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MOMENT GENERATING FUNCTIONS OF SAMPLE MINIMUM OF ORDER STATISTICS FROM GEOMETRIC DISTRIBUTIONS

ABSTRACT

More advance, it has studied the paper that finding of expected values for sample range of order statistics from the geometric distribution. In this paper, for sample minimum of order statistics from geometric distributions, moment generating function is obtained.

Keywords: Order Statistics, Moment Generating Functions, Geometric Distribution, Distribution Function, Sample Extremes

GEOMETRİK DAĞILIMDAKİ SIRA İSTATİSTİKLERİN ÖRNEK MİNİMUMUNUN MOMENT ÇIKARAN FONKSİYONU

ÖZET

Geometrik dağılımdaki sıra istatistiklerin örnek aralığının beklenen değerinin bulunmasıyla ilgili çalışmalar daha önce yapılmıştır. Bu makalede ise geometrik dağılımdaki sıra istatistiklerin örnek minimumunun moment çıkaran fonksiyonu bulunmuştur.

Anahtar Kelimeler: Sıra İstatistikleri, Moment Çıkaran Fonksiyon, Geometrik Dağılım, Dağılım Fonksiyonu, Örnek Ekstremleri.

1. INTRODUCTION (GİRİŞ)

Let $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ be the order statistics obtained from a sample from a discrete distribution.
Order statistics from the geometric distribution have been studied by many author, see Abdel-Aty (1954) and Morgolin & Winokur (1967). In particular, characterizations of the geometric distribution using order statistics have received great attention; for example, see Ferguson (1967), Srivastava (1974) and Galambos (1975).

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

Geometric distribution has an important place in order statistics. Therefore, moments obtained from this distribution plays role in applications areas from order statistics. But, moment generating function of sample minimum of order statistics from geometric distribution can be given, only. In further studies, finding the moment generating function of sample maximum of order statistics from geometric distribution will aim.

3. THE DISTRIBUTIONS OF ORDER STATISTICS (SIRA İSTATİSTİKLERİN DAĞILIMI)

Let $F_{r:n}(x)\,(r=1,2,\ldots,n)$ denote by the *cdf* of $X_{r:n}$. Then can be seen easily,

$$F_{r:n}(x) = \Pr\{X_{r:n} \le x\}$$

$$= \sum_{i=r}^{n} \binom{n}{i} [F(x)]^i [1-F(x)]^{n-i}, \ -\infty < x < \infty \quad (3.1)$$

Thus, we find that the *cdf* of $X_{r:n}$ $(1 \le r \le n)$ is simply the tail probability of a binomial distribution with $F(x)$ success and *n* trials.

The cumulative distribution function of the 1 *th* and n *th* order statistics follow from (2.1) (when *r=1* and *r=n)* to be

$$F_{1:n}(x) = 1-[1-F(x)]^n, \qquad -\infty < x < \infty$$

and

$$F_{n:n}(x) = [F(x)]^n \qquad -\infty < x < \infty$$

respectively. Furthermore, by using the identity that

$$\sum_{i=r}^{n} \binom{n}{i} p^i (1-p)^{n-i} = \int_0^p \frac{n!}{(r-1)!(n-r)!} t^{r-1}(1-t)^{n-r}\, dt \qquad 0 < p < 1. \quad (3.2)$$

we can write the *cdf* of $X_{r:n}$ from (2.1) equivalently as

$$F_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!} \int_0^{F(x)} t^{r-1}(1-t)^{n-r}\, dt \qquad -\infty < x < \infty. \quad (3.3)$$

(Arnold et.al, 1992 and David, 1981).

Observe that all the expressions given above hold for any arbitrary population whether continuous or discrete. For discrete population, the probability mass function of $X_{r:n}\,(1 \le i \le n)$ may be obtained from (2.3) by differencing as

$$f_{r:n}(x) = \Pr\{X_{r:n} = x\} = F_{r:n}(x) - F_{r:n}(x-)$$
$$= C(r:n) \int_{F(x-)}^{F(x)} t^{r-1}(1-t)^{n-r}\,dt \quad (3.4)$$

where $C(r:n) = \dfrac{n!}{(r-1)!(n-r)!}$ (Balakrishnan and Rao, 1998). (3.5)

In particular, we also have

$$f_{1:n}(x) = \int_{F(x-)}^{F(x)} n(1-t)^{n-1}dt = [\overline{F}(x-)]^n - [\overline{F}(x)]^n \quad , \quad \overline{F}(x) = 1 - F(x). \quad (3.6)$$

and

$$f_{n:n}(x) = \int_{F(x-)}^{F(x)} nt^{n-1}dt = [F(x)]^n - [F(x-)]^n. \quad (3.7)$$

(Arnold et.al, 1992 and David,1981, Balakrishnan and Rao, 1998).

4. ORDER STATISTICS FROM GEOMETRIC DISTRIBUTIONS (GEOMETRİK DAĞILIMDAKİ SIRA İSTATİSTİKLERİ)

Let, X be geometric distribution. Note that its pmf and cdf is given by $f(x) = pq^{x-1}$ and $F(x) = 1 - q^x$, $x \in Z^+$, respectively. Consequently the cdf of r *th* order statistics is given by

$$F_{r:n}(x) = \sum_{i=r}^{n} \binom{n}{i} (1-q^x)^i q^{x(n-i)} \quad (4.1)$$

From (2.4), the pmf of minimum order statistics is given by

$$f_{1:n}(x) = (1-q^n) q^{n(x-1)} \quad (4.2)$$

5. THE MOMENT GENERATING FUNCTIONS OF GEOMETRIC ORDER STATISTICS GEOMETRIK SIRA İSTATİSTİKLERİN MOMENT ÇIKARAN FONKSİYONLARI)

$X_{1:n} \le X_{2:n} \le \ldots \le X_{n.n}$ are indicate order statistics. Let $M_{X_{r:n}}$ be moment generating function of r *th* order statistics

$$M_{X_{r:n}}(t) = \sum_x e^{tx} f_{r:n}(x)$$

Thus, moment generating function of sample minimum of order statistics from geometric distribution expressed by the following theorem.

Theorem 1 (Teorem 1): Let $X_1, X_2, \ldots, X_n$ be random variables from a geometric distributions and $X_{1:n}$ be 1 th order statistics corresponding to these random variables. Then, moment generating function of $X_{1:n}$,

$$M_{X_{1:n}}(t) = (1-q^n)\frac{e^t}{1-e^t q^n}$$

Proof (İspat): If expression in (3.2) of $f_{1:n}(x)$ is written in definition of moment generating function, first and second moments of $X_{1:n}$

$$M_{X_{1:n}}(t) = (1-q^n)\sum_{x=1}^{\infty} e^{tx} q^{n(x-1)}$$
$$= (1-q^n)q^{-n}\sum_{x=1}^{\infty}(e^t q^n)^x$$
$$= (1-q^n)\frac{e^t}{1-e^t q^n}$$

Therefore, the proof is complete.

In particular, if sequential derivatives of $M_{X_{1:n}}(t)$ are taken, in Theorem 1

$$E(X_{1:n}) = \frac{1}{1-q^n}$$
$$Var(X_{1:n}) = \left[\frac{1+q^n}{(1-q^n)^2}\right] - \left[\frac{1}{1-q^n}\right]^2 = \frac{q^n}{(1-q^n)^2}$$

are obtained.

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