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TEACHING THE ORDER OF OPERATIONS TOPIC TO FOURTH-GRADERS USING CODE.ORG

RESEARCH ARTICLE

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Abstract:

Coding education has been included in the education programs of many countries and is taught to students in schools or outside of school hours to gain them learning outcomes. This study attempts to use coding as a context for mathematical learning at grade level 4 to support and enhance students' mathematics learning. It examines the codes that emerged as a result of the order of operations topic learning process with coding. A case study approach was used in the study, and participants consist of three fourth-grade students with high, medium, and low achievements in mathematics. The data were collected through video records, photographs, worksheets, and observation notes and analyzed using a constant comparative coding method. In the applications, the participants first learned coding through the code.org platform. Then the expressions from the order of operations were studied with coding. As a result of data analysis, the codes were merged under the themes of code.org, student, and mathematical process. The analysis results showed that this coding activity enabled students to learn through modeling by concretizing the subject and developed their mathematical competencies by participating in many cognitive skills such as reasoning, analyzing, exploring, and verifying. Besides, it was found that problem-solving and generalizing from mathematical thinking skills were used in this learning activity.

Keywords: coding, computational thinking, fourth-graders, the order of operations topic

CODE.ORG İLE DÖRDÜNCÜ SINIF ÖĞRENCİLERİNE İŞLEM ÖNCELİĞİ KONUSUNUN ÖĞRETİLMESİ

Öz: Kodlama eğitimi birçok ülkenin eğitim programlarına dâhil edilmiştir ve öğrencilere okul saatlerinde veya okul dışı saatlerde kodlama eğitimi verilmektedir. Bu çalışma, dördüncü sınıf öğrencilerinin matematik öğrenimini desteklemek ve geliştirmek için kodlamayı bir bağlam olarak kullanmayı amaçlamaktadır. Çalışmada, işlem önceliği konusunun kodlama ile öğretilmesi süreci incelenmektedir. Durum çalışması deseninde gerçekleştirilen araştırmanın katılımcıları üç dördüncü sınıf öğrencisinden oluşmaktadır. Veriler video kayıtları, fotoğraflar, çalışma yaprakları ve gözlem notları aracılığıyla toplanmış ve karşılaştırmalı analiz yöntemi kullanılarak analiz edilmiştir. Uygulamalarda katılımcılar önce code.org platformu üzerinde kodlama yapmayı öğrenmiş, daha sonra işlem önceliği konusu kodlama ile çalışılmıştır. Verilerin analizi sonucunda ortaya çıkan kodlar; code.org, öğrenci ve matematiksel süreç olmak üzere üç temada toplanmıştır. Analiz sonuçları, bu kodlama etkinliğinin konuyu somutlaştırdığını, öğrencilerin modelleme yoluyla öğrenmelerini sağladığını, akıl yürütme, analiz etme, keşfetme, doğrulama gibi birçok bilişsel etkinlikte bulunarak matematiksel yeterliliklerini geliştirdiğini göstermiştir. Ayrıca öğrencilerin bu öğrenme etkinliğinde matematiksel düşünme süreçlerinden genelleme yapma ve problem çözme becerilerini kullandıkları tespit edilmiştir.

Anahtar Kelimeler: bilgi işlemsel düşünme, dördüncü sınıf öğrencileri, işlem önceliği konusu, kodlama

