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Abdülkadir Şengür
İbrahim Türkoğlu
Melih Cevdet İnce
University of Firat
ksengur@firat.edu.tr
Elazig-Turkiye

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AN UNSUPERVISED IMAGE SEGMENTATION USING POISSON MARKOV RANDOM FIELDS

ABSTRACT

Markov random field (MRF)-based image segmentation methods have gained considerable interest over the last few decades. The ubiquitous of MRF is the conditional model that has a joint Gaussian distribution so it is called Gaussian MRF (GMRF). On the other hand, Pal and Pal (1991), proposed that image histograms were more appropriately modeled by the mixture of Poisson distributions. Therefore, in this paper, we proposed a simple unsupervised Poisson MRF (PMRF) for gray level image segmentation. The proposed PMRF has been tested on a variety of images including artificial images and real world images. Experimental results show that by visually and by numerically comparing, it is obvious that using PMRF model generates much more accurate results than the GMRF.

Keywords: Unsupervised Image Segmentation, Poisson Distribution, Markov Random Fields, Expectation Maximization

POİSSON MARKOV RASSAL ALANLARI İLE EĞİTİCİSİZ GÖRÜNTÜ BÖLÜTLEME

ÖZET

Son zamanlarda Markov Rassal Alanları temelli görüntü bölütleme yöntemleri bir hayli ilgi çekmiştir. MRF'ler genellikle Gauss dağılımlı şartlı modeller olup, bundan dolayı çoğunlukla Gauss MRF (GMRF) olarak adlandırılırlar. Diğer taraftan, Pal ve Pal (1991)'de, gri seviyesi görüntülerin histogramlarının modellenmesinde karma Poisson dağılımının kullanılmasının daha uygun olduğunu göstermiştir. Böylece bu çalışmada, basit eğiticisiz bir yapı olan Poisson MRF (PMRF) önerilmiştir. Önerilen PMRF başarımı, birçok yapay ve gerçek dünya görüntüleri üzerinde test edilmiştir. Deneysel sonuçlar hem görsel hem de sayısal olarak önerilen bu yeni yaklaşımın etkinliğini ve GMRF'ye olan üstünlüğünü göstermiştir.

Anahtar Kelimeler: Eğiticisiz Görüntü Bölütleme, Poisson Dağılımı, Markov Rassal Alanları, Beklentilerin Maksimizasyonu

1. INTRODUCTION (GİRİŞ)

One of the important tasks of early vision problem is image segmentation. Image segmentation can be viewed as the process of separating an image into some disjoint homogenous regions [1]. In other words, image segmentation is the process of grouping pixels of a given image into regions with respect to certain features and with semantic content [2].

Its purpose is to extract labeled regions or boundaries for targeted objects for subsequent applications such as object recognition.

Intensive research and various segmentation methods have been proposed over the last few decades. Thresholding is the most popular approach [3]. Clustering methods [4], region growing and splitting methods [5] and multi resolution [6] techniques are the other proposed approaches. Among these methods, Markov random field (MRF)-based image segmentation methods have gained considerable interest. Using MRF models for image segmentation has a number of advantages. First, the spatial relationship can be seamlessly integrated into a segmentation procedure. Second, MRF based segmentation model can be inferred in the Bayesian framework which is able to utilize various kind of image features. Third, the label distribution can be obtained when maximizing the probability of the MRF model [7]. Since the seminal paper of Besag [8], MRF has been introduced to image processing and the computer vision community and there have been many methodological developments accompanied by important applications [9]. MRFs are powerful tools for image processing because it describes an image as the local interactions of the neighboring pixels. During the past years, many articles presented about the MRF image segmentation. Liu et al. [10], proposed a multiresolution color image segmentation algorithm which uses MRF. The proposed approach is a relaxation process that converges to the MAP estimate of the segmentation. They also proposed an evaluation function. The main advantage of this is that it incorporates the heuristic criteria used to evaluate segmentation result without requiring any threshold value. Dubes et al. [11], carried out an empiric comparative study for three MRF-based segmentation algorithms. Çeşmeli et al. [12], proposed a Gaussian MRF and Locally Excitatory Globally Inhibitory Oscillator Networks (LEGION) for texture analysis. Their algorithm is composed of two main parts. A set of GMRF based texture features is the first part of the algorithm. The second part is LEGION which is a 2D array of neural oscillators. Sarkar et al. [13], proposed a simple technique, which has been suggested to obtain optimal segmentation based on tonal and textural characteristics of an image using MRF model. The technique takes an initially over segmented image as well as the original image as its inputs and defines MRF over the region adjacency graph of the initially segmented regions. Kato et al. [14], proposed an unsupervised MRF model for color image segmentation. Their algorithm estimates initial mean vectors if the image histogram does not have clearly distinguishable peaks. Yang et al. [15], presented an unsupervised texture segmentation method. They used boundary MRFs for refining the coarse segmentation. Ibanez et al. [16], made a comparative study of MRF image segmentation and parameter estimation problem. Deng et al. [17], presented as simple MRF schema which automatically estimate the model parameters and produce accurate unsupervised segmentation results. Clausi et al. [18], compared the discrimination ability of two texture analysis methods. These methods are MRFs and gray-level co-occurrence probabilities. Kim et al. [19], proposed an unsupervised method for segmenting video sequences degraded by noise. Each frame is modeled using MRF and the energy function of each MRF is minimized by a genetic algorithm. Yu et al. [20], presented a Metropolis-Hastings algorithm and gradient method to estimate the MRF parameters.

