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THE SOLUTION OF DIFFERENTIAL EQUATIONS USING DUAL FUNCTIONS

ABSTRACT

In this paper a differential equation is considered. This equation is separated into dual and real parts such that the dual part of the equation is the higher order differential of each term in the real part. Solving these equations separately, one may observe that the solution of the dual part is obtained by adding a constant to the solution of the real part. Making use of this method, the solution of a higher order differential equation can be attained by reducing each term of it.

Keywords: Dual Numbers, Dual Vectors, Dual Function, Differential Equations, Linear Differential Equation

DUAL FONKSİYONLARI KULLANARAK DİFRERANSİYEL DENKLEMLERİN ÇÖZÜMÜ

ÖZET

Bu çalışmada herhangi bir diferensiyel denklem göz önüne alınmıştır. Bu denklem dual kısımdaki diferensiyel denklem reel kısımdaki diferensiyel denklemin her teriminin bir üst mertebeden diferensiyel olacak şekilde reel ve dual kısımlara ayrılmıştır. Bu kısımlardaki denklemler ayrı ayrı çözüldüğünde dual kısmın çözümünün, reel kısmın çözümüne bir sabit ilave edilerek bulunduğu görülmüştür. Bu metoddan faydalanarak yüksek mertebeden bir diferensiyel denklemin çözümünün, diferensiyel denklemin her teriminin mertebelerini indirgemek suretiyle kolayca elde edilmiş olacaktır.

Anahtar Kelimeler: Dual Sayılar, Dual Vektörler, Dual Fonksiyonlar, Diferansiyel Denklemler, Lineer Diferansiyel Denklem

1. INTRODUCTION (GİRİŞ)

A linear differential equation of order n, in the dependent variable y and the independent variable x, is an equation that can be expressed in the form

$$a_0(x)\frac{d^n y}{dx^n}+a_1(x)\frac{d^{n-1}y}{dx^{n-1}}+a_2(x)\frac{d^{n-2}y}{dx^{n-2}}+\ldots+a_{n-1}(x)\frac{dy}{dx}+a_n(x)y=f(x)$$

where $a_0(x)$ is not identically zero. An equation which is not linear is called a non-linear differential equation, [4].

If a and a^* are real numbers and $\varepsilon\neq 0$ $\varepsilon^2=0$ the combination, [5 and 6]:

$$A=a+\varepsilon a^* \tag{1}$$

is called a dual number. Hence ε dual unit . Dual numbers are considered as polynominal in ε, subject to the defining relation $\varepsilon^2=0$. Clifford defined the dual numbers and showed that they form algebra, not a field. The pure dual numbers εa^* are zero divisors $(\varepsilon a^*)(\varepsilon b^*)=0$. No numbers εa^* has an inverse in the algebra. However, the other laws of the algebra of dual numbers are the same as the laws of algebra of complex numbers. This means dual numbers form a ring over the real number field. For example, two dual numbers $A=a+\varepsilon a^*$ and $B=b+\varepsilon b^*$ are added component wise:

$$A+B=(a+b)+\varepsilon(a^*+b^*) \tag{2}$$

In addition, they are multiplied by

$$ab=ab+\varepsilon(a^*b+ab^*) \tag{3}$$

For the equality of A and B we have

$$A=B\Leftrightarrow a=b \text{ and } a^*=b^* \tag{4}$$

An oriented line in E^3 may be given by two points x and y on it. If λ is any non-zero constant, the parametric equation of the line is:

$$y=x+\lambda a \tag{5}$$

a is a unit vector along the line. The moment of a with respect to the origin coordinates is

$$a^*=x\times a=y\times a \tag{6}$$

This means that a and a^* are not indepented of choice of the points on the line. The two vectors a and a^* are not independent of one another, they satisfy the following equations;

$$\langle a,a\rangle=1,\quad \langle a,a^*\rangle=0 \tag{7}$$

The six components a_i, $a_i^*(i=1,2,3)$ of the vectors a and a^* are Plucker homogeneous line coordinates. Hence, two vector a and a^* determine the oriented line. A point z is on this line if and only if

$$z\times a=a^* \tag{8}$$

The set of all oriented lines E^3 is one-to-one correspondence with pairs of vectors subject to the conditions (1.7), and so we may expect to represent i, t as a certain four-dimensional set in R^6 of six components of real numbers; we may take the space D^3 of triples of dual numbers with coordinates;

$$X_i = x_i + \varepsilon x_i^* \quad \text{(i=1,2,3)} \tag{9}$$

Each line E^3 is represented by the dual vector in D^3

$$A = a + \varepsilon a^* \tag{10}$$

$$\langle A, A\rangle = \langle a, a\rangle + 2\varepsilon\langle a, a^*\rangle = 1 \tag{11}$$

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

Making use of this method, the solution of a higher order differential equation can be attained by reducing each term of it.