Introduction

Children need to learn 21st-century skills including creativity, innovation, critical thinking, and problem-solving, to thrive in our interconnected and constantly changing world (Geist, 2016). Coding activities support an environment where students can develop 21st-century competencies (Miller, 2019; Wing, 2006). While coding is a term of creating in complex programming languages, now it is used to describe creating a sequence of instructions with tools basic enough for young children (Hutchinson et al., 2016). Researchers have suggested that the inclusion of coding in the school curriculum has many cognitive benefits for learners (Arfe et al., 2020; Di Lieto et al., 2017; Kalelioğlu, 2015; Moreno-León et al., 2016; Resnick et al., 2009) as well as mathematical gains (e.g., Benton et al., 2017; Hutchinson et al., 2016; Hoyles and Noss, 1992; Miller, 2019). According to Gadanidis (2015), coding activities focus mostly on coding for their own sake or developing generic problem-solving and digital literacy skills, but the most important thing is to design mathematical activities with coding concepts. As mentioned, coding activities focus on coding or problem-solving skills, but it is crucial to teach mathematical concepts with coding activities. Therefore, this study focuses on teaching the order of operations with coding and explores how coding may be used as a context for mathematical learning at grade level 4 to support mathematics learning. Thus, coding activities will be saved from focusing only on developing programming and problem-solving skills, and students will be able to learn mathematics with these activities. Accordingly, this study attempts to use coding as a context for mathematical learning at grade level 4 to support and enhance students' mathematics learning. It examines the codes that emerged as a result of the learning process of the order of operations with coding.

STEM – Coding – Computational Thinking

Global revival requires teaching coding to children for three reasons: to enable them to gain a 21st-century competency, learn to be 'producers' of innovation, and prepare the future workforce with knowledge of computer programming (Gadanidis, 2015; Popat and Starkey, 2019). These reasons are compatible with the aims of integrated STEM education because it also aims to raise a STEM literate society, a workforce with 21st-century skills, and advanced research and development focused on innovation (Bybee, 2013). In the context of STEM education, coding and computational thinking take place in the school curriculums as a basic skill/literacy for all students (Miller, 2019), and children who have a strong foundation in computational thinking are found to be more effective problem solvers and critical thinkers (Wing, 2006). Computational thinking refers to "the thought processes involved in formulating a problem and expressing its solution in a way that a computer-human or machine can effectively carry out" (Wing, 2006, p. 7). It was found that computational thinking is related to problem-solving skills, spatial ability, and reasoning ability (Di Lieto et al., 2017; Fessakis et al., 2013; Roman Gonzalez et al., 2017; Wing, 2006). It also includes concepts to design and evaluate complex systems and understand human reasoning and behavior (Florez et al., 2017). Therefore, coding education is accepted within the scope of integrated STEM education, and it also develops computational thinking skills related to students' higher-level cognitive skills.

Mathematics Education and Coding Activities

The thinking required to code is quite mathematical; therefore, coding is a natural fit for mathematics teaching and learning (Gadanidis, 2014). Coding activities provide an opportunity for developing students' mathematical knowledge and cognition (Papert, 1980). The studies showed that in the learning process of coding, students learned mathematical skills containing problem-solving, measurement, geometry, and spatial concepts (Fessakis et al., 2013; Popat and Starkey, 2019; Savard and Highfeld, 2015). Moreover, adding coding as a context for mathematics learning, rather than a separate content area, provides experience, representing, investigating, and modeling mathematics concepts and relationships in new ways (Gadanidis, 2014; Moreno Leon, et al., 2021). In the experimental studies examining the effects of coding activities, it was found that learning coding improved children planning and inhibition skills (Arfe et al., 2020), visio-spatial working memory (Di Lieto et al., 2017), academic performance (Hayes and Stewart, 2016; Moreno-León et al., 2016), computational thinking (Saez Lopez et al., 2016) and higher-level of mathematical thinking skills (Miller, 2019). However, it was also determined that teaching programming did not cause any significant differences between experimental and control groups' scores for mathematical modeling and procedural comprehension (Bernardo and Morris, 1994) and reflective thinking skills (Kalelioğlu, 2015). There are contradictory findings of problem-solving skills; a significant increase in the problem-solving abilities of the treatment group (Palumbo and Michael Reed, 1991) and non-significant results (Falloon, 2016; Kalelioğlu and **Gülbahar, 2014; Psycharis** and Kallia, 2017) were reported. Qualitative studies were also executed in this field; the results of an exploratory case study revealed that 5–6 years old kindergarten children had opportunities to develop mathematical concepts, problem-solving, and social skills through programming activities (Fessakis et al., 2013). Taylor et al. (2010) also found that coding activities created an environment where the children used problem-solving processes such as goal setting, generating, and testing ideas. As a continuation of the studies, this study explores the outcomes of a mathematical learning activity executed with coding and will contribute to few qualitative studies in this field.

