POLİTEKNİK DERGİSİ

JOURNAL of POLYTECHNIC

ISSN:1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE)
URL: http://dergipark.gov.tr/politeknik

Introducing the *λ-Scale* DEMATEL method: Developing the classical DEMATEL technique

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To cite to this article: Ebrahimi S.H, "Introducing the *λ-Scale* DEMATEL method: Developing the classical DEMATEL technique", ***Journal of Polytechnic***, 26(2): 751-758, (2023).

Bu makaleye şu şekilde atıfta bulunabilirsiniz: Ebrahimi S.H, "Introducing the *λ-Scale* DEMATEL method: Developing the classical DEMATEL technique", ***Politeknik Dergisi***, 26(2): 751-758, (2023).

Erişim linki(To link to this article): http://dergipark.gov.tr/politeknik/archive

DOI: **10.2339/politeknik.961157**

Introducing the *λ-Scale* DEMATEL method: Developing the classical DEMATEL technique

Highlights

- *A new fuzzy linguistic scale for DEMATEL method is introduced using the Euler's series.*
- *The main steps of the classic DEMATEL has been reconfigured and modeled again.*
- *A new threshold value determination formula has been introduced for filtering out the final total matrix.*

GraphicalAbstract

In this research study, the Euler's series is usd to introduce a new fuzzy linguistic scale along with supposing a threshold value determination to develop the classic DEMATEL method.

$$0 < \lambda = \lim_{n \to \infty} \left[\sum_{k=1}^{n} \frac{1}{k} - \mathrm{Ln}(n) \right] \cong 0.577215 < 1 \qquad (4)$$

Aim

The main purpose of this study is to reform the input scale which applied to classic DEMATEL method.

Design &Methodology

Mathematical modeling of fuzzy linguistic terms by Euler's series.

Originality

Euler's series has been used to symmetrise the classic input scale applied in classic DEMATEL while given flexibility to threshold value determination.

Findings

Reforming the classic DEMATEL method by Euler's series explicitly provides a more realistic linguistic symmetrical terms to build the initial influence matrix by experts.

Conclusion

As a clear modofication, the applicability of the mathematical series is presented to involve the participate the number of experts and the number of the criteria associated to the real world problem understudied.

Declaration of Ethical Standards

The author of thi sarticle declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Introducing the *λ-Scale* DEMATEL method: Developing the classical DEMATEL technique

Araştırma Makalesi / ResearchArticle

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(Geliş/Received : 02.07.2021; Kabul/Accepted : 10.02.2022 ; Erken Görünüm/Early View : 21.02.2022)

ABSTRACT

DEMATEL method is a methodology to deal with complex systems. Its core process is to visualize structurally the graphical interrelation among some critical factors known as the most important factors affecting on the performance or the decision making issues. The initial direct influence matrix in DEMATEL methodology would be assessed by the integer scale of 0,1,2,3,4 for "no influence" , "low influence", "medium influence", "high influence" and "very high influence" respectively. Linguistically this scale is not symmetric because of that there has been no value considered for the term of " very low influence". Additionally there is a challenge for the infinity test of the normalized matrix in DEMATEL procedure. In this research, it would be intended to redefine the input scale of DEMATEL and propose a modificationto redesign the DEMATEL method. Eventually, a threshold value formula has been presented for filtering out the final total influence matrix.

Keywords: DEMATEL method, Euler's series, fuzzy linguistic term, mathematical modeling

1.INTRODUCTION

Structural modeling is nearly a view of a system that emphasizes the structure of the objects, including their classifiers, relationships, attributes and operations.A structural model is inherently purposed to depict a graphical diagram which consists of a set of nodes and connections between the nodes.It should be noticed that in this diagram the nodes are usually considered as the main factors influencing the total operation of a system and besides the connections are shown by numbers or some directed vectors which logically derived from the mathematical model that deployed to simulate a real complicated problem.Generally speaking, structural diagrams are being used to understand some aspects of a system complexity particularly the interrelationships between its components.

In this sense, the casual inter-dependency and inter-relationship among a group of factors are the two vital and mandatory features which have been proposed by structural modeling diagrams.Briefly, we are keen to realize and visualize the critical cause and effect relationships between the factors/criteria influencing the whole system performance to better manipulating of decisions and evaluating some major factors/criteria. Therefore a great deal of attention has been set by researchers to design useful techniques for presenting comprehensive approaches to scientifically investigate about the cause and effect pattern recognition using graphs and matrixes.