The ubiquitous of MRF is the conditional model, which has a joint Gaussian distribution. This is perhaps because aggregated data are often Gaussian due to the Central limit theorem and spatial data often exhibit dependence that increases with their proximity each other. Moreover the gray level histogram is often modeled as a mixture of Gaussian distributions.

On the other hand, Pal and Pal [21], proposed that image histograms are more appropriately modeled by the mixture of Poisson distributions. In reference 21 modeling of gray level histogram by a mixture of Poisson distributions has been derived based on the theory of formation of image. Thus in this paper, we propose a Poisson MRF (PMRF) for gray level image segmentation. Mainly we want to show the application of the PMRF which performs better segmentation results than Gaussian MRFs. We present some experimental results for real and artificial images.

The rest of the paper is organized in as following way: Section 2 presents the simple PMRF based segmentation model. Section 3 discusses how to implement the segmentation model. Section 4 presents the experimental study and results. Conclusions are drawn in section 5.

2. IMAGE SEGMENTATION MODEL (GÖRÜNTÜ BÖLÜTLEME MODELİ)

This section introduces the general framework to MRF image analysis and gives a brief overview of the MRF theory. MRF is n-dimensional random process defined on a discrete lattice. Usually the lattice is a regular 2-dimensional grid in the plane [22]. A random field can be considered as a MRF, if its probability distribution at any site depends only upon its neighborhood [8]. According to the Cliff-Hammersley theorem, any MRF can be described by a probability distribution of the Gibbs form:

$$p(x) = \frac{1}{Z} e^{(-U(x))} \qquad (1)$$

Where x is the random field, Z is the normalization constant and the energy function $U(x)$ is defined as;

$$U(x) = \sum_{c \in C} V_c(x) \qquad (2)$$

Where $V_c(x)$ is the potential function. We assume that the image is defined on an MxN rectangular lattice $L = \{(i,j), 1 \le i \le M, 1 \le j \le N\}$ and c is a set of pixels, called a clique that consists of either a single pixel or a group of pixels. Figure 1-(a), demonstrates the first-order spatial neighbors of a site t as 1, second order neighbor as 2 so on and figure 1-(b) provides a convenient labeling for neighbors of each pixel.

5	4	3	4	5
4	2	1	2	4
3	1	t	1	3
4	2	1	2	4
5	4	3	4	5

(a)

t:-11	t:-7	t:-6	t:+8	t:+12
t:-9	t:-3	t:-2	t:+4	t:+10
t:-5	t:-1	t	t:+1	t:+5
t:-10	t:-4	t:+2	t:+3	t:+9
t:-12	t:-8	t:+6	t:+7	t:+11

(b)

Figure 1. Definition of neighborhoods and relative neighborhoods
(Şekil 1. Komşulukların tanımlanması ve bağıl komşuluklar)

The image observed is denoted by the *MN*-vector random variable *Y* and is obtained by adding a noise process to the true image. Therefore, the density model for Y given the true image is;

$$f(y \mid X = x) = \prod_{t=1}^{MN} f_t(y_t \mid x_t) \tag{3}$$

Note that $f_t(. \mid x_t)$ is the conditional density function for Y_t, the gray level at pixel *t*. We take $f_t(y_t \mid x_t)$ to be the Poisson density function with sample mean λ_{x_t} and it is defined as;

$$f_t(x_t \mid \lambda) = \frac{\lambda^{x_t}}{x_t!} e^{-\lambda} \tag{4}$$

The a posteriori probability mass function for the pixel labels *X*, given the observed image *Y=y* also has the form of a Gibbs random fields respect to a neighborhood system cliques.

$$P(X = x \mid Y = y) = \frac{e^{-U(x|y)}}{Z} \tag{5}$$

Where Z is the normalizing constant and the energy function is as follows;

$$U(x \mid y) = \sum_{t=1}^{MN} [x_t \ln(\lambda_{x_t}) - \lambda_{x_t} - \ln(x_t!) + \sum_{r=1}^{c} [\theta_r J(x_t, x_{t:+r})]] \tag{6}$$

Where $J(a, b) = -1$ if $a=b$, 0 if $a \neq b$ and $c=2$ for first-order neighbor model. $[\theta_1, ..., \theta_c]$ are the clique parameters. The local properties of an MRF can be derived from Gibbs random fields. Let $X_{\partial t}$ be a random variable presenting the gray level of neighbor of pixel *t* denoted by $[x_{t:+r}\ x_{t:-r}]$ for *r* from *1* to *c*. The conditional probability of X_t can be written as [11];

$$P(X_t = x_t \mid X_{\partial t} = x_{\partial t}, Y = y) = \frac{e^{-U_t(x_t, x_{\partial t}|y)}}{Z} \tag{7}$$

and

$$U(x_t, x_{\partial t} \mid y) = \sum_{t=1}^{MN} [x_t \ln(\lambda_{x_t}) - \lambda_{x_t} - \ln(x_t!) + \sum_{r=1}^{c} \theta_r [J(x_t, x_{t:+r}) + J(x_t, x_{t:-r})]] \tag{8}$$

Now the segmentation problem is considered as observing y and estimating the labels in the true image. The Maximum A-Posteriori

(MAP) estimate is the vector x' which maximizes *P(X=x | Y=y)* with respect to true image *x*.