Another critical point is that countries around the world show different approaches to providing coding education to students. While countries such as Finland, France provide coding education interdisciplinary (mathematics, crafts); the countries like Portugal, Denmark, Australia, Italy, and Turkey serve coding education as part of the information technologies course (Bocconi et al., 2016). Because no model or framework is provided for coding to demonstrate how this can be integrated into curriculum areas and of concern is the lack of explicit links to mathematics in the new digital technologies curriculum; there is a need for more researches integration of coding effectively into mathematics or other curriculum areas (Miller, 2019). Accordingly, this study integrates coding education into the mathematics program and offers a different perspective to mathematics education.

Easy Way to Teach Coding for Kids: Code.org

Programming languages such as Scratch, ScratchJr, Code Spells, Tynker, Code.org, and Lego Mindstorms are more like games, enable students to code without learning complex traditional programming languages, and they can open up the world of coding and programming to young children (Geist, 2016; Taylor et al., 2010). A block-based visual programming code.org is one of the most effective options for children from preschool to primary school (Saez-Lopez et al., 2016). It is an open-source programming platform established as a non-profit site founded in the USA in 2013 that aims to teach students the basic programming concepts via challenges in the tasks. These tasks are asked to complete to pass levels using the drag-and-drop method, and it provides a visual programming language that is particularly appropriate for young learners (Kalelioglu, 2015; Saez-Lopez et al., 2016). Although the code.org platform is spread worldwide and is available in a growing number of national languages, its efficacy in developing coding and cognitive skills remains unexplored mainly (Arfe et al., 2020). This study will fill the gap in this field by examining the learning process of a mathematics subject with coding.

Teaching an Algebraic Topic Using Coding: The Order of Operations

The order of operations in mathematical expressions is not always the same as in the order in which they are written; if brackets, addition, subtraction, multiplication, division, and exponential numbers exist in the same mathematical expression, in such cases, it is necessary to operate according to the order of operation rule (Blando et al., 1989). Order of operations is one of the five obstacles identified in the algebraic context, and it causes students difficulties with the algebraic structure due to the lack of understanding of structural notions in arithmetic (Linchevski and Livneh, 1999; Vanderbeek, 2007). A significant part of the students' mistakes in arithmetic operations is also caused by the violation of the order of operation rule in secondary schools (Blando et al., 1989). Moreover, prospective teachers also have difficulties in terms of the order of operations (Glidden, 2008). Therefore, students' success in arithmetic operations depends on their understanding of the order of operation well. The topic of order of operation is taught to students in the fourth-grade in a limited way. The mathematics curriculum includes teaching that in addition and multiplication, changing the places of additives and multipliers does not change the result (Ministry of National Education [MoNE], 2018). However, Gadanidis (2015) states that teaching children complex math topics is possible through coding. In line with this view, this study taught a high-level topic to fourth-grade students by coding.

Method

A case study approach was used to investigate the learning environment that emerged in teaching the order of operations with coding. This approach attempted to describe, analyze and interpret the learning process of a mathematical subject with coding.

Participants

The research was executed with 10-year-old (fourth-grade) academically diverse children from a state, culturally non-mixed, rural, midrange Turkey city school to explore the mathematical results of the code.org platform. Participants of the study consist of three fourth-grade students with a diverse range of learners (low, average, and high achievers) according to their mathematics grades and the classroom teacher's opinions. The first participant, Yunus Emre, was identified as a child with advanced learning abilities, high mathematics achievement who generally exhibited poor social skills. It was observed that he was a determined boy because difficult tasks motivated him to achieve. He is the son of a farmer family, there was no computer at his home, but he knew how to use a computer. The other, Tarık, has an average level of achievement in mathematics. He was evaluated as a child with normal learning abilities. His father was a teacher. He had his computer at home and learned to code with code.org for a short time, so he had some previous knowledge about code.org. This ensured Tarık to finish the coding education in a shorter time but not another advantage in learning the order of operations with coding. The last one, Ece, was a child with low capabilities against her peers' achievements. She was a talkative child and had deficiencies in mathematical knowledge. There was a computer in her home, but she had no previous coding experience. Before the study started, the students were informed about the study and confidentiality. Their preferences about keeping their names confidential were asked and permission was obtained from each of them to be used their real names in the final report. It was also stated excerpts from audio and videotapes and selected pictures of worksheets would be included in the final report with their permission.