Decision making trial and evaluation laboratory (DEMATEL) introduced by Fontela&Gabus [1] as a kind of structural modeling has been widely used for analyzing the cause and effect relationships among elements of a system. The DEMATEL methodology has been also extended to delineate graphically the real-world systems especially those associated with uncertainty and imprecise data involved in problems. Nowadays DEMATEL method is recognized as an effective structural technique due to its identification of cause and effect relationship among a group of known factors in a complex system. The DEMATEL technique can convert the interrelations between factors into an intelligible structural model of the system and divide them into a cause group and an effect group [2]. Additionally, over the past decades DEMATEL method and its related applications have been particularly applied in a lot of scientific areas to better making decision.Finding the key barriers to the implementation of green supply chain management by Wang et al [3] , evaluating the key factors affecting customer satisfaction in an internet banking system by Asad et al [4] and identify the key performance evaluation criteria for achieving customer satisfaction through balanced scorecard (BSC) by Pan and Ngnyen [5] are typical application of DEMATEL method. Sheng-Li et.al [6] reviewed the DEMATEL methodologies and its applications comprehensively and technically. Their study showed vastly and pervasively the DEMATEL usability for different situations of decision making and various industrial applications. Computer science, engineering, business and managements and decision sciences are the four major categories which bolded in this study exploiting from DEMATEL methodologies. So, it would be not much far from the truth to say that DEMATEL has found its relevant utilization routes besides its basic mathematical approaches is still being improved.

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The remain part of this study consists of four sections. Section 2 is devoted to review the DEMATEL methodology and challenges accompanied with. In section 3, the new scale for DEMATEL method and the modified DEMATEL algorithm have been introduced. The numerical study and conclusion parts are followed in section 4 and 5 respectively.

2. CLASSİCAL DEMATEL METHOD

As stated before, it would be clearly perceived that DEMATEL technique has been pervasively adopted in many areas of scientific interests. Accordingly, in this section, it is intended to review the classical DEMATEL method. Here are the following steps of DEMATEL listed below:

Step 1: Building the primary relationship evaluation matrix *M* between the n-factors in a system.

(which identified and averaged by some experts. The integer scale of "no influence (0),""low influence (1),""medium influence (2),""high influence (3)," and "very high influence (4)" is used to determine the group direct influence matrix *M*= $[m_{ij}]_{n\times n}$).

Step 2: Normalizing the direct influence matrix.

(The normalized direct influence matrix N can be achieved as follows:

$$N = {}^{M}/_{k} \quad , \quad k = \max\left(\max_{1\le i\le n} \sum_{j=1}^{n} m_{ij} \ , \max_{1\le j\le n} \sum_{i=1}^{n} m_{ij} \right) \qquad (1)$$

Where, $\max_{1\le i\le n} \sum_{j=1}^{n} m_{ij}$ represents the total direct effect of criterion *i* applied on other criteria and $\max_{1\le j\le n} \sum_{i=1}^{n} m_{ij}$ represents the total direct effect which the criterion *j* receives from other criteria.)

Step 3: Calculate the total influence matrix *T*= $[t_{ij}]_{n\times n}$by the following equation.

(The total influence matrix *T* is computed by summation of consecutive powers of normalized matrix *N*.

$$T = \lim_{k\to\infty} (N + N^2 + \cdots + N^k) = N(I - N)^{-1} \qquad (2)$$

Where *I*is the identity matrix with the same dimension of matrix *N*.)

Step 4: Set a threshold value and draw the graphical influential relation map (GIRM) showing the final influential relationship between the criteria.

(In this step we have the following values:

r = sum of the rows of total influence matrix *T*.

c= sum of the columns of total influence matrix *T*.

Thus the cause and effect groups can be separated from each other by the element of $(r_i - c_i)$. The criterion *i* would be the effect factor if $(r_i - c_i)$ is negative and vice-versa it would be the cause factor if $(r_i - c_i)$ is positive. (For i ϵ { 1,2 ,.....,n})

For filtering out the negligible effects in matrix T , a threshold value is determined to revise the matrix T.)

Completing these steps is resulted in depicting a valuable map as a structural explanation of prominent factors that can be used helpfully for decision making.

2.1.Challenges with classical DEMATEL method

2.1.1. Infeasibility of normalized initial-direct relation matrix N

Mathematically, DEMATEL method can successfully operate to compute the total relation matrix *T*, when the infinite-indirect influence of N^{∞} is converged to null matrix$[0]_{n\times n}$.Lee et al [7] have argued about this topic. They completely proved that for the normal matrix *N* whose some columns sum to unity, then the N^{∞} might or might not converge to the null matrix. So they added a sufficient condition in the second step of the DEMATEL method as follows to make the N^{∞} be always converged to null matrix $[0]_{n\times n}$.

$$N = {}^{M}/_{k} \quad , \quad k = \max\left(\max_{1\le i\le n} \sum_{j=1}^{n} m_{ij} \ , \varepsilon + \max_{1\le j\le n} \sum_{i=1}^{n} m_{ij} \right) \qquad (3)$$

where ε is a very small positive number for example10^{-4}.