3. ITERATED CONDITIONAL MODES (ICM) (ŞARTLI İTERATİF KİPLER)

ICM is an optimization method. Besag [8] proposed the ICM method as a computationally feasible alternative to MAP. In ICM, all sites are visited iteratively without restriction where the label that yields the maximum a posterior probability accepted as the estimate for the site. It is motivated for reducing the computational time produced by using the stochastic techniques such as Gibbs sampler. The ICM method can be summarized by the following equation where the label of the pixel *t,* given the observed image y and the current estimates $x_{\partial t}$ of the labels of all pixels in the neighborhood of pixel *t*.

$$\begin{aligned} &P(X_t = x_t \mid y, X_{S|r} = x_{S|r}) \\ &= f_t(y_t \mid x_t) P(X_t = x_t \mid X_{\partial t} = x_{\partial t}) \end{aligned} \tag{9}$$

Maximizing the conditional probability in eq. (9) is equivalent to minimizing the energy function which is given in eq. (8). The ICM algorithm can be represented as follows;

- **Step 1:** Initialize x' by maximizing $f_t(y_t \mid x_t)$ for all pixels.
- **Step 2:** For t=1 to MN, update $x_t^{'}$ to the value of x_t which maximizes energy function in eq. (8).
- **Step 3:** Go to the step 2 for N times.

4. PARAMETER ESTIMATION WITH EM ALGORITHM (EM ALGORITMASI İLE PARAMETER TAHMİNİ)

Our goal is to segment the observed image using an unsupervised PMRF-based classification algorithm and compare its performance to the GMRF. For estimating the Poisson and Gaussian probability distributions of the labels in the observed image, we need to estimate the sample mean λ_{x_t} , for Poisson distribution and μ_{x_t} and the variance $\sigma_{x_t}^2$ for Gaussian distribution of the each class label. There is no prior information so we can not use maximum likelihood approach for estimating the parameters of the probability distributions of the each class. In statistics, this problem is called as the incomplete data problem [23]. Here, we only give the parameter estimation of the Poisson distribution; Gaussian parameters estimation can be found in [24]. EM algorithm, which has been proposed by Dempster et al., aims to find these parameters [25]. EM algorithm consists of an E-step and an M-step and it starts with initial values $p_m^0 \; and \; \lambda_m^0$ for the parameters and iteratively performs these two steps until convergence. Suppose that θ^t denotes the estimation of θ obtained after the *t* th iteration of the algorithm. Then at the *(t+1)* th iteration the E-step computes the expected complete log-likelihood function;

$$Q(\theta, \theta^t) = \sum_{k=1}^{K} \sum_{m=1}^{M} \{\log \alpha_m p(x_k \mid \theta_m)\} P(m \mid x_k; \theta^t) \tag{10}$$

where $P(m \mid x_k; \theta^t)$ is a posterior probability and it is computed as follows;

$$P(m \mid x_k; \theta^t) = \frac{\alpha_k^t p(x_k \mid \theta_m^t)}{\sum_{l=1}^{M} \alpha_l^t p(x_k \mid \theta_l^t)} \tag{11}$$

The M-step finds the *(t+1)* estimation θ^{t+1} of θ by maximizing Q (θ, θ^{t})

$$\alpha_m^{t+1} = \frac{1}{K}\sum_{k=1}^{K} P(m \mid x_k;\theta^t) \tag{12}$$

$$\lambda_m^{t+1} = \frac{\sum_{k=1}^{K} x_k P(m \mid x_k;\theta^t)}{\sum_{k=1}^{K} P(m \mid x_k;\theta^t)} \tag{13}$$

5. EXPERIMENTAL RESULTS AND EVALUATION OF THE ALGORITHMS (DENEYSEL SONUÇLAR VE ALGORİTMANIN DEĞERLENDİRİLMESİ)

Segmentation results are often evaluated only on the basis of observers' impressions and the effectiveness of the segmentation results in the subsequent application steps of the processing. To appraise the performance of the algorithms more objectively, we have used the evaluation function proposed by Liu et al. [10]. The advantage of this method is that it incorporates the heuristic criteria used to evaluate the segmentation result without requiring any threshold value. The function is as follows;

$$F(I) = \frac{1}{1000xMxN}\sqrt{R}\sum_{i=1}^{R}\frac{e_i^2}{\sqrt{A_i}} \tag{14}$$

Where *I* is the image to be segmented, *N x M* is the size of image and *R* the number of the region of the segmented image. A_i, the area, or the number of pixels of the *i*th region, and e_i the color error of region *i*. e_i is calculated as the sum of the Euclidean distance of the color vectors between the original image and the segmented image of each pixel in the region. The smaller the value of the *F (I)* , the better the segmentation results.

The proposed PMRF has been tested on a variety of images including artificial images and real world images. The test program has been implemented in MATLAB. We present the examples of these results and compare GMRF segmentation results with PMRF segmentation results.

Figure 2 shows a block diagram of our study. Before starting the MRF segmentation, the conditional probability density functions of the each region in the observed image has to be calculated. So, we employ the EM algorithm for estimating the parameters of the probability density functions of the regions. The number of the regions in the image is given by the user. The user may look at histogram of image for deciding about the number of region in the image. But this situation may not be true for all time because of the noise.

