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INCORPORATING THE FUNDAMENTAL ANALYSIS INTO THE ROBUST MEAN – VARIANCE ANALYSIS: AN APPLICATION ON THE TURKISH BANKING STOCKS

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Abstract

Robust optimization is an important tool to deal with the uncertainty of parameters. However, due to the worst-case orientation, the existing robust mean – variance (MV) models ignore the plausible portfolio choices, backed by additional criteria or subjective judgements. Thus, we propose a way to incorporate the fundamental analysis into the robust MV analysis under the assumption that the risk-free asset and short positioning are allowed. After laying down the theoretical points, we give an explanatory example by using the real data set of six banking stocks trading on the Borsa Istanbul (BIST).

Keywords: *Portfolio Selection, Fundamental Analysis, Principal Components Analysis, Robust Optimization, Mean - Variance Model.*

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1. INTRODUCTION

The mean - variance (MV) model introduced in (Markowitz, 1952), has had a profound influence on the portfolio management theoretically (Goldfarb and Iyengar, 2003). However, it is not generally used in the practice due to the statistical errors in the estimation of its parameters (Breuer, 2006; De Miguel et al., 2009). The mean vector's estimation is harder and thus a more important problem (De Miguel et al., 2009; Garlappi et al., 2006). Furthermore, this vector may change in the future because of the market shocks (Berkowitz, 2000). Recently, Goktas and Duran (2020) introduce a new robust MV model to overcome these problems. The new robust MV model may be preferred to the other robust MV models for several reasons such as the ease of use. However, this model, which depends on the Principal Components Analysis (PCA), may not be suitable for the non-conservative investors. Because the worst-case situation is not very likely to occur and there is a cost of to be more conservative than necessary (Huang et al., 2010).

Portfolio selection based on the fundamental analysis is another alternative for the investors. The Analytical Hierarchy Process (AHP) or its fuzzy extensions can be used in bringing the information about the stocks together and then determining the optimal portfolio allocation (Saaty et al., 1980; Tiryaki and Ahlatcioglu, 2009). We believe that it is a valuable approach since it enables to consider the important concepts such as the profitability of the companies and the cheapness of the stocks. However, it is an appropriate alternative only when the short positioning is not allowed by the regulators or not preferred by the investors.

To fill in the gaps as mentioned above, we propose a way to incorporate the fundamental analysis into the robust MV analysis, which provides the flexibility to the investors unlike the classical MV analysis. To be more specific, we use the priority vector found with AHP in picking a fundamentally backed solution from the infinitely many plausible solutions obtained with the robust MV analysis. This fundamentally backed solution (FBS) simply corresponds to the portfolio that maximize the investor's utility function under the assumption that the utility function is linear and the utility vector of the stocks is equal to the priority vector found with AHP. The major shortcoming of the proposed approach is that it is applicable only when the risk-free asset and short positioning are allowed.

The rest of paper is organized as follows. In Section 2, we give the theory of the proposed approach. In Section 3, we give an explanatory example to illustrate our approach.

Here, we use a real data set of all banking stocks listed on the BIST 30 where the training and testing periods covers the complete year of 2016 and the first quarter of 2017 respectively. We conclude the paper with Section 4.

2. THE THEORY OF THE PROPOSED APPROACH

In this paper, we take the initial values of the portfolios as 1 for simplicity where the weight of the risk-free asset and the total weight of the stocks sum to 1. We also ignore the extra cost of short positioning. We prefer the excess logarithmic return vector of the stocks (r) to their simple return vector because the logarithmic returns are summable. This choice also brings us several advantages empirically and theoretically (Levy and Robinson, 2016). We show its mean vector and positive definite[1] covariance matrix with μ and $\sum$ respectively.

We assume that the covariance matrix of *r* is equal to the sample covariance matrix of *r*. Then, the principal components vector (x) is defined as below. Here, Λ is a diagonal matrix[2] of which its positive eigenvalues given in the ascending order and *V* is an orthogonal matrix[3], of which ith column is the corresponding orthonormal eigenvector to the ith eigenvalue (Johnson and Wichern, 2007; Jolliffe, 2002).

$$\Sigma = V\Lambda V^T \mapsto x := V^T r \qquad (1)$$

PCA is an orthogonal coordinate transformation. The linearized profit function $p(r)$ is expressed on the new orthogonal coordinate system as below where *w* is the weight vector of the stocks and $\tilde{w}$ is the weight vector of the principal components.

$$f(x) = \tilde{w}^T x = \left(V^T w\right)^T x = w^T (Vx) = w^T r = p(r) \qquad (2)$$

Remark: $\Lambda_{i,i}$ is the ith eigenvalue of the sample covariance matrix or equivalently the sample variance of the ith principal component. $\widehat{\mu}_x = V^T \widehat{\mu}$ is the sample mean vector of *x* where $\widehat{\mu}$ is the sample mean vector of *r*.