Indeed , using this modification, would guarantee that normalized initial-direct relation matrix N having the column sum of each column less than one so that $\lim_{k\to\infty} N^k = [0]_{n\times n}$.But, as it was expressively mentioned before, this modification would be a sufficient condition for DEMATEL feasibility, not an imperative and necessary condition. It means that once obtaining the normalized matrix of N, the infinity test must be done to clarify whether N^{∞} is converged to the null matrix or not. In this study this test will be added to the classical DEMATEL procedure to save and manage the computerized time and cost relevant to DEMATEL technique. Besides, by checking the test, the normalized initial-direct relation matrix N is to revised to make condition for which the N^{∞} will converge to the null matrix.

2.1.2. Non-symmetry of integer scaling to indicate the initial direct influence

Classically in DEMATEL method it is assumed that there are some factors closely relevant to a known complicated system which then their affecting relationships must be assessed by some experts. Therefore the integer scale of "no influence (0),""low influence (1),""medium influence (2),""high influence (3)," and "very high influence (4) is used to indicate the initial relation matrix between these factors. If this scaling is depicted (figure 1) as below , then it would be easily perceived that the scale is not linguistically symmetric.

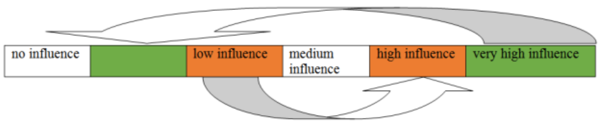

Figure1. The non-symetric extension of DEMATEL scale

Linguistically and logically, such terms of "low& high" and "very low& very high" are simultaneously brought in human's mind to interpret relationships, interconnections and many other inferential statements. As shown in figure (1) there is no symmetric term for the "very high influence" item. On the other hand, it may be valued to consider the "influence" term as a fuzzy linguistic variable. A fuzzy linguistic variables often consists of the two main parts. First the fuzzy predicate (primary term such as influence) and second the fuzzy modifier (such as very or low).Actually, the fuzzy modifier is used to change the meaning of predicate. Hence, in this study it would be first intended to define the "very low influence" material which added to the scale of classical DEMATEL method and specially it is particularly supposed to mathematically formulate the "influence" term by fuzzy linguistic term.

3.Λ-SCALE DEMATAL METHOD

3.1.The supposed Euler scale or the λ-Scale

In this section, it is aimed to introduce the λ-scale approach as a brand new and a flexible one for making the initial direct relationship matrix given by some experts in DEMATEL methodology. The intrinsic core of the λ-scale mechanism is to convert the linguistic term of "influence" into the fuzzy mode interval. Basically, the proposed fuzzy interval belongs to (0,1). Fundamentally, the Euler's series is deployed to define the linguistic variable of "influence" along with its all modifier such as the "low influence" and "very high influence" terms. It must be noted that the Euler's series which shown below, is a convergent series due to condition of infinity.

$$0 < \lambda = \lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right] \cong 0.577215 < 1 \qquad (4)$$

The approximate value of 0.577215 is called the Euler's number. According to Euler's series, we are to generally formulate the "influence" as a linguistic term. These formulas are categorizes as below.

$$5)\ no\ influence = 0 = \lambda^4 - \lambda^4$$

$$6)\ very\ low\ influence = \lambda^3 - \lambda^4 = \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^3 - \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^4$$

$$7)\ low\ influence = \lambda^2 - \lambda^4 = \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^2 - \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^4$$

$$8)\ medium\ influence = \lambda^{1.5} - \lambda^4 = \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^{1.5} - \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^4$$

$$9)\ high\ influence = \lambda^1 - \lambda^4 = \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^1 - \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^4$$

$$10)\ very\ high\ influence = \lambda^0 - \lambda^4 = 1 - \left(\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \mathrm{Ln(n)}\right]\right)^4$$

Where *n* is the natural number. It must be paid attention that we don't replace the infinite value of the Euler's series 0.577215 into the formulas (5) to (10) just because it would be dependent to the problem accuracy level and the opinion of experts who want to apply the *λ-scale* up to any desired decimal number. The typical proposed *λ-scale* values are gathered in the table (1).