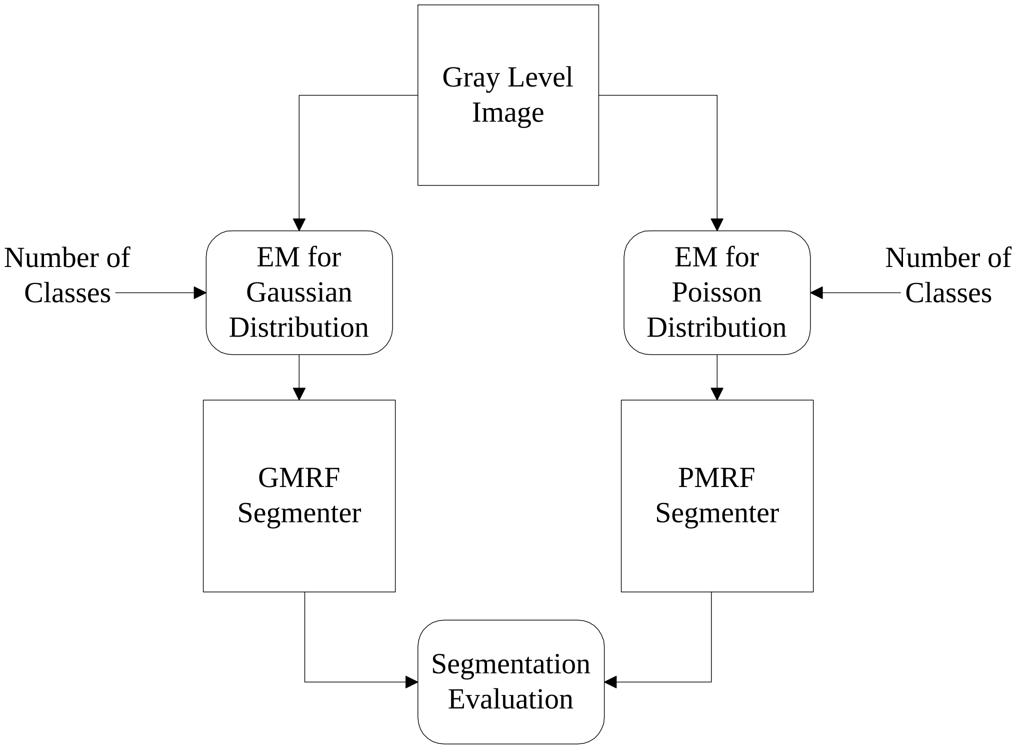

Figure 2. The block diagram of the proposed system
(Şekil 2. Önerilen systemin blok diyagramı)

The initial parameters for EM algorithm can be chosen randomly. But, this can cause a slow convergence, so we use K-means algorithm for initialization. For MRF parameters, we assign fixed value 2 for all $[\theta_1,...,\theta_c]$ clique parameters. We generate several artificial gray level images of size 128 x128 on MATLAB environment. Several real world images are taken from the web site of *caip.rutgers.edu* [26]. Color images are than converted to the gray level images. Figure 3 shows a three-class artificial image, its histogram and segmentation results obtained by GMRF and PMRF. Here, when we by visually count the misclassified pixels, the PMRF performs only one pixel better segmentation than GMRF. The *F* value for both segmentation algorithms is given at Table 1. This value also shows the PMRF superiority for this image sample. Figure 4, 5, 6, 7 and 8 shows the real world images and the segmented versions according to the GMRF and PMRF. By visually comparing, it is obvious that using PMRF model, much more accurate results are obtained than the GMRF. The PMRF segmentation accuracy has been supported by the F values for each image (Table 1).

Figure 9 and Figure 10 shows the noisy real world images SNR =20 dB and SNR = 10 dB respectively. The segmentation results are also given at Table 1. From these results, it is seen that PMRF once more produces good results than GMRF.

For sake of justice, we state the all conditions same and the images pass through the same process during the segmentation period. ICM is an iterative algorithm. So, the stopping criterion for ICM algorithm is that the segmentation procedure stops when there is no changing for the label every pixel. When we compare the both algorithms according to the computation time, EM+PMRF is less than EM+GMRF process. Moreover, while estimating the parameters of the each region distribution with EM, we run the simulations several times for guarantying the convergence.

Table 1. Evaluation of the segmentation algorithms
(Tablo 1. Bölüteme algoritmalarının değerlendirilmesi)

Image	GMRF	PMRF
Fig. 3	2.2482e+003	2.2463e+003
Fig. 4	1.5750e+004	3.7916e+003
Fig. 5	3.2445e+004	1.2124e+004
Fig. 6	1.2821e+004	8.8572e+003
Fig. 7	1.5627e+004	1.2334e+004
Fig. 8	2.0588e+004	8.8572e+003
Fig. 9	1.1499e+005	3.0437e+004
Fig. 10	9.3786e+004	5.1147e+004

6. CONCLUSIONS (SONUÇLAR)

In this paper, we have proposed a simple unsupervised gray level image segmentation algorithm. The segmentation model is based on PMRF model. Most of the MRF based segmentation method uses Gaussian mixture model for conditional probability density fitting on the other hand, image histograms are more appropriately modeled by the mixture of Poisson distributions [21]. So we consider that PMRF structure performs better segmentation results than GMRF structure. The proposed PMRF has been tested on a variety of images including artificial images and real world images. The test program has been implemented in MATLAB. We present the segmentation results and compare GMRF segmentation results with PMRF segmentation results. Experimental results show that by visually and by numerically comparing, it is obvious that using PMRF model generates much more accurate segmentation results than the GMRF model.