[1] A symmetric matrix is positive definite if all of its eigenvalues are positive.
[2] Its non-diagonal elements are equal to 0.
[3] Its inverse is equal to its transpose, which is shown with V^T.

Remark: We assume that the eigenvalues of the sample covariance matrix are distinct as in our case to provide that our robust MV analysis is well-defined**.** They are distinct with probability 1 if its entries have a joint probability density (Girko, 1998).

The uncertainty set of the mean vector of *x* can be determined as the following box type set where z_τ is the τ quantile of the standard normal distribution and *m* is the number of the return data per stock (Goktas and Duran, 2020).

$$U = \left\{ \mu_x \mid \mu_{x,i}^L := \widehat{\mu}_{x,i} - \frac{z_{(1+\tau)/2}}{\sqrt{m}} \sqrt{\Lambda_{i,i}} \le \mu_{x,i} \le \mu_{x,i}^U := \widehat{\mu}_{x,i} + \frac{z_{(1+\tau)/2}}{\sqrt{m}} \sqrt{\Lambda_{i,i}}, \forall i \right\} \quad \textbf{(3)}$$

The robust MV model is formulated with the following MaksMin problem where η is the nonnegative coefficient of risk aversion (Goktas and Duran, 2020).

$$\underset{\tilde{w} \in \mathbb{R}^n}{\text{maks}} \min_{\mu_x \in U} \tilde{w}^T \mu_x - 0.5\eta \left(\tilde{w}^T \Lambda \tilde{w} \right) \quad \textbf{(4)}$$

Under certain assumptions, the robust MV analysis is independent from the duration of the testing period. Furthermore, the solution of (4) for the i[th] principal component is uniquely found as below where sgn () shows the signum function[4] (Goktas and Duran, 2020). Then, the worst-case solution (WCS) is found as $V\tilde{w}_R^*(\eta, \tau)$.

$$\tilde{w}_{R,i}^*(\eta, \tau) = \begin{cases} \dfrac{\mu_{x,i}^L}{\eta \Lambda_{i,i}}, \text{sgn}\left(\mu_{x,i}^L\right) = \text{sgn}\left(\mu_{x,i}^U\right) = 1 \\ \dfrac{\mu_{x,i}^U}{\eta \Lambda_{i,i}}, \text{sgn}\left(\mu_{x,i}^L\right) = \text{sgn}\left(\mu_{x,i}^U\right) = -1 \\ 0, \text{sgn}\left(\mu_{x,i}^L \mu_{x,i}^U\right) \le 0 \end{cases} \quad \textbf{(5)}$$

Since the i[th] principal component's mean is interval-valued as in (3), its robust MV optimal weight set is found as below.

[4] Signum function takes the value 1 for positive values, -1 for negative values and 0 for 0. Clearly, worst-case orientation brings the i[th] principal component's optimal weight closer to 0.

$$MPS = \frac{\Sigma^{-1}\hat{\mu}}{\eta} = \frac{V\Lambda^{-1}V^T\hat{\mu}}{\eta} = V\left(\frac{\Lambda^{-1}\hat{\mu}_x}{\eta}\right) = V\tilde{w}_R^*(\eta, 0) \quad \textbf{(6)}$$

Since the i^th principal component's mean is interval-valued as in (3), its robust MV optimal weight set is found as below

$$\tilde{w}_i^*(\eta,\tau) \in \left[\frac{\hat{\mu}_{x,i}}{\eta\Lambda_{i,i}} - \frac{z_{(1+\tau)/2}}{\eta\sqrt{m\Lambda_{i,i}}}, \frac{\hat{\mu}_{x,i}}{\eta\Lambda_{i,i}} + \frac{z_{(1+\tau)/2}}{\eta\sqrt{m\Lambda_{i,i}}}\right] \quad \textbf{(7)}$$

. (7) gives infinitely many plausible solutions. In this paper, we use AHP in picking a fundamentally backed solution from them. Here, we determine the criteria as below.

- The return on average equity ratio (C1) as a proxy of the company's profitability.
- The dividend per earnings ratio (C2) as a proxy of the company's investor centeredness.
- The book to market ratio (C3) as a proxy of the stock's cheapness.
- The long-term domestic credit note given by Moody's (C4) as a proxy of the company's credibility.