Table1.*λ-scale* values

λ *parameter*- Rounded to three decimal digits	*λ-scale* values					
	no influence	Very low influence	Low influence	Medium influence	High influence	Very high influence
0.577	0	0.081	0.222	0.327	0.466	0.889

3.2. The Euler scale or the λ-Scale DEMATEL algorithm

In this point, it is planned to present the process of the supposed DEMATEL method using the *λ-scale* values as a new version of DEMATEL. Besides, there would be some notifications which to enhance the classical DEMATEL methodology specially for the challenge discussed in section 2.1.1. The steps of the *λ-scale* DEMATEL are described as follows:

Step 1: Building the primary relationship evaluation matrix *M* between the n-factors in a system. Suppose that there are also *E* expert who should assess the direct influence of factor n_i has on factor n_j. So the matrix *M* has to be identified and averaged by some experts.

The *λ-scale* of "no influence (0)", "very low influence (0.081)" , "low influence (0.222)" ,"medium influence (0.327)", "high influence (0.466)" and "very high influence (0.889)" is used to determine the group direct influence matrix *M*= $[m_{ij}]_{n\times n}$.

Step 2: Normalizing the direct influence matrix.

The normalized direct influence matrix *N* can be achieved as follows:

$$N = {}^{M}/_{k} \;\;, \;\; k = \max\left(\max_{1\le i\le n} \sum_{j=1}^{n} m_{ij} \;, \max_{1\le j\le n} \sum_{i=1}^{n} m_{ij} \right) \qquad (11)$$

Step 3: Test the following infinite limitation for the normalized direct influence matrix *N*.

$$\lim_{k\to\infty} N^k = [0]_{n\times n} \qquad (12)$$

If the infinite-indirect influence of N^{∞} is converged to null matrix$[0]_{n\times n}$,go to **Step 5,** else go to **Step 4**.

Step 4: For the normalized direct influence matrix *N*, calculate the following quantity at first then subtract it from all the non- zero elements in matrix *N* .This can cause all the columns of matrix *N* sum to less than unity.

$$\varepsilon = \left(\left[\sum_{\boldsymbol{k=1}}^{\boldsymbol{E\times n}} \frac{1}{k^{1.732}} \right] \right)^{-(5)} , E = number\ of\ experts, \quad n = number\ of\ criteria \qquad (13)$$

Step 5: Calculate the total influence matrix *T*= $[t_{ij}]_{n\times n}$by the following equation.

The total influence matrix *T* is computed by summation of consecutive powers of normalized matrix *N*.

$$T = \lim_{k\to\infty}(N + N^2 + \cdots + N^k) = N(I-N)^{-1} \qquad (14)$$

Where *I* is the identity matrix with the same dimension of matrix *N*.

Step 6: Set the threshold value θ using the following formula and draw the graphical influential relation map (GIRM) showing the final influential relationship between the criteria.

$$\theta = (Ave\ \ [t_{ij}]_{n\times n}) \times (1 - \left(\left[\sum_{\boldsymbol{k=1}}^{\boldsymbol{E\times n}} \frac{1}{k} - \mathrm{Ln}(\boldsymbol{E}\times\boldsymbol{n}) \right] \right)^4),$$
$$E = number\ of\ experts,$$
$$n = number\ of\ criteria \qquad (15)$$

That is to say the threshold value θ is determined by calculating the averaged value of all the elements of matrix T which would be adjusted by amount of $\beta = (1 - \left(\left[\sum_{k=1}^{E\times n} \frac{1}{k} - \mathrm{Ln(n)}\right]\right)^4)$. Indeed β is operated as a adjuster coefficient that varies for different values of number of experts E and number of criteria n dealing with each system.

4. NUMERICAL STUDY

Three numerical examples and also a real case study are illustrated in this section.

Example 1: Assume that the initial average influence matrix determined by three experts for four criteria is as follows:

$$A = \begin{bmatrix} 0 & 0.118 & 0.201 & 0.402 \\ 0.231 & 0 & 0.261 & 0.261 \\ 0.196 & 0.331 & 0 & 0.331 \\ 0.374 & 0.183 & 0.231 & 0 \end{bmatrix}$$

The maximum amount for the rows and columns sum would be 0.99. So the normalized direct influence matrix *N* is then calculated as follows:

$$N = \begin{bmatrix} 0 & 0.119 & 0.202 & 0.404 \\ 0.232 & 0 & 0.263 & 0.263 \\ 0.197 & 0.333 & 0 & 0.333 \\ 0.376 & 0.186 & 0.232 & 0 \end{bmatrix}$$

The infinite limit of N^{∞} would be the null matrix $[0]_{n\times n}$.

Thus the total influence matrix is calculated as:

$$T = \begin{bmatrix} 0.754 & 0.689 & 0.804 & 1.158 \\ 0.958 & 0.606 & 0.871 & 1.099 \\ 1.021 & 0.927 & 0.737 & 1.235 \\ 1.073 & 0.769 & 0.865 & 0.924 \end{bmatrix}$$

For this example, we have the values listed below to determine the threshold value θ.