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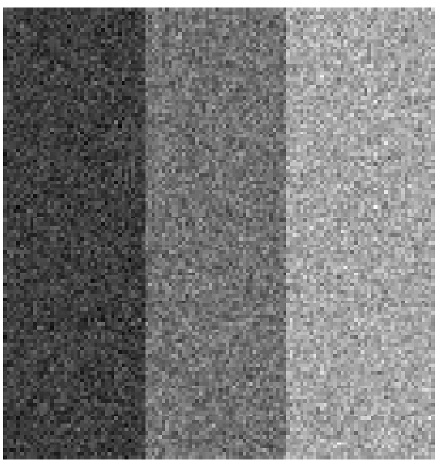
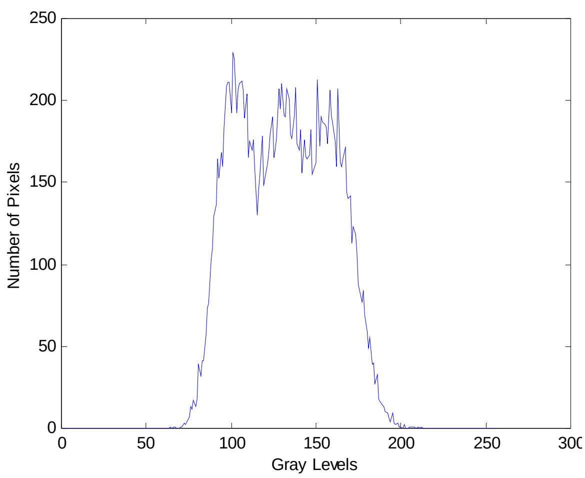
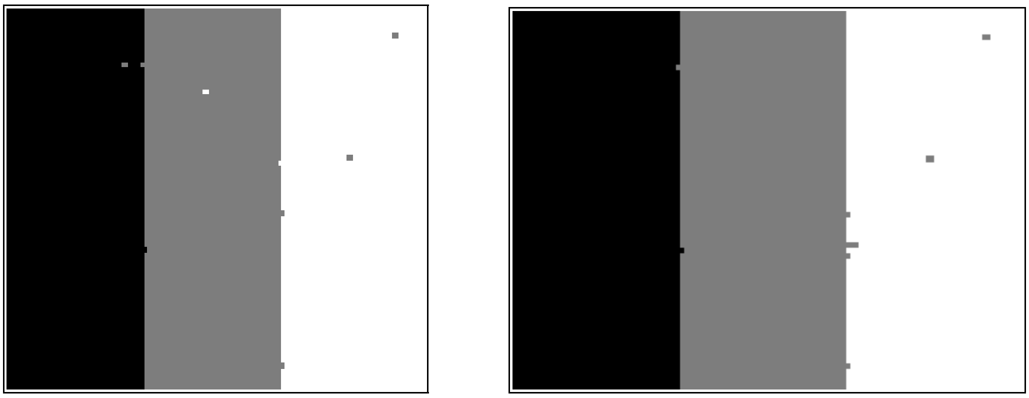

(a)

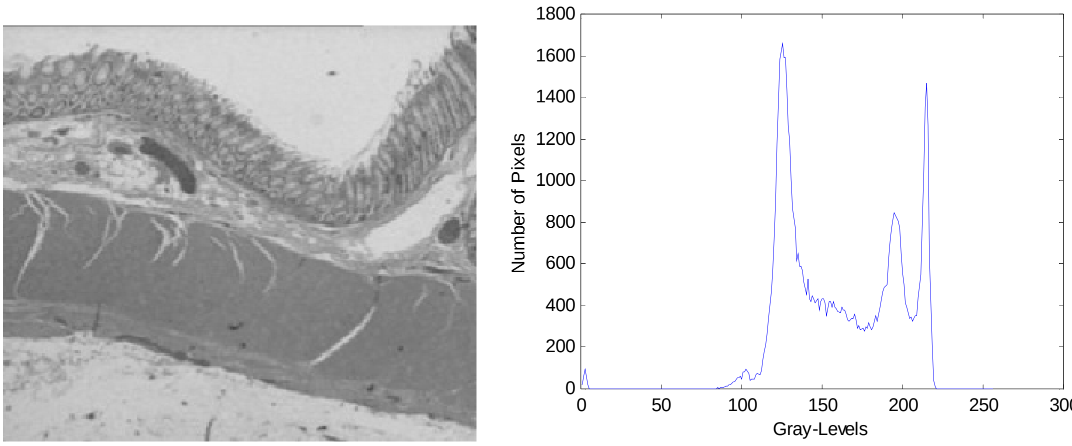


(b)

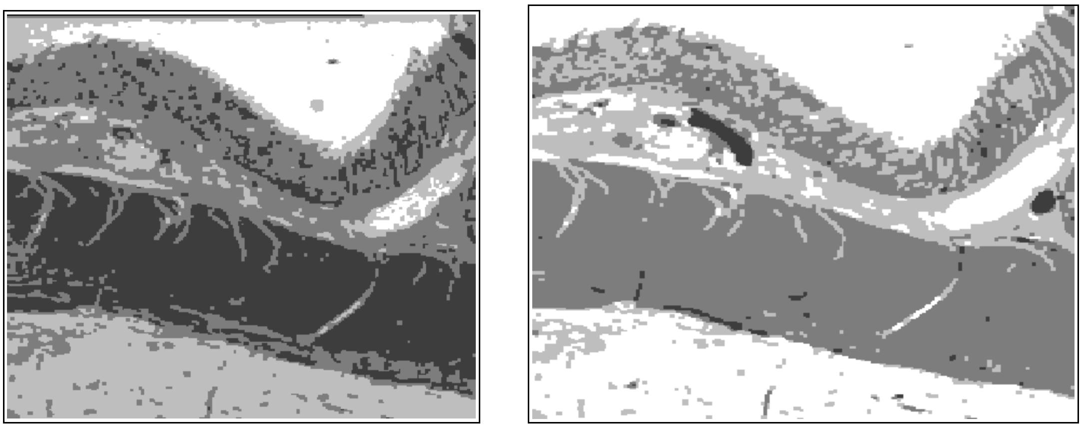


(c) (d)

Figure 3. (a) Original three-class synthetic image (b) Histogram (c) GMRF segmentation result (d) PMRF segmentation result
(Şekil 3. (a) Orijinal üç sınıf yapay görüntü (b) Histogramı (c) GMRF bölütleme sonucu (d) PMRF bölütleme sonucu)

