ISSN:1306-3111
e-Journal of New World Sciences Academy
2011, Volume: 6, Number: 2, Article Number: 3A0035

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PHYSICAL SCIENCES
Received: November 2010
Accepted: February 2011
Series : 3A
ISSN : 1308-7304

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ON ORDER STATISTICS OF DISCRETE RANDOM VARIABLES

ABSTRACT

In this study, the distributions of rth order statistic of innid discrete random variables are obtained. Then, the results related to pf and df of minimum and maximum of innid discrete random variables are given.

Keywords: Order Statistics, Discrete Random Variable, Probability Function, Distribution Function, Permanent

KESİKLİ TESADÜFİ DEĞİŞKENLERİN SIRALI İSTATİSTİKLERİ ÜZERİNE

ÖZET

Bu çalışmada, innid kesikli tesadüfi değişkenlerin r-inci sıralı istatistiğinin dağılımları elde edilmiştir. Sonra, innid kesikli tesadüfi değişkenlerin minimum ve maksimumunun pf ve df ’si ile ilgili sonuçlar verilmiştir.

Anahtar Kelimeler: Sıralı İstatistikler, Kesikli Tesadüfi Değişken, Olasılık Fonksiyonu, Dağılım Fonksiyonu, Permanent

1. INTRODUCTION (GİRİŞ)

Several identities and recurrence relations for probability density function (*pdf*) and distribution function(*df*) of order statistics of independent and identically distributed(*iid*) random variables were established by numerous authors including Arnold et al. [1], Balasubramanian and Beg [4], David [14], and Reiss[21]. Furthermore, Arnold et al. [1], David [14], Gan and Bain [15], and Khatri [18] obtained the probability function (*pf*) and *df* of order statistics of *iid* random variables from a discrete parent. Balakrishnan [2] showed that several relations and identities that have been derived for order statistics from continuous distributions also hold for the discrete case. Nagaraja [19] explored the behavior of higher order conditional probabilities of order statistics in a attempt to understand the structure of discrete order statistics. Nagaraja [20] considered some results on order statistics of a random sample taken from a discrete population. Corley [12] defined a multivariate generalization of classical order statistics for random samples from a continuous multivariate distribution. Expressions for generalized joint densities of order statistics of *iid* random variables in terms of Radon-Nikodym derivatives with respect to product measures based on *df* were derived by Goldie and Maller [16]. Guilbaud [17] expressed the probability of the functions of independent but not necessarily identically distributed(*innid*) random vectors as a linear combination of probabilities of the functions of *iid* random vectors and thus also for order statistics of random variables.

Recurrence relationships among the distribution functions of order statistics arising from *innid* random variables were obtained by Cao and West [10]. In addition, Vaughan and Venables [22] derived the joint *pdf* and marginal *pdf* of order statistics of *innid* random variables by means of permanents. Balakrishnan [3], and Bapat and Beg [8] obtained the joint *pdf* and *df* of order statistics of *innid* random variables by means of permanents. Using multinomial arguments, the *pdf* of $X_{r:n+1}$ ($1\leq r\leq n+1$) was obtained by Childs and Balakrishnan [11] by adding another independent random variable to the original *n* variables $X_1, X_2, \ldots, X_n$. Also, Balasubramanian et al.[7] established the identities satisfied by distributions of order statistics from non-independent non-identical variables through operator methods based on the difference and differential operators. In a paper published in 1991, Beg [9] obtained several recurrence relations and identities for product moments of order statistics of *innid* random variables using permanents. Recently, Cramer et al. [13] derived the expressions for the distribution and density functions by Ryser's method and the distribution of maxima and minima based on permanents. In the first of two papers, Balasubramanian et al. [5] obtained the distribution of single order statistic in terms of distribution functions of the minimum and maximum order statistics of some subsets of $\{X_1, X_2, \ldots, X_n\}$ where X_i's are *innid* random variables. Later, Balasubramanian et al. [6] generalized their previous results [5] to the case of the joint distribution function of several order statistics.

As far as we know, these approaches have not been considered in the framework of order statistics from *innid* discrete random variables.

From now on, the subscripts and superscripts are defined in the first place in which they are used and these definitions will be valid unless they are redefined.

If $\mathbf{a}_1, \mathbf{a}_2, \ldots$ are defined as column vectors, then the matrix obtained by taking m_1 copies of $\mathbf{a}_1$, m_2 copies of $\mathbf{a}_2$,… can be denoted as $[\underset{m_1}{\mathbf{a}_1}\ \ \underset{m_2}{\mathbf{a}_2}\ \ \ldots]$ and *per*A denotes the permanent of a square matrix A, which is defined as similar to determinants except that all terms in the expansion have a positive sign.

Let $X_1, X_2, \ldots, X_n$ be *innid* discrete random variables and $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ be the order statistics obtained by arranging the *n* X_i's (*i=1, 2,…, n*) in increasing order of magnitude. Let F_i and f_i be *df* and *pf* of X_i, respectively.