E=3 (number of experts) , n=4(number of criteria) , $Ave\ \ [t_{ij}]_{n\times n} = 0.906$

$$\left(\left[\sum_{k=1}^{E\times n}\frac{1}{k} - \text{Ln}(\boldsymbol{E}\times\boldsymbol{n})\right]\right)^4 \cong 0.146$$

Therefore the threshold value of θ is approximately obtained as 0.774.

Thus the total influence matrix T should be filtered out by omitting the numbers less than 0.774. The final influence matrix is shown as follows.

$$\tilde{T} = \begin{bmatrix} 0 & 0 & 0.804 & 1.158 \\ 0.958 & 0 & 0.871 & 1.099 \\ 1.021 & 0.927 & 0 & 1.235 \\ 1.073 & 0 & 0.865 & 0.924 \end{bmatrix}$$

***Example 2*:**There are three experts who presented the averaged initial relation matrix A for three criteria.

$$A = \begin{bmatrix} 0 & 0.709 & 0.709 \\ 0.709 & 0 & 0.709 \\ 0.709 & 0.709 & 0 \end{bmatrix}$$

In the next step, the normalized direct influence matrix N is then determined. The maximum amount for the rows and columns sum would be 1.418.

$$N = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

Note that the infinite limit of N^{∞} would not be the null matrix $[0]_{n\times n}$.

$$N^{\infty} = \begin{bmatrix} 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \end{bmatrix} \neq [0]_{3\times 3}$$

Thus as a modification, all the non- zero elements of matrix N must be subtracted from 0.0072.

$$\varepsilon = \left(\left[\sum_{k=1}^{9}\frac{1}{k^{1.732}}\right]\right)^{-(5)} \cong 0.064, E = 3, \qquad n = 3$$

The revised normalized matrix N is as follows:

$$N = \begin{bmatrix} 0 & 0.436 & 0.436 \\ 0.436 & 0 & 0.436 \\ 0.436 & 0.436 & 0 \end{bmatrix}$$

So the total influence matrix is calculated as:

$$T = \begin{bmatrix} 2.068 & 2.372 & 2.372 \\ 2.372 & 2.068 & 2.372 \\ 2.372 & 2.372 & 2.068 \end{bmatrix}$$

For threshold value calculation, we have the values listed below.

E=3 (number of experts) , n=3(number of criteria) , $Ave\ \ [t_{ij}]_{n\times n} \cong 2.271$

$$\left(\left[\sum_{k=1}^{E\times n}\frac{1}{k} - \text{Ln}(\boldsymbol{E}\times\boldsymbol{n})\right]\right)^4 \cong 0.159$$

At last the threshold value of θ is approximately obtained as 1.909.

Thus the total influence matrix T has been filtered out by omitting the numbers less than 1.909. As it is clear there would be no element less than 1.909 in matrix T.

***Example 3*:** Assume that we have given the initial average influence matrix determined by five experts for five criteria as follows:

$$A = \begin{bmatrix} 0 & 0.159 & 0.203 & 0.286 & 0.08 \\ 0.159 & 0 & 0.262 & 0.298 & 0.459 \\ 0.163 & 0.189 & 0 & 0.257 & 0.1 \\ 0.397 & 0.218 & 0.189 & 0 & 0.203 \\ 0.109 & 0.248 & 0.262 & 0.159 & 0 \end{bmatrix}$$

The normalized direct influence matrix N is then determined. The maximum value for the rows and columns sum would be 1.178.

$$N = \begin{bmatrix} 0 & 0.135 & 0.172 & 0.243 & 0.068 \\ 0.135 & 0 & 0.222 & 0.253 & 0.389 \\ 0.138 & 0.160 & 0 & 0.218 & 0.085 \\ 0.337 & 0.185 & 0.160 & 0 & 0.172 \\ 0.093 & 0.211 & 0.222 & 0.135 & 0 \end{bmatrix}$$

The infinite limit of N^{∞} is the null matrix $[0]_{n\times n}$.So the final total matrix T would be resulted as shown below.

$$T = \begin{bmatrix} 0.400 & 0.490 & 0.551 & 0.644 & 0.444 \\ 0.688 & 0.561 & 0.794 & 0.852 & 0.868 \\ 0.512 & 0.504 & 0.400 & 0.619 & 0.456 \\ 0.766 & 0.633 & 0.663 & 0.575 & 0.626 \\ 0.492 & 0.572 & 0.619 & 0.589 & 0.410 \end{bmatrix}$$

The threshold θ is calculated as 0.514 using the following data.