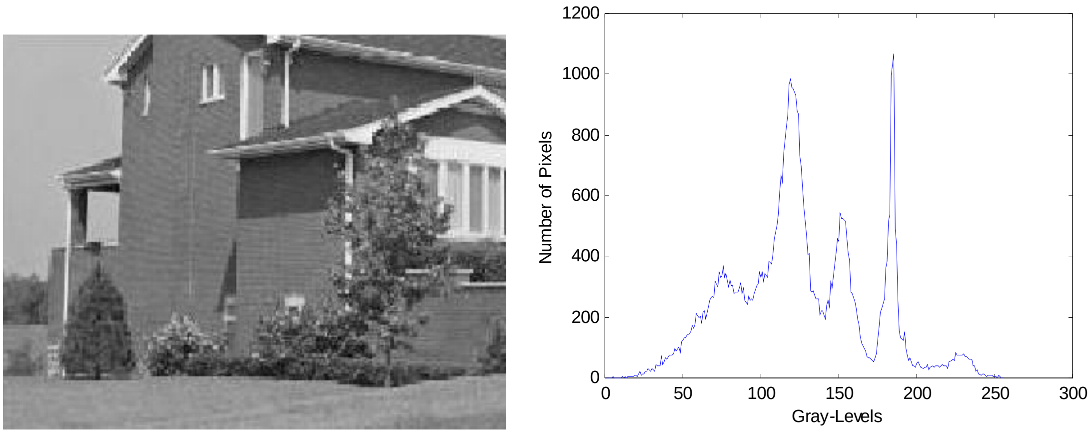


(a) (b)



(b) (d)

Figure 4. (a) Original 'path' image (b) Histogram (c) GMRF segmentation result (d) PMRF segmentation result
(Şekil 4. (a) Orijinal 'path' görüntüsü (b) Histogramı (c) GMRF bölütleme sonucu (d) PMRF bölütleme sonucu)

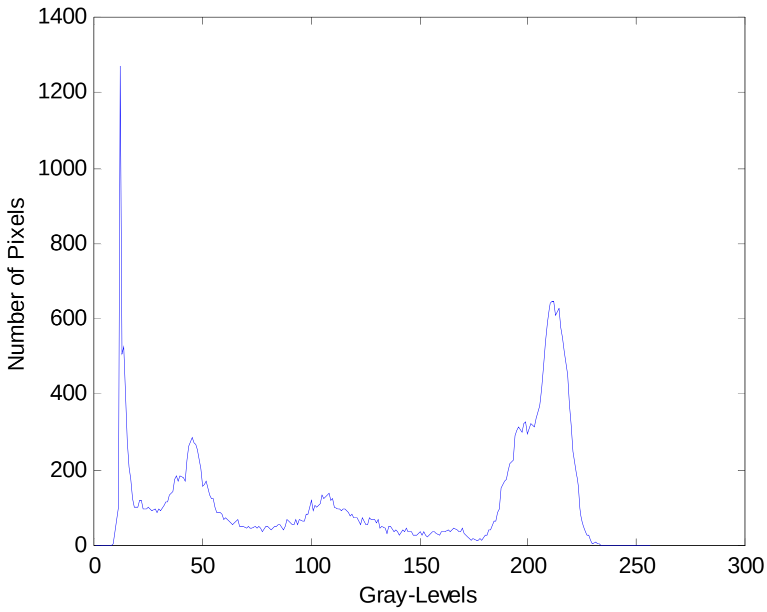


(a) (b)

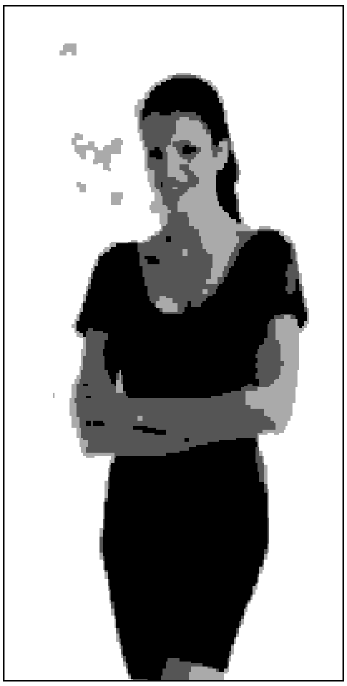
(c) (d)

Figure 5. (a) Original 'smhouse' image (b) Histogram (c) GMRF segmentation result (d) PMRF segmentation result
(Şekil 5. (a) Orijinal 'smhouse' görüntüsü (b) Histogramı (c) GMRF bölütleme sonucu (d) PMRF bölütleme sonucu)

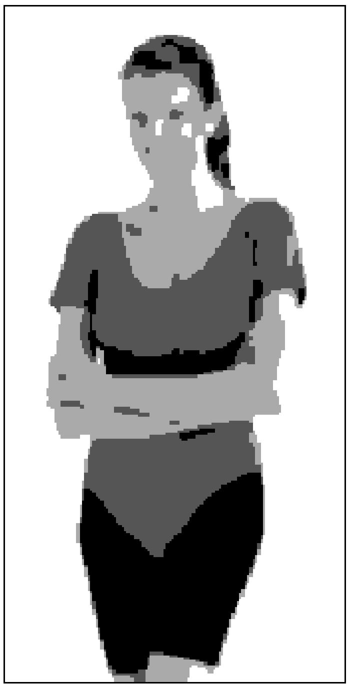
(a)



(b)



(c)



(d)

Figure 6. (a) Original ‘woman’ image (b) Histogram (c) GMRF segmentation result (d) PMRF segmentation result
(Şekil 6. (a) Orijinal ‘Kadın’ görüntüsü (b) Histogramı (c) GMRF bölütleme sonucu (d) PMRF bölütleme sonucu)