3. THE SOLUTION OF DIFFERENTIAL EQUATION WITH DUAL VARIABLE (DUAL DEĞİŞKENLİ DİFERANSİYEL DENKLEMLERİN ÇÖZÜMÜ)

Definition 12. If x and x* are real variable and $\varepsilon^2=0$, the combination

$$\begin{aligned}\Gamma\left(X, Y, Y', \ldots, Y^{(n)}\right) &= G(X) \qquad (12)\\ &= F\left(x, f(x), f'(x), \ldots, f^{(n)}(x)\right) + \varepsilon x^* \left[F'\left(x, f(x), f'(x), \ldots, f^{(n)}(x)\right)\right]\\ &= g(x) + \varepsilon x^* g'(x)\end{aligned}$$

is called a differential equation with dual variable where, from

$$\begin{aligned}X &= x + \varepsilon x^*\\ Y &= f(x) + \varepsilon x^* f'(x)\\ Y' &= f'(x) + \varepsilon x^* f''(x)\\ &.\\ &.\\ &.\\ Y^{(n)} &= f^{(n)}(x) + \varepsilon x^* f^{(n+1)}(x)\end{aligned}$$

If $y = f(x)$ is solution of real partial of differential equation (12), $y = f(x) + c$ is solution of dual partial where c is constant.

Lemma 12. If Y_{gi} is general solution of a differential equation of order i, then

$$\begin{aligned}&Y_{g_1}\\ &Y_{g_2} = Y_{g_1} + c_1\\ &Y_{g_3} = Y_{g_2} + c_2 x\\ &Y_{g_4} = Y_{g_3} + c_3 x^2\\ &.\\ &.\\ &.\\ &Y_{g_n} = Y_{g_{n-1}} + c_{n-1} x^{n-2}\end{aligned}$$

where dependent variable Y_{g_i} and independent variable x.

4. EXAMPLES (ÖRNEK)

Example 4.1. Under the above method, we have

$$\frac{d^2Y}{dX^2}+Y=\sin X$$

$$\Rightarrow \frac{d^2y}{dx^2}+y+\varepsilon x*\left(\frac{d^3y}{dx^3}+\frac{dy}{dx}\right)=\sin x+\varepsilon x*\cos x$$

$$\frac{d^2y}{dx^2}+y=\sin x \quad \text{(real section)}$$

$$\frac{d^3y}{dx^3}+\frac{dy}{dx}=\cos x \quad \text{(dual section)}$$

And therefore, we are obtained following:

$$y_{real}=c_1\cos x+c_2\sin x-\frac{1}{2}x\cos x$$

$$y_{dual}=y_{real}+c_3$$

$$y_{dual}=c_1\cos x+c_2\sin x-\frac{1}{2}x\cos x+c_3$$

$$Y_g=y_{real}+\varepsilon x*y_{dual}$$

Example 4.2. We consider a differential equation

$$\frac{d^4Y}{dX^4}+2\frac{d^2Y}{dX^2}+Y=X^2+X$$

then,

$$\left(\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y\right)+\varepsilon x*\left(\frac{d^5y}{dx^5}+2\frac{d^3y}{dx^3}+\frac{dy}{dx}\right)=x^2+x+\varepsilon x*(2x+1)$$

$$\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=x^2+x \quad \text{(real section)}$$

$$\frac{d^5y}{dx^5}+2\frac{d^3y}{dx^3}+\frac{dy}{dx}=2x+1 \quad \text{(dual section)}$$

and thus, following obtained:

$$y_{real}=(c_1x+c_2)\sin x+(c_3x+c_4)\cos x+x^2+x-4$$

$$y_{dual}=y_{real}+c_5$$

$$y_{dual}=(c_1x+c_2)\sin x+(c_3x+c_4)\cos x+x^2+x-4+c_5$$

$$Y_g=y_{real}+\varepsilon x*y_{dual}$$

5. RESULT (SONUÇ)

As a result, this method for the solution of differential equations with higher order will provide a great convenience.

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