E=5 (number of experts) , n=5(number of criteria) , $Ave\ \ [t_{ij}]_{n\times n} \cong 0.589$

$$\left(\left[\sum_{k=1}^{\boldsymbol{E}\times\boldsymbol{n}}\frac{1}{k}-\mathrm{Ln}(\boldsymbol{E}\times\boldsymbol{n})\right]\right)^{4}\cong 0.127$$

Lastly the final total matrix T must be filtered out by replacing the elements less than 0.514 with zero.

$$\tilde{T}=\begin{bmatrix}0 & 0 & 0.551 & 0.644 & 0\\ 0.688 & 0.561 & 0.794 & 0.852 & 0.868\\ 0 & 0 & 0 & 0.619 & 0\\ 0.766 & 0.633 & 0.663 & 0.575 & 0.626\\ 0 & 0.572 & 0.619 & 0.589 & 0\end{bmatrix}$$

Example 4 (A real case study): In this section, it is intended to apply classical DEMATEL method and our proposed λ-scale DEMATEL method for a real case industrial problem of high pressure pipeline gas compressor where there are five important factors (criteria) including inlet gas temperature (T_{in}), inlet gas pressure (P_{in}), outlet gas temperature (T_{out}), outlet gas pressure (P_{out}) and ambient temperature (T_{amb}). Hence, for the first part ,two technical experts are asked to set their opinions expressing initial influence matrix using classical integer scale as follows.

$$E_1=\begin{array}{c}\\ \text{Tin}\\ \text{Pin}\\ \text{Tout}\\ \text{Pout}\\ \text{Tamb}\end{array}\begin{array}{c}\begin{array}{ccccc}\text{Tin} & \text{Pin} & \text{Tout} & \text{Pout} & \text{Tamb}\end{array}\\ \begin{bmatrix}0 & 3 & 4 & 4 & 0\\ 3 & 0 & 2 & 4 & 0\\ 1 & 2 & 0 & 3 & 0\\ 2 & 1 & 2 & 0 & 0\\ 1 & 1 & 0 & 0 & 0\end{bmatrix}\end{array},$$

$$E_2=\begin{array}{c}\\ \text{Tin}\\ \text{Pin}\\ \text{Tout}\\ \text{Pout}\\ \text{Tamb}\end{array}\begin{array}{c}\begin{array}{ccccc}\text{Tin} & \text{Pin} & \text{Tout} & \text{Pout} & \text{Tamb}\end{array}\\ \begin{bmatrix}0 & 3 & 3 & 4 & 0\\ 2 & 0 & 2 & 3 & 0\\ 1 & 1 & 0 & 2 & 0\\ 2 & 1 & 1 & 0 & 0\\ 2 & 1 & 1 & 0 & 0\end{bmatrix}\end{array}$$

Thus, the averaged initial relation matrix A is obtained as:

$$A=\begin{bmatrix}0 & 3 & 3.5 & 4 & 0\\ 2.5 & 0 & 2 & 3.5 & 0\\ 1 & 1.5 & 0 & 2.5 & 0\\ 2 & 1 & 1.5 & 0 & 0\\ 1.5 & 1 & 0.5 & 0 & 0\end{bmatrix}$$

The normalized direct influence matrix N must be then calculated. The maximum value for the rows and columns sum would be 10.5.

$$N=\begin{bmatrix}0 & 0.286 & 0.333 & 0.381 & 0\\ 0.238 & 0 & 0.190 & 0.333 & 0\\ 0.095 & 0.143 & 0 & 0.238 & 0\\ 0.190 & 0.095 & 0.143 & 0 & 0\\ 0.143 & 0.095 & 0.048 & 0 & 0\end{bmatrix}$$

The infinite limit of N^{∞} is the null matrix $[0]_{n\times n}$.So the final total matrix T would be finalized as shown below.

$$T=\begin{bmatrix}0.369 & 0.573 & 0.690 & 0.877 & 0\\ 0.495 & 0.286 & 0.515 & 0.739 & 0\\ 0.284 & 0.304 & 0.224 & 0.500 & 0\\ 0.348 & 0.274 & 0.355 & 0.308 & 0\\ 0.256 & 0.219 & 0.206 & 0.219 & 0\end{bmatrix}$$

Ultimately, to get the final total matrix the threshold value is determined by the simple arithmetic mean of elements of matrix T. Average number is approximately 0.322, so by filtering matrix T we have:

$$\tilde{T}=\begin{bmatrix}0.369 & 0.573 & 0.690 & 0.877 & 0\\ 0.495 & 0 & 0.515 & 0.739 & 0\\ 0 & 0 & 0 & 0.500 & 0\\ 0.348 & 0 & 0.355 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