The paper is organized as follows. In section 3, we give the theorems concerning *pf* and *df* of order statistics of *innid* discrete random variables. In the last section, some results related to *pf* and *df* will be given.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In general, the distribution theory for order statistics is complicated when the discrete random variables are *innid*. In this study, the distributions of the *r*th order statistics from *innid* discrete random variables are obtained easily by using permanent.

3. THEOREMS FOR PROBABILITY AND DISTRIBUTION FUNCTIONS (OLASILIK VE DAĞILIM FONKSİYONLARI İÇİN TEOREMLER)

In this section, the theorems related to *pf* and *df* of $X_{r:n}$ will be given. We will now express the following theorem for the *pf* of *r*th order statistic of *innid* discrete random variables.

- **Theorem 1.**

$$f_{r:n}(x) = \sum_{k=0}^{r-1} \sum_{m=0}^{n-r} \frac{1}{(r-1-k)!(k+1+m)!(n-m-r)!} per[\underset{r-1-k}{\mathrm{F}(x-)}\ \ \underset{k+1+m}{\mathrm{f}(x)}\ \ \underset{n-m-r}{1-\mathrm{F}(x)}], \tag{1}$$

where $\mathrm{F}(x-) = (F_1(x-), F_2(x-), \ldots, F_n(x-))'$, $\mathrm{f}(x) = (f_1(x), f_2(x), \ldots, f_n(x))'$, $1-\mathrm{F}(x) = (1-F_1(x), 1-F_2(x), \ldots, 1-F_n(x))'$ are column vectors and $F_i(x-) = P(X_i < x)$.

Proof. Consider the event

$$\{X_{r:n} = x\}, \quad r = 1, 2, \ldots, n .$$

It can be realized mutually exclusive ways, as follows: $r-1-k$ observations are less than x, $k+1+m$ observations are equal to x and $n-m-r$ observations are exceed x with respective probabilities $F(x-)$, $f(x)$ and $1-F(x)$ $(k = 0, 1, 2, \ldots, r-1$ and $m = 0, 1, 2, \ldots, n-r)$.

Therefore, the probability function of the above event can be written as

$$f_{r:n}(x) = P\{X_{r:n} = x\} . \tag{2}$$

(2) can be expressed as (1). Thus, the proof is completed.

We will now express the following theorem to obtain the *df* of *r*th order statistic from *innid* discrete random variables.

- **Theorem 2.**

$$F_{r:n}(x) = \sum_{x=0}^{x} \sum_{k=0}^{r-1} \sum_{m=0}^{n-r} \frac{1}{(r-1-k)!(k+1+m)!(n-m-r)!} per[\underset{r-1-k}{\mathrm{F}(x-)}\ \ \underset{k+1+m}{\mathrm{f}(x)}\ \ \underset{n-m-r}{1-\mathrm{F}(x)}] . \tag{3}$$

Proof. It can be written

$$F_{r:n}(x)=\sum_{x=0}^{x} f_{r:n}(x) \tag{4}$$

and using (1) in (4), (3) is obtained.

4. RESULTS (SONUÇLAR)

In this section, the results related to *pf* and *df* of $X_{r:n}$ will be given. We will now express the following result for *pf* of minimum order statistic.

- **Result 1.**

$$f_{1:n}(x)=\sum_{m=0}^{n-1}\frac{1}{(1+m)!(n-m-1)!}per[\underset{1+m}{\mathrm{f}(x)}\ \underset{n-m-1}{1-\mathrm{F}(x)}].$$

(5)

Proof. In (1), if $r=1$, (5) is obtained.

We will express the following result for *pf* of maximum order statistic.

- **Result 2.**

$$f_{n:n}(x)=\sum_{k=0}^{n-1}\frac{1}{(n-1-k)!(k+1)!}per[\underset{n-1-k}{\mathrm{F}(x-)}\ \underset{k+1}{\mathrm{f}(x)}]. \tag{6}$$

Proof. In (1), if $r=n$, (6) is obtained.

In the following result, we will express *df* of minimum order statistic.

- **Result 3.**

$$F_{1:n}(x)=\sum_{x=0}^{x}\sum_{m=0}^{n-1}\frac{1}{(1+m)!(n-m-1)!}per[\underset{1+m}{\mathrm{f}(x)}\ \underset{n-m-1}{1-\mathrm{F}(x)}]. \tag{7}$$

Proof. In (3), if $r=1$, (7) is obtained.

We will express the following result for *df* of maximum order statistic.

- **Result 4.**

$$F_{n:n}(x)=\sum_{x=0}^{x}\sum_{k=0}^{n-1}\frac{1}{(n-1-k)!(k+1)!}per[\underset{n-1-k}{\mathrm{F}(x-)}\ \underset{k+1}{\mathrm{f}(x)}]\ . \tag{8}$$

Proof. In (3), if $r=n$, (8) is obtained.

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