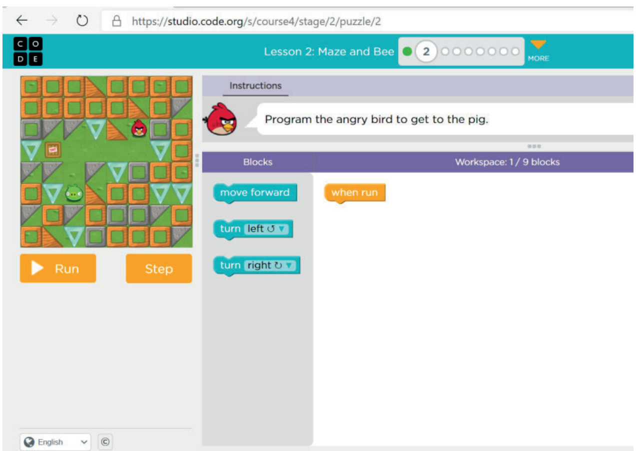
Data Collection Tools

Data were collected using a triangulation method that included children's reflective statements about their learning by digital video records, digital photographs, examples of the children's work on the worksheets, and direct observations that were written reflections by the researcher (Creswell and Miller, 2000). Think aloud protocols and interviews were also used to identify and record children's thinking processes. They were asked to express their thoughts aloud during the applications, and all applications were recorded with a digital camera. When they were silent, the researcher guided them with open-ended interview questions and tried to reveal their thoughts. In the study, participants firstly created expressions with the coding on the computer, and then they wrote the expression created with the coding on the worksheet. Worksheets consist of blank papers; thus, it was measured whether they could write the mathematical expression of the operation created with code blocks on the worksheet. In this whole process, the worksheets functioned as a means of checking what has been learned. The photographs of the operations created on the computer with coding and written on the worksheets were taken and used in the data analysis process. Direct observation notes were also a data collection tool in the study. Observations were made in the form of simultaneous recordings with video recordings and consist of short notes showing the time of observation. The researcher took part as an observer in the process and took notes of the important events. These observation notes were used in the analysis.

Applications

The study was carried out in two stages with each student. In the first stage, the study started by teaching students how to code using the challenges of the code.org platform. Code.org platform provides online coding stages according to the grade levels of students. Course 4, which was presented for students aged ten on code.org (after updating code.org, Course E is currently recommended for fourth-grade), was studied. In this course, students solve the challenges in the task by moving the code blocks to the workspace to make the bird move. The challenges of the tasks include coding concepts such as moving forward, turning left/right, looping, conditionals, angles. To deliver the challenges and solve the problems, they learned to create codes, tried different techniques, reached the goals, and completed the tasks. They learned the coding concepts to encode the workspace of the calc part for teaching the order of operations, and the first stage of the applications was completed. One of the challenges in Course 4 is presented in Figure 1.

Figure 1. The second challenge of course 4 in code.org



In the second stage, students learned the order of operations with coding using the calc part of code.org. The application lasted for three weeks. Each student participated in the study two days a week. In the first week, Yunus Emre completed the tasks. Then in the second week, coding was worked with Ece, Tarık joined the application last week. The application of the study consisted of eight lessons, separately for each student, the first four lessons with a teaching coding focus, and the last four lessons were used to teach the order of operations with coding. Yunus Emre and Tarık learned coding in a shorter time than Ece. All applications were conducted during school time, in the computer laboratory in a quiet environment. Each of the students worked with the researcher alone. While the other students in the class were attending the normal class lessons, the selected students participated in the study. The applications were carried out by the researcher who is working on studies that integrate coding and mathematics education.

Teaching the Order of Operations Using Coding

After teaching coding, the applications focused on teaching the order of operations topic using coding. Teaching the order of operations including brackets has been carried out in four stages.