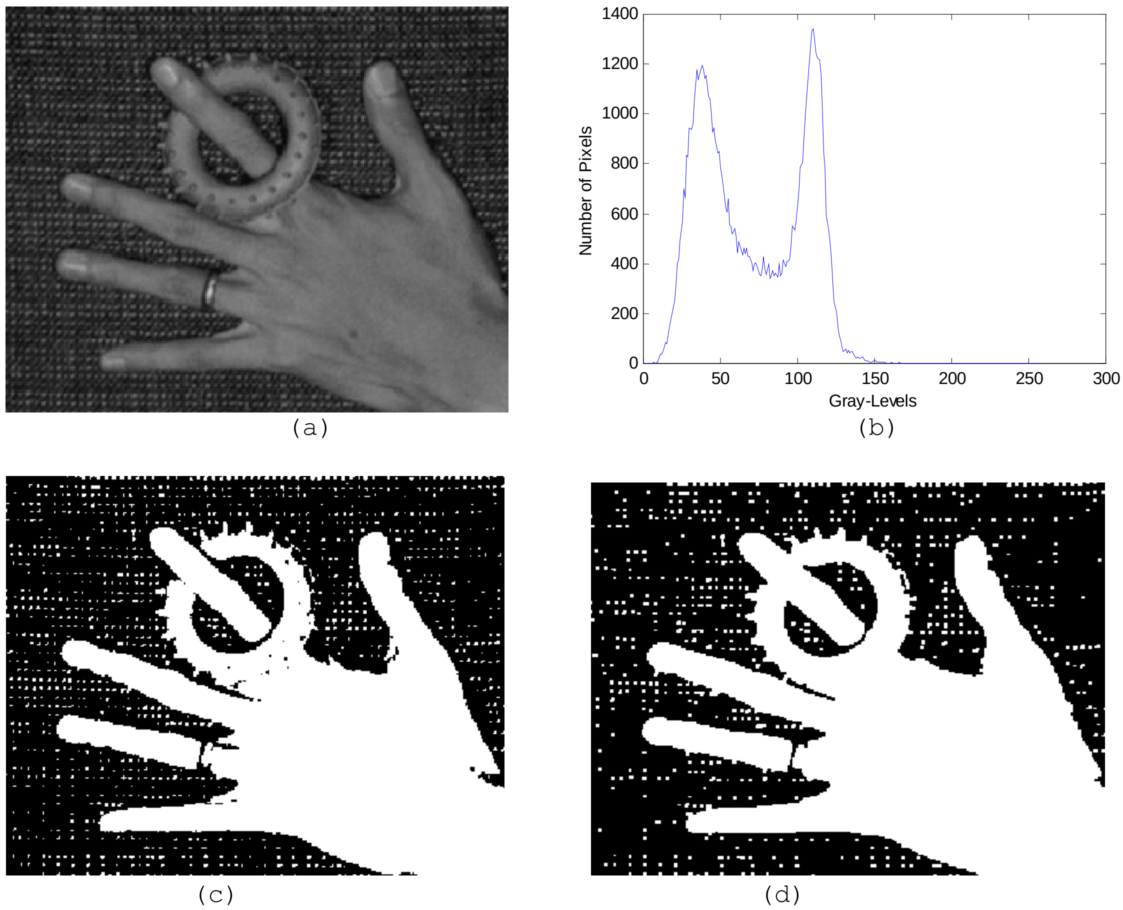


Figure 7. (a) Original ‘Hand’ image (b) Histogram (c) GMRF segmentation result (d) PMRF segmentation result
(Şekil. 7 (a) Orijinal ‘El’ görüntüsü (b) Histogramı (c) GMRF bölütleme sonucu (d) PMRF bölütleme sonucu)