It is claimed that Saaty's original AHP method has several shortcomings (Buckley at al., 2001). On the other hand, we prefer it to the other (fuzzy) AHP methods for several reasons. First, it is widely accepted and used in many areas (Buckley at al., 2001; Saaty and Vargas, 2012). Second, fuzzifying the judgements is simply a perturbation, which does not improve the overall results (Saaty and Tran, 2007). Third, for each pairwise comparision matrix, its Perron vector[5] should be used in obtaining the priorities to control the inconsistencies in the judgments (Saaty, 2003). Hence, after finding the stocks' priority vector (p) by using this method, we determine the i^th principal component's weight as below. Then, fundamentally backed solution (FBS) is equal to $V\tilde{w}_F^*(\eta,\tau)$.

[5] Its Perron vector is equal to the normalized eigenvector corresponding to its maximum eigenvalue. We find the all Perron vectors by using the MATLAB.

$$\tilde{w}^{*}_{F,i}\left(\eta,\tau\right):=\frac{\widehat{\mu}_{x,i}}{\eta\Lambda_{i,i}}+\operatorname{sgn}\left(\left(V^{T}p\right)_{i}\right)\frac{z_{(1+\tau)/2}}{\eta\sqrt{m\Lambda_{i,i}}} \quad \textbf{(8)}$$

The priority vector of the alternatives (stocks) can be thought as the utility vector of the alternatives (Malakooti, 2013). Under the linearity assumption, the utility vector of the principal components is found as follows. Then, (8) gives the plausible portfolio that maximizes the utility based on the linearity in (2) and boundaries given in (7). (8) also indicates that when the utility of a principal component is negative (positive), its weight is minimized (maximized).

$$p_{x}=V^{T}p \quad \textbf{(9)}$$

Remark: In practice, the priority vectors can be found with approximations. There are two steps in the mostly used approximation. In the first step, the columns of the pairwise comparison matrix are normalized by using the Manhattan distance. In the final step, the priority vector is found by averaging each row. However, we do not prefer such an approach and make the priority vector equal to the exact Perron vector of the pairwise comparison matrix. Since approximations may lead important problems such as rank reversal (Saaty and Vargas, 2012).

3. AN APPLICATION ON THE TURKISH BANKING STOCKS

The all-banking stocks listed on BIST 30 are GARAN, AKBNK, YKBNK, ISCTR, VAKBN and HALKB. We calculate their logarithmic returns for the 52 weeks in 2016 by using the Friday closing prices. We set the Bloomberg benchmark interest rate at the end of 30.12.2016, which is equal to 0.1063, as the yearly risk-free rate. Then, we obtain the excess logarithmic returns by subtracting the weekly risk-free rate from the logarithmic returns. We give their summary statistics in Table 1 where SSD is the sample standard deviation.

Table 1: The summary statistics.

	Average	SSD	Skewness	Kurtosis	Median	Minimum	Maximum
GARAN	0.000	0.043	-2.240	6.749	0.010	-0.177	0.072
AKBNK	0.001	0.043	-1.876	6.509	0.005	-0.183	0.067
YKBNK	-0.001	0.050	-1.998	7.105	0.006	-0.224	0.086
ISCTR	0.001	0.043	-1.365	4.614	0.002	-0.164	0.083
VAKBN	0.001	0.050	-1.448	5.140	0.004	-0.210	0.090
HALKB	-0.003	0.051	-1.800	6.309	-0.001	-0.223	0.092

We give the positive definite sample linear correlation matrix in the Table 2. Clearly, linear correlations between them are close to 1. The positive definite sample covariance matrix can be obtained by using this matrix and sample standard deviations given in Table 1.

Table 2: The sample linear correlation matrix.

	GARAN	AKBNK	YKBNK	ISCTR	VAKBN	HALKB
GARAN	1	0.926	0.868	0.899	0.822	0.854
AKBNK	0.926	1	0.870	0.894	0.851	0.876
YKBNK	0.868	0.870	1	0.827	0.873	0.816
ISCTR	0.899	0.894	0.827	1	0.862	0.907
VAKBN	0.822	0.851	0.873	0.862	1	0.845
HALKB	0.854	0.876	0.816	0.907	0.845	1

After employing the Principal Components Analysis of the sample covariance matrix, we form the boxplots of the principal components as in the Chart 1 respectively. Here, the plotted whiskers extend to the adjacent values (the extremes of non-outliers) and the principal components are shown with PC1, PC2, PC3, PC4, PC5 and PC6 respectively. We find that the all eigenvalues i.e., the sample variances of the principal components are distinct and the %88.6 of the total variance is result from PC6.