1. Coding a simple expression [e.g., 5×3=?]

2. Coding an expression including brackets on one side [e.g., 5×(7-2)=? or (3+2)×4=?]

3. Coding an expression including brackets on both sides [e.g., (6/2)×(3+4)=?]

4. Coding an expression including nested operations [e.g., 4+(8/(4×2))=?]

Students firstly created the expressions with the coding on the computer in these four stages, and then they wrote the coded expression on the worksheets. This is a more abstract stage for children because writing the expressions created with code blocks to a worksheet using mathematical symbols requires comprehending and abstracting the expression. While teaching the order of operations, these stages were followed;

1. Creating an expression by selecting the code blocks.

2. Completing the coding of expression by entering the numbers.

3. Explaining the expression created with code blocks.

4. Writing the coded expression on the worksheet.

5. Solving the expression on the worksheet.

6. Displaying the expression solution by pressing run on the computer and controlling the result on the worksheet.

7. Closing the solution window on the computer to correct the mistakes and solving again.

The execution of an example of "coding an expression including brackets on one side (e.g., 5×(7-2)=?)" was carried out as follows; the students coded the expression and entered the numbers. They explained the coded expression on the computer, then they wrote it on the worksheet and solved. They monitored the result of the expression by pressing run, and compared the results on the computer and worksheet. In these stages, the students have a better understanding of creating expressions with the coding. By explaining the coded expression mathematically and writing the mathematical expressions of these coding patterns on the worksheets, it was determined whether they comprehended the usage and role of brackets in the order of operation rule. In this process, to record the students' thoughts about coding activities and their learning of expressions with coding, the researcher asked questions to probe student understanding. The role of the researcher in this process was an observer and guide for students as they participated in learning experiences by making them think and speak loudly.

Data Analysis

In this paper, only findings from the courses about teaching the order of operations topic using coding were presented. The video records, photographs, worksheets, and observation notes were used for in-depth analysis. The video records of the applications were transcripted in the NVivo program. The transcripts were analyzed following the steps of Creswell (2007); preparing and organizing the data for analysis, reducing the data into themes through coding and condensing the codes, and finally representing the data in figures, tables, or a discussion. A constant comparative method was also used for coding. This methodology incorporates four stages: "(1) comparing incidents applicable to each category, (2) integrating categories and their properties, (3) delimiting the theory, and (4) writing the theory" (Glaser and Strauss, 1967, p. 105). Three coding procedures were undertaken: open coding, selective coding, and axial coding (Creswell et al., 2007). The transcripts were re-read, important points were determined to identify code patterns. A thematic approach was used when synthesizing the data. Similar skills and actions were identified, grouped into the same themes, and refined codes and themes. Then a framework of three main themes was created, and a model relating the themes was created from the codes (nodes in Nvivo). Checking reliability and consistency of the codes between the researcher and another lecturer working in the mathematics field were conducted to verify the coding of the transcripts. Photographs and worksheets were analysed using document anaysis and submitted with the expressions and quotations in the findings.

Trustworthiness of the Study

In qualitative research, qualitative criteria and techniques are described based on trustworthiness (Lincoln and Guba, 1986). The measures taken in this context can be listed as follows: A triangulation method was used in the data collection. Creswell and Miller (2000) suggest triangulation as a popular practice for qualitative inquirers to corroborate evidence collected through multiple methods for the validity of the research. The data was collected with video records, photographs, worksheets, observation notes, and the analysis results of transcripts were supported with photographs, worksheets, and direct quotations. Another procedure for establishing credibility was applied in the study by describing the setting, the participants, and the themes in rich detail. To practice "audit trial" the analysis results were submitted to an external expert, a lecturer working in the field of mathematics (Creswell and Miller, 2000). He read all the data and results, expressed the incongruent points.

