gamma=-1.00$	$\theta=0.50$	0.673	0.241	0.823	0.973	0.753	0.990	0.983	0.819	0.991
$\gamma=-1.00$	$\theta=1.00$	0.077	0.001	0.085	0.156	0.003	0.189	0.163	0.018	0.196
$\gamma=-0.50$	$\theta=0.01$	0.310	0.103	0.366	1.000	0.907	1.000	1.000	1.000	1.000
$\gamma=-0.50$	$\theta=0.05$	0.305	0.101	0.361	1.000	0.897	1.000	1.000	1.000	1.000
$\gamma=-0.50$	$\theta=0.10$	0.294	0.100	0.350	0.999	0.878	1.000	1.000	1.000	1.000
$\gamma=-0.50$	$\theta=0.50$	0.190	0.009	0.195	0.544	0.144	0.596	0.626	0.128	0.641
$\gamma=-0.50$	$\theta=1.00$	0.060	0.002	0.063	0.057	0.001	0.057	0.056	0.001	0.099
$\gamma=-0.01$	$\theta=0.01$	0.051	0.002	0.041	0.049	0.002	0.029	0.080	0.012	0.112
$\gamma=-0.01$	$\theta=0.05$	0.052	0.002	0.037	0.052	0.001	0.037	0.080	0.010	0.098
$\gamma=-0.01$	$\theta=0.10$	0.048	0.001	0.042	0.053	0.002	0.039	0.068	0.003	0.085
$\gamma=-0.01$	$\theta=0.50$	0.047	0.001	0.048	0.050	0.001	0.040	0.058	0.001	0.085
$\gamma=-0.01$	$\theta=1.00$	0.052	0.002	0.049	0.050	0.002	0.043	0.058	0.004	0.067

Note: In the ADF test, the critical values were used -3.502 for 50 observations, -3.432 for 200 observations and -3.414 for 1000 observations. The critical value was used -3.40 in the KSS test. In the developed test, the critical values were used 4.74 for 50 observations, 4.56 for 200 observations and for 3.62 1000 observations.

It was determined that the power characteristics of the three tests were similar. However, when all the results were evaluated under the selected conditions, it was seen that the developed test $F_{LSTAR,c=0}$ stood out a more. $F_{LSTAR,c=0}$ was found to be as successful as its alternatives under the selected conditions. As the γ parameter of the LSTAR model approaches zero, the power of the evaluated tests decreases. As the speed parameter θ of the LSTAR model approaches zero, the power of the $F_{LSTAR,c=0}$ increases. To summarize, the performance of the developed test is revealed when the transition speed slows down, that is, the transition becomes smoother. The high speed of the transition indicates that the model transitions suddenly between regimes. Under model structures with zero mean, non-zero mean, and deterministic trend variables, the evaluated tests show similar power behaviors.

3. Empirical results

$F_{LSTAR,c=0}$ is applied to the four-quarter differences industrial production growth rates of Austria, Belgium, Canada, Germany, Norway, Sweden, United States of America, Japan and Europe (OECD) which are sourced from the OECD database[2]. In this study, industrial production growth rates were used only as an application tool. The importance, its elements or its effects of these economic data for the national economies are not included.

The macroeconomic rates from 1961 (i) to 1986 (iv) which were used by Teräsvirta and Anderson (1992). The results of this study show that these series can be successfully expressed using the LSTAR model structure. The logarithmic index series was assumed to be stationary by after taking the seasonal difference in the main paper, so we applied the same procedure. The main difference in the series used is that the base year taken for the creation of the index is 2015; the behavior of the data will remain the same, and only their values will change. The unit root structure of the series was analyzed using the developed test, ADF and KSS, and the results obtained were tabulated. Teräsvirta and Anderson (1992) chose a specific lag length by using Akaike information criteria. In all cases, we accept their modeling to investigate the unit root structure.