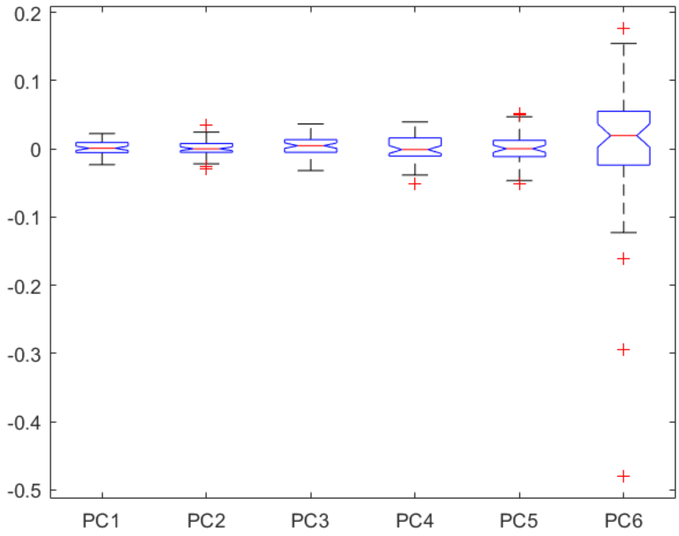

Chart 1: Boxplots.

The evaluation matrix of the alternatives is given in Table 3 under the assumption that the investor correctly foresees the companys' equities with net profits and the stocks' dividends for the year of 2016. Clearly, this assumpiton is very strict but we eliminate most of the human-based errors in this paper with this assumption.

Table 3: The evaluation matrix of the alternatives.

	C1	C2	C3	C4
GARAN	0.154	0.210	0.854	Ba1
AKBNK	0.164	0.199	0.917	Ba1
YKBNK	0.119	0	0.561	Ba1
ISCTR	0.146	0.249	0.598	Ba1
VAKBN	0.156	0.050	0.553	Ba1
HALKB	0.125	0.205	0.535	Ba1

The positive-valued pairwise comparision matrix of the criteria with respect to the goal and its Perron vector are as in the Table 4. We also find that its consistency ratio is lower than 0.10 as in the other cases. Hence, our pairwise comparions are consistent (Saaty and Vargas, 2012).

Table 4: The pairwise comparision matrix of the criteria with respect to the goal.

	C1	C2	C3	C4	Perron vector
C1	1	2	3	3	0.455
C2	1/2	1	2	2	0.263
C3	1/3	1/2	1	1	0.141
C4	1/3	1/2	1	1	0.141

The positive-valued pairwise comparision matrix of the alternatives with respect to C1 and its Perron vector are as in the Table 5.

Table 5: The pairwise comparision matrix of the alternatives with respect to C1.

	GARAN	AKBNK	YKBNK	ISCTR	VAKBN	HALKB	Perron v.
GARAN	1	1	5	2	1	5	0.252
AKBNK	1	1	5	2	1	5	0.252
YKBNK	1/5	1/5	1	1/4	1/5	1	0.047
ISCTR	1/2	1/2	4	1	1/2	4	0.149
VAKBN	1	1	5	2	1	5	0.252
HALKB	1/5	1/5	1	1/4	1/5	1	0.047

The positive-valued pairwise comparision matrix of the alternatives with respect to C2 and its Perron vector are as in the Table 6.

Table 6: The pairwise comparision matrix of the alternatives with respect to C2.

	GARAN	AKBNK	YKBNK	ISCTR	VAKBN	HALKB	Perron v.
GARAN	1	1	7	1/3	5	1	0.173
AKBNK	1	1	7	1/3	5	1	0.173
YKBNK	1/7	1/7	1	1/9	1/2	1/7	0.028
ISCTR	3	3	9	1	7	3	0.410
VAKBN	1/5	1/5	2	1/7	1	1/5	0.042
HALKB	1	1	7	1/3	5	1	0.173

The positive-valued pairwise comparision matrix of the alternatives with respect to C3 and its Perron vector are as in the Table 7.

Table 7: The pairwise comparision matrix of the alternatives with respect to C3.