Now, the same two technical experts are asked to set their opinions expressing initial influence matrix using λ-scale method as follows:

$$E_1=\begin{array}{c}\\ \text{Tin}\\ \text{Pin}\\ \text{Tout}\\ \text{Pout}\\ \text{Tamb}\end{array}\begin{array}{c}\begin{array}{ccccc}\text{Tin} & \text{Pin} & \text{Tout} & \text{Pout} & \text{Tamb}\end{array}\\ \begin{bmatrix}0 & 0.466 & 0.889 & 0.327 & 0\\ 0.466 & 0 & 0.222 & 0.466 & 0\\ 0.081 & 0.081 & 0 & 0.081 & 0\\ 0.327 & 0.081 & 0.081 & 0 & 0\\ 0.081 & 0.081 & 0.081 & 0 & 0\end{bmatrix}\end{array},$$

$$E_2=\begin{array}{c}\\ \text{Tin}\\ \text{Pin}\\ \text{Tout}\\ \text{Pout}\\ \text{Tamb}\end{array}\begin{array}{c}\begin{array}{ccccc}\text{Tin} & \text{Pin} & \text{Tout} & \text{Pout} & \text{Tamb}\end{array}\\ \begin{bmatrix}0 & 0.222 & 0.889 & 0.081 & 0\\ 0.222 & 0 & 0.081 & 0.466 & 0\\ 0 & 0.081 & 0 & 0.222 & 0\\ 0 & 0.222 & 0.466 & 0 & 0\\ 0.081 & 0.222 & 0.222 & 0.327 & 0\end{bmatrix}\end{array}$$

Therefore the average matrix is then calculated as:

$$A=\begin{bmatrix}0 & 0.344 & 0.889 & 0.204 & 0\\ 0.344 & 0 & 0.152 & 0.466 & 0\\ 0.041 & 0.081 & 0 & 0.152 & 0\\ 0.164 & 0.152 & 0.274 & 0 & 0\\ 0.081 & 0.152 & 0.152 & 0 & 0\end{bmatrix}$$

The normalized direct influence matrix N must be then obtained. The maximum value for the rows and columns sum would be 1.467.

$$N=\begin{bmatrix}0 & 0.234 & 0.606 & 0.139 & 0\\ 0.234 & 0 & 0.104 & 0.318 & 0\\ 0.028 & 0.055 & 0 & 0.104 & 0\\ 0.112 & 0.104 & 0.187 & 0 & 0\\ 0.055 & 0.104 & 0.104 & 0 & 0\end{bmatrix}$$

The infinite limit of N^{∞} is the null matrix $[0]_{n\times n}$.Thus the total matrix T would be resulted as shown below.

$$T=\begin{bmatrix}0.143 & 0.348 & 0.794 & 0.352 & 0\\ 0.330 & 0.147 & 0.404 & 0.453 & 0\\ 0.068 & 0.091 & 0.079 & 0.151 & 0\\ 0.175 & 0.175 & 0.333 & 0.115 & 0\\ 0.104 & 0.148 & 0.198 & 0.082 & 0\end{bmatrix}$$

The threshold θ is calculated as 0.159 using the following data.

E=2 (number of experts) , n=5(number of criteria) , $Ave\ [t_{ij}]_{n\times n} \cong 0.188$

$$\left(\left[\sum_{k=1}^{E\times n}\frac{1}{k}-\mathrm{Ln}(\boldsymbol{E}\times\boldsymbol{n})\right]\right)^{4}\cong 0.154$$

Finally the final total matrix T must be filtered out by substituting the elements less than 0.159 with zero.

$$\tilde{T}=\begin{bmatrix}0 & 0.348 & 0.794 & 0.352 & 0\\ 0.330 & 0 & 0.404 & 0.453 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0.175 & 0.175 & 0.333 & 0 & 0\\ 0 & 0 & 0.198 & 0 & 0\end{bmatrix}$$

5.DISCCUSION

Classic DEMATEL method has been used in many different scientific decision making applications as some mentioned in literature section. But, its initial scale, algorithm and its way of threshold determination is static. In this study, λ-scale DEMATEL is introduced to comprehensively extend and compromise its mathematical modeling to be more realistic linguistically and additionally to be more non-static due to number of experts and criteria. Furthermore, threshold value in our proposed refined DEMATEL methodology is dependently determined as a function of number of experts and number of criteria being modeled in a complicated system. Numerical case study have logically improved its feasibility and applicability. Although , comparing classical DEMATEL with λ-scale DEMATEL method could not be as mathematically proofed as it is not usually done for other structural modeling or multi criteria methods. Thus its feasibility would be the most important features that satisfy its design.