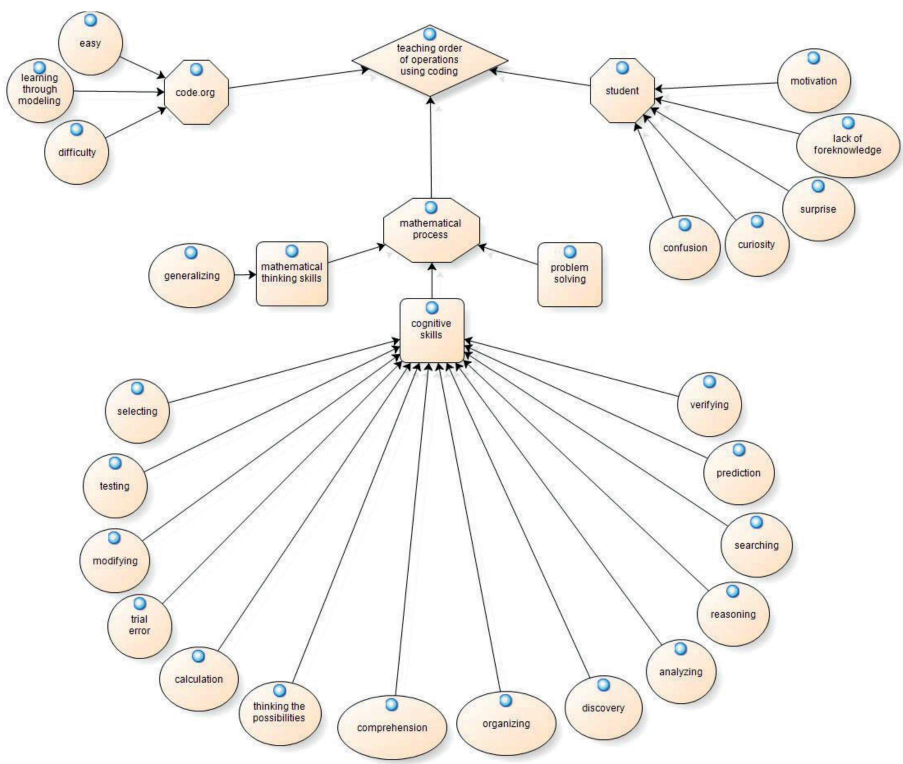
Ethical Procedures

All ethical principles were complied with during the implementation of this study. Participants were informed about the study before the application and participated in the study on a voluntary basis. The application of this study was completed before 2020. Since the study was produced from data collected before 2020, retrospective ethics committee approval is not required according to the 2020 TR Index Journal Evaluation criteria.

Findings

The analysis results showed that the codes of the applications were merged under the themes of code.org, student, and mathematical process. Figure 2 represents the themes and codes that emerged from the data analysis.

Figure 2. The codes and themes emerged from the data analysis

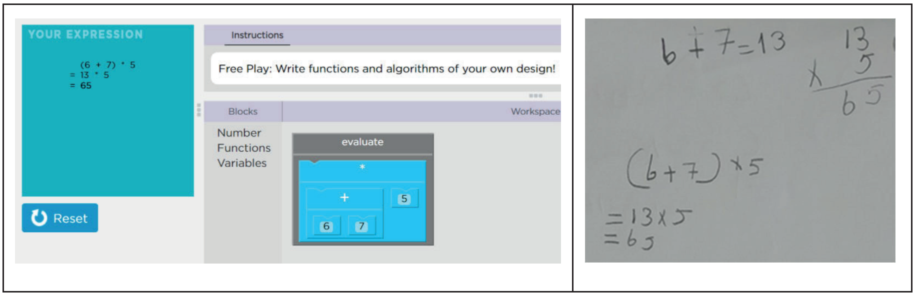


The data analysis results revealed three codes of code.org showing the program's features: easy, difficulty, learning through modeling. The examples of these codes are presented in Table 1. The easy code has emerged as the students easily comprehend the order of operations with coding. But in this learning activity, especially while teaching the nested operations with coding, students sometimes had difficulties. It was also determined that the code.org environment enabled students to learn through modeling by visualizing and concretizing the order of operations topic. Because, the expression is modeled with code blocks on the coding screen, and then the expression is solved with the solving steps when the run key is pressed. The example of Ece is presented in Table 1. Moreover, similar examples were also observed in the learning processes of Tarık and Yunus Emre.