[2] The data were taken from https://data.oecd.org/industry/industrial-production.htm

Figure 2: The four-quarter differences Industrial production growth rates of selected countries

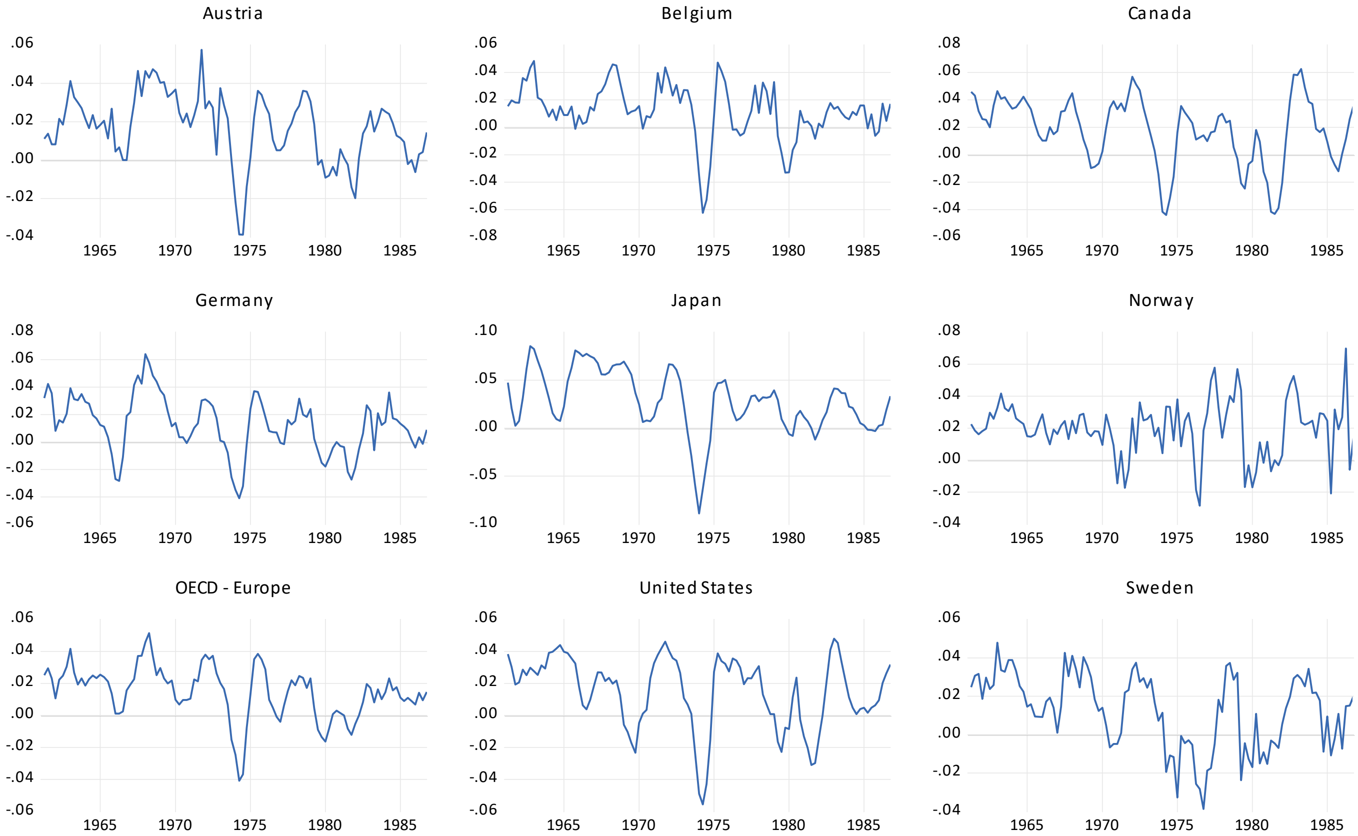


Table 6: The Unit root tests results of Industrial production growth rates

Countries	Zero mean model			Non-zero mean model			Deterministic trend model		
	KSS	**ADF**	$F_{LSTAR,c=0}$	**KSS**	**ADF**	$F_{LSTAR,c=0}$	**KSS**	**ADF**	$F_{LSTAR,c=0}$
United States	-1.670	-1.655***	5.337**	-2.773***	-2.477	3.909***	-2.679	-2.357	3.913***
	k=12	k=12	k=12	k=12	k=12	k=12	k=12	k=12	k=12
Sweden	-2.002***	-2.079**	2.758	-2.252	-2.568	3.283***	-2.245	-2.598	3.480
	k=8	k=8	k=8	k=8	k=8	k=8	k=8	k=8	k=8
Norway	-1.041	-1.041	1.042	-4.693*	-4.642*	10.769*	-4.641*	-4.629*	10.855*
	k=12	k=12	k=12	k=7	k=7	k=7	k=7	k=7	k=7
Japan	-1.778	-0.936	2.586	-2.759***	-1.286	2.553	-2.791	-2.666	4.228***
	k=12	k=12	k=9	k=12	k=12	k=9	k=12	k=10	k=10
Germany	-1.254	-2.556**	5.297**	-2.360	-2.854*	5.706**	-2.740	-3.169***	8.976*
	k=4	k=4	k=4	k=4	k=8	k=4	k=7	k=8	k=6
Europe (OECD)	-1.344	-1.231	1.386	-1.843	-1.792	2.392	-1.875	-2.033	3.601
	k=8	k=12	k=8	k=8	k=12	k=8	k=8	k=12	k=8
Canada	-0.279	-1.562	4.399***	-2.626	-3.176**	5.193**	-2.253	-3.526**	6.575*
	k=12	k=12	k=9	k=9	k=9	k=9	k=9	k=9	k=9
Belgium	-3.204*	-1.526	2.502	-2.338	-2.664**	3.507	-2.406	-3.140	5.030**
	k=8	k=12	k=8	k=8	k=8	k=8	k=8	k=8	k=8
Austria	-2.451**	-1.476	1.192	-1.712	-2.347	3.092	-2.668	-2.805	4.350***
	k=8	k=8	k=8	k=8	k=8	k=8	k=4	k=8	k=8

Note: In all tests, the relevant maximum number of lags was set at 12. The most appropriate lag value (k) in terms of model structure is given under the test statistics. The results obtained for the developed test were compared with the critical value obtained for 100 samples. Statistical significance of 1%, 5% and 10% is indicated by ***, ** and *.