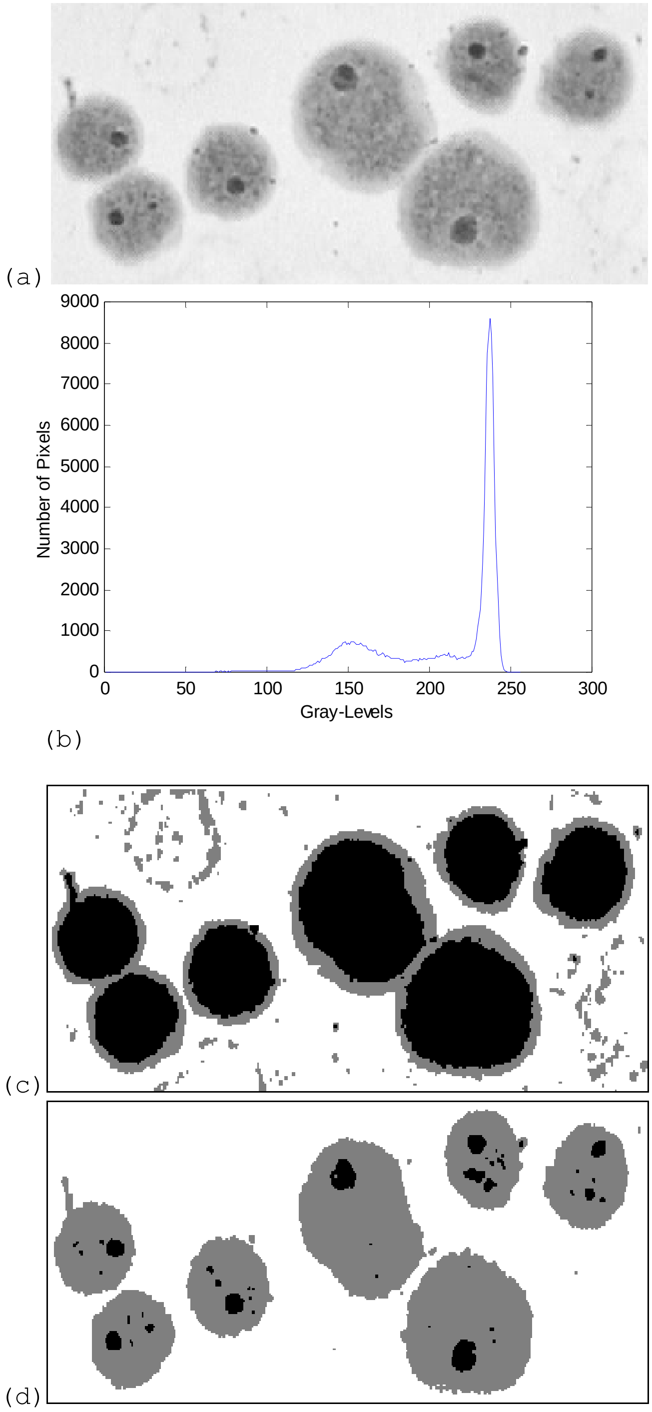


Figure 8. (a) Original 'Leucemia' image (b) Histogram (c) GMRF segmentation result (d) PMRF segmentation result
(Şekil 8. (a) Orjinal 'Lösemi' görüntüsü (b) Histogramı (c) GMRF bölütleme sonucu (d) PMRF bölütleme sonucu)

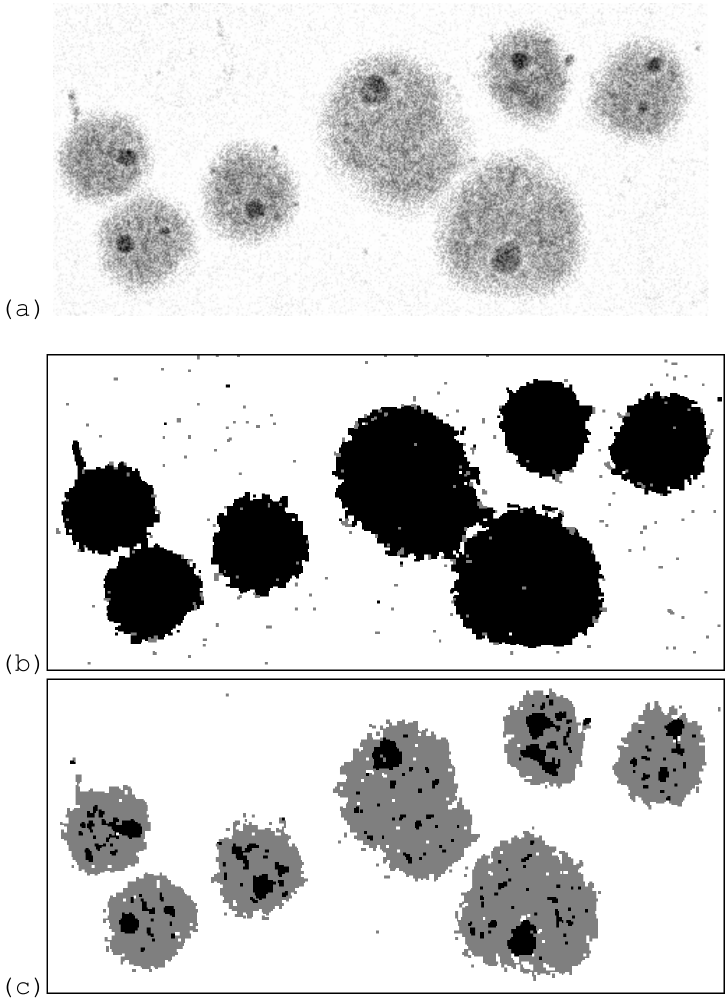


Figure 9. (a) SNR = 20 dB Original noisy Leucemia image (b) GMRF segmentation result (c) PMRF segmentation result
(Şekil 9. (a) SNR=20 Orijinal gürültülü 'Lösemi' görüntüsü (b) GMRF bölütleme sonucu (c) PMRF bölütleme sonucu)

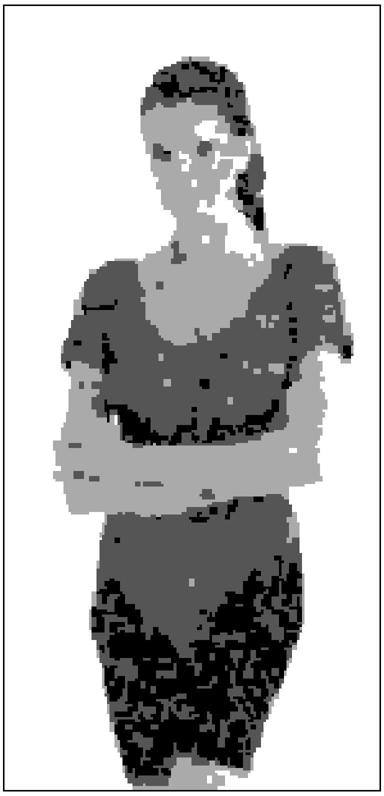
(a) 
(b) 
(c) 

Figure 10. (a) SNR = 10 dB Original noisy woman image (b) GMRF segmentation result (c) PMRF segmentation result
(Şekil 10. (a) SNR=10 dB orjinal gürültülü kadın görüntüsü (b) GMRF bölütleme sonucu (c) PMRF bölütleme sonucu)