	GARAN	AKBNK	YKBNK	ISCTR	VAKBN	HALKB	Perron v.
GARAN	1	2	1/8	1/8	1/8	1/8	0.033
AKBNK	1/2	1	1/9	1/9	1/9	1/9	0.024
YKBNK	8	9	1	1	1	1	0.236
ISCTR	8	9	1	1	1	1	0.236
VAKBN	8	9	1	1	1	1	0.236
HALKB	8	9	1	1	1	1	0.236

Since the all elements of the pairwise comparision matrix with respect to C4 is 1, the all elements of the Perron vector found for C4 is equal to 1/6. Then, the priority vector of the alternatives (stocks) are found by taking a simple weighted average of the Perron vectors found for the criteria where the weight vector of the criteria is equal to the Perron vector given in the Table 4.

We determine η and τ as 13.866 and 0.5 respectively. Then, we find the portfolios as in the Chart 2 where the weight vector of AHPS is equal to the priority vector of the stocks. We see that only ISCTR has the positive weight in each portfolio.

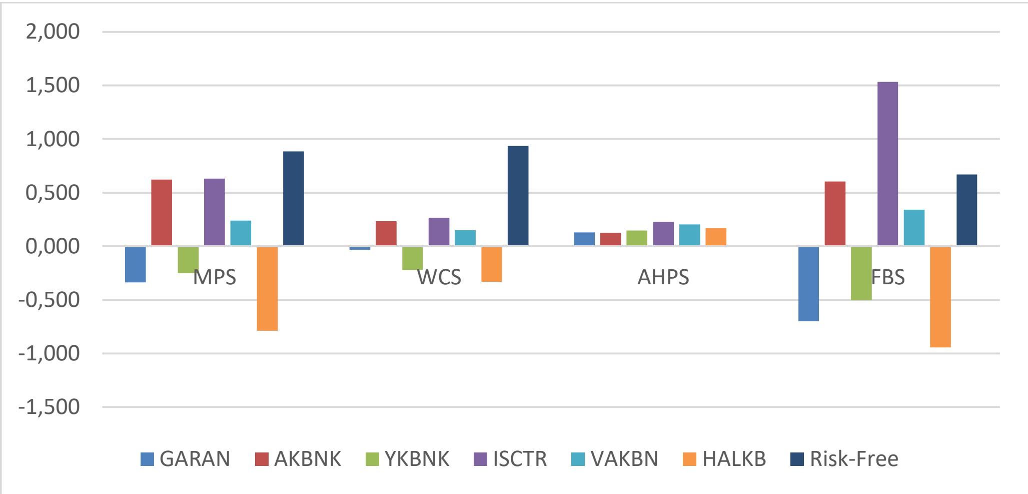

Chart 2: Weight vectors.

Sharpe ratio (SR), defined as the mean or the realized profit per the standard deviation, is a commonly used performance measure. Since, the mean is interval-valued, Sharpe ratio is also interval-valued. Then, we have the following predictions for the first quarter of 2017 based on the square root of time rule[6]. We see that FBS promises the vaguer performance than WCS whereas the predictions about AHPS's performance is not satisfactory.

Table 8: Performance predictions.

	MPS	WCS	AHPS	FBS
Lower Bound of SR	0.342	0.418	-0.386	0.076
Upper Bound of SR	1.439	1.279	0.337	1.460

We find the realized results for the first quarter of 2017 in the Table 9. We see that all portfolios give better performance than the performance predictions due to the better market conditions. We also see that the FBS has the best results in each criterion whereas AHPS has the satisfactory profit but not the performance. WCS is the worst (second best) portfolio in the profitability (performance) criterion whereas MPS is the third one in each criterion.

	MPS	WCS	AHPS	FBS
Realized profit	0.172	0.091	0.202	0.371
Sharpe ratio	2.685	3.009	1.272	3.525

4. CONCLUSION

Although we believe that the worst-case solution is the best choice in the robust MV analysis for the conservative investors or financial institutions, it may not be suitable for the non-conservative investors. Because it may not provide the sufficiently high profit due to the worst-case orientation. On the other hand, the fundamentally backed solution (FBS) provides the best profit and performance in our example. It also conveys more information based on the fundamental analysis of the stocks. Hence, it may be a better choice for the non-conservative investors especially when they want to consider both the quantitative analysis and fundamental analysis in the portfolio selection. On the other hand, it should not be forgotten that proposed approach is applicable under certain conditions. Furthermore, in the real life, it may not give good results due to the human-based errors or inefficient market.

[6] Under certain assumptions, the mean and variance are the linear function of time. Thus, the standard deviation and Sharpe ratio increase by square root of time.

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