6.CONCLUSION

The classic DEMATEL technique as a kind of structural modeling approach can obtain the interdependencies between the known factors in a complicated system ,besides it is especially useful in analyzing the cause and effect relationships among components of a system. But, there are challenges linked to its relevant procedure due to non -symmetry of the input integer scale and the infeasibility. In this investigation, we have reconfigured the DEMATEL's main steps and its related procedures using the series application especially the Euler' series. Fundamentally, the Euler's series has been adequately deployed to introduce a new fuzzy linguistic scale along with supposing a threshold value determination. It would be worthy to note that in this research the applicability of the mathematical series is presented to involve the participate the number of experts and the number of the criteria associated to the problem understudied. This modification caused to introduce a flexible and more importantly a feasible version of DEMATEL method for structural modeling. Additionally, the supposed λ-scale approach would provide a more realistic linguistic symmetrical terms to build the initial influence matrix by experts. As a core result, it would be expressed that the full mathematical modeling of DEMATEL method has improved it forever and for each system with several criteria assessed by given numbers of experts.

DECLARATION OF ETHICAL STANDARDS

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

Seyed Hossain Ebrahimi: Performed the mathematical modeling and wrote the manuscript.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

REFERENCES

[1] Fontela, E., &Gabus, A. The DEMATEL observer, DEMATEL 1976 report. Switzerland Geneva: Battelle Geneva Research Center,(1976).

[2] C.-Y. Huang, J. Z. Shyu, and G.-H. Tzeng. Reconfiguring the innovation policy portfolios for Taiwan's SIP mall industry, ***Technovation***, vol. 27, no. 12, pp. 744–765,(2007).

[3] Wang Z, Mathiyazhagan K, Xu L, Diabat A. A decision making trial and evaluation laboratory approach to analyze the barriers to Green Supply Chain Management adoption in a food packaging company. ***J Clean Prod*** 117:19–28,(2016).

[4] Asad MM, Mohajerani N, NoursereshM.Prioritizing factors affecting customer satisfaction in the internet banking system based on cause and effect relationships. ***Proc Econ Finance*** 36:210–219,(2016).

[5] Pan JN, NgnyenHTN.Achieving customer satisfaction through product–service systems. ***Eur J Oper Res*** 247:179–190,(2015).

[6] Sheng-Li Si, Xiao-Yue You, Hu-Chen Liu , and Ping Zhang. DEMATEL Technique: A Systematic Review of the State-of-the-Art Literature on Methodologies and Applications. ***Mathematical Problems in Engineering*** Volume 2018, Article ID 3696457, 33 pages https://doi.org/10.1155/2018/3696457,(2018).

[7] Hsuan-Shih Lee, Gwo-HshiungTzeng, WeichungYeih, Yu-Jie Wang, Shing-Chih Yang. Revised DEMATEL: Resolving the Infeasibility of DEMATEL. ***Applied Mathematical Modelling*** Volume 37 .pp 6746–6757,(2013).

[8] G. H. Tzeng, C. H. Chiang, and C. W. Li, "Evaluating intertwined effects in e-learning programs: a novel hybrid MCDM model based on factor analysis and DEMATEL,"***Expert Systems with Applications***, vol. 32, no. 4, pp. 1028–1044, (2007).

[9] C. Lin and G. Tzeng, "A value-created system of science (technology) park by using DEMATEL,"***Expert Systems withApplications***, vol. 36, no. 6, pp. 9683–9697, (2009).

[10] P. T.-W. Lee and C.-W. Lin, "The cognition map of financial ratios of shipping companies using DEMATEL and MMDE,"***Maritime Policy&Management***, vol. 40, no. 2, pp. 133–145, (2013).

[11] J. Sara, R. M. Stikkelman, and P. M. Herder, "Assessing relative importance and mutual influence of barriers for CCS deployment of the ROAD project using AHP and DEMATEL methods,"***International Journal of Greenhouse Gas Control***, vol. 41, pp. 336–357, (2015).

[12] Lenth RV. Quick and easy analysis of unreplicated factorials. ***Technometrics*** 31:469–473,(1989).

[13]J.I. Shieh, H.H. Wu, K.K. Huang, A DEMATEL method in identifying key success factors of hospital service, ***Knowl.-Based Syst***. 23 (3) 277–282,(2010).

[14] J.K. Chen, I.S. Chen, Using a novel conjunctive MCDM approach based on DEMATEL, fuzzy ANP, TOPSIS as an innovation support system for Taiwanese higher education, ***Expert Sys. Appl***. 37 1981–1990,(2010).