Table 1. The Codes of Code.org Theme

Theme	Codes	Quotations
Code.org	Easy	- I will ask you to do another expression with coding. - It is very easy (Yunus Emre).
	Learning through modeling	Ece created the expression of (6+7)×5 =? with code blocks. - Can you explain it? - It will add 6 to 7, add 7 to the number it found. It will multiply 13 by 7, sorry multiply 5 by the number it found (She confused the order of operations). - Can you write this expression on the worksheet? She examined the coded expression for a while and solved it correctly on the worksheet (see Figure 3, the one above). - Can you write the operations together? - No. I can't do it. She examined the coded expression again, then pressed run and viewed the solution steps on the code screen. After closing the solution, she could write the operations together (see below operation in Figure 3). This result showed that Ece learned how to write the operations together by examining the modeled operation on code.org because the code blocks made it easier to understand the order of operation rule by modeling and concretizing it (Ece).
	Difficulty	At the last stage, Tarık created a nested operation 7+(12/(6-2))=? but he could not explain the solution. - Did you understand? - I did not understand, it is too difficult. He displayed the solution steps by pressing run and examined it. Then, after closing the solution, he was able to write all the operations sequentially on the paper. Besides, he clarified the solution: - I'll divide 12 by the difference of 6 minus 2, I go on sequentially, I will divide 12 by 4, the result is 3, add 3 to 7. (Tarık).

Figure 3. The Expression of Ece About "Coding an Expression Including Brackets on One Side"

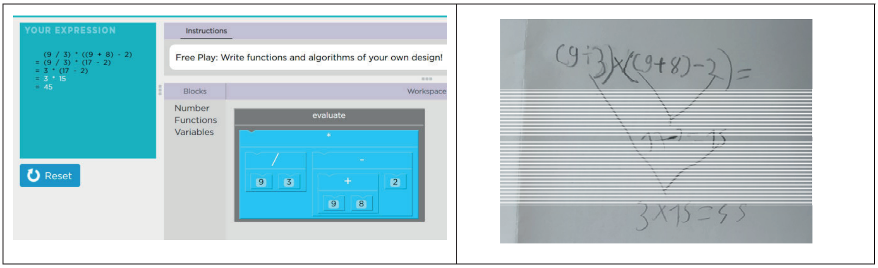


The codes of student theme are curiosity, surprise, motivation, confusion, and lack of foreknowledge, as in Table 2. The applications were first started by introducing the calc section to the students, and it was observed that they were curious about the program. Ece created a nested operation with code blocks, but it was difficult to comprehend for her; she confused the order of operations. Besides, the learning process of students revealed the importance of lack of foreknowledge in mathematics.

Table 2. The Codes of Student Theme

Theme	Codes	Quotations
Student	Confusion	Ece created a nested operation of (5+3)×(8/(6-4))=? with code blocks. - What kind of expression is this? - This expression is both addition, multiplication, and division, but too complicated. - Can you express the solution? - 8 divided by 6, minus 4, equals, I cannot solve it; if we divide 8 by 8, 1, then I added it up, I am confused (Ece).
	Curiosity	Code blocks were introduced to Tarık at the beginning of the application. - You can create an expression with code blocks yourself. - What is being done? (He wondered) - I don't know, you will see by trying (Tarık).
	Lack of fore-knowledge	Ece created the expression of (7/5)-8=? and tried to solve it but she could not. Because she did not know which numbers divided perfectly. - We divide 7 by 5. - What is the result? - We add 7 to 5, we multiply 7 by 5. - But we have to divide, why are we multiplying? - Because division can not be done, so we multiply these numbers to find the result (Ece).
	Motivation	At the last stage, Yunus Emre created a nested operation with code blocks. He entered the numbers by calculating [(9/3)×((9+8)-2)=?] (see Figure 4). Then he started to write it on the paper, and he explained the procedures: - Divide 9 by 3, 9 plus 8, minus 2. Yunus Emre wrote the expression sequentially on the paper. He liked to create expressions with coding, and challenging operations motivated him. Then he created a different example with code blocks by calculating the result [(9×(6-1))/(8+7)=?], wrote it on the worksheet, and solved it correctly (Yunus Emre).
	Surprising	After the calc section was introduced to Yunus Emre, he created the expression of 4+6=? He realized that he created an addition with code blocks when he pressed run. - Oh! The result is 10, it added up, you know (Yunus Emre).