Teräsvirta and Anderson (1992), determined that Japan and Europe (OECD) DGPs from 1961 (i) to 1986 (iv) can be explained by LSTAR or ESTAR structures. Other countries, as Austria, Belgium, Canada, Germany, Norway, Sweden and the United States, are suitably explained by the LSTAR model structure. In the original study, the seasonal effect was removed by taking the difference of four quarters of the series. Therefore, the series is indexed values that become all ratios. Under the assumption of non-zero mean and deterministic trend structures for time series, the most successful test is shown as a developed test $F_{LSTAR,c=0}$ by comparison with each other.

4.Discussion

This study aims to determine a simple unit root test against the alternative of logistic smooth transition autoregressive nonlinearity with a non-threshold. Recently, the re-definition of nonlinear models, such as ESTAR, based on expressed alternatively by using Taylor expansion. This auxiliary regression which prevents the unknown coefficient issue of nonlinear structure with Taylor expansions, it is aimed at determining the unit root structure. After obtaining critical values, the developed test, in which assumptions are extended, examined its size and power characteristics. In particular, the power characteristics of this developed test gave better results in any conditions from comparing ones. The empirical application of data driven by Teräsvirta and Anderson (1992) supports this, but also shows the shortcomings of the test. In the main study, all the series were used in the form of LSTAR. Shortcomings can be expressed by neglecting regime changes caused by structural breakage. If structural breaks are not taken into account, it is reasonable to find the non-stationary for time series.

Another shortcoming is that the threshold value is assumed to be zero. It is beneficial to re-define the LSTAR model for Taylor expansion, but smooth transaction models are based on behavioral changes. This behavioral change occurs when the time series approximates the threshold value. As a result, the developed test $F_{LSTAR,c=0}$ could be used as an alternative to the questioning of unit root structure by considering the LSTAR structure in a simple way. It has been observed in the empirical experiment that the developed model is useful in practice.

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