Figure 4. The Expression of Yunus Emre About "Coding an Expression Including Nested Operations"

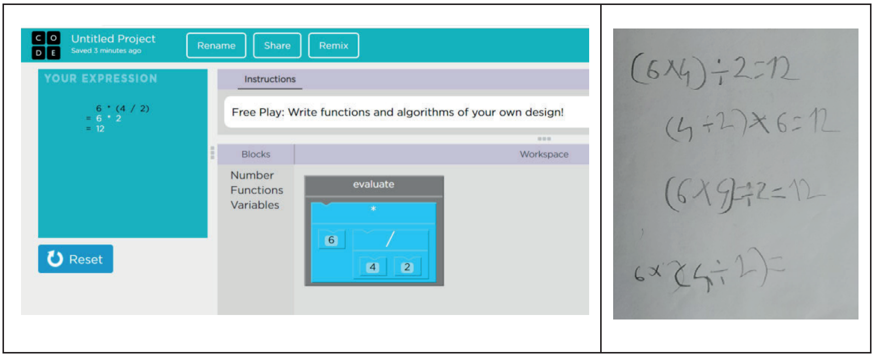


There are three sub-themes under the theme of mathematical process: mathematical thinking skills, problem-solving and cognitive skills. The analysis results showed that generalizing from mathematical thinking skills was used by students in this learning environment. One example of this code is presented in Table 3.

Table 3. The Example of Generalizing Code

Theme	Codes	Quotations
Mathematical thinking skills	Generalizing	This example shows the understanding and generalization of Yunus Emre that the second side of a code block can include a different operation with brackets. The conversation between the researcher and Yunus Emre about the expression of (6×4)/2=? was as follows. - This is an expression including brackets on one side and a single number on the other. Can you place the parenthetical operation to the other side? - It cannot be on that side, it can be placed only on this side (After trying, he created the expression of (4/2)×6=? with code blocks and also wrote this expression on the worksheet; see Figure 5). - But this is on the first side, again. - Look, I divided 4 by 2, I multiplied 2 by 6, the result is 12 (He told the solution of the expression). - OK. I want you to take this parenthetical operation on the other side. - The result is 12 again, I don't know why it is 12 (He still thought about the result). - I want this operation is on the second side, take this over there. (He waited for a while, then created the expression of 6×(4/2)=? with code blocks. He started to write this expression on the worksheet). - There is also such an expression, 6 cross, open brackets (He created an expression including brackets on the other side, but he could not comprehend where he had to put brackets when writing it). - Open brackets here, OK? (The researcher showed it on the computer) - 4 divided by 2 (He inserted brackets around 6×4 operation, see Figure 5). - But you still put brackets here. - But it always happens that way. - No, you can change it. (He wrote the expression correctly this time, see Figure 5, the one below). - Well done. - The result is 12 again. The location of the brackets can change… This is the first time I see it like this... The positions of the operations can change, the place of the parenthesis can change, but the result does not change, such examples can be made. (Yunus Emre).

Figure 5. The Expression That Yunus Emre Took The Expression Including Brackets to The Other Side



Problem-solving was another code that was seen extensively in the applications. One of the examples of Yunus Emre's problem-solving process is presented in Table 4.

Table 4. The Problem-Solving Code of Mathematical Process Theme

Theme	Codes	Quotations
Mathematical process	Problem-solving	Yunus Emre tried to determine the numbers to enter into the spaces of the addition code block when creating the expression of (9×(6-1))/(8+7) =? -What numbers should you enter in the addition? - 45 can be divided by 9. - Then what should be there? - It can also be divided by 15, let's change that number, 7, 14, 15. I'll write 7 here. The result is 3. (He changed the numbers in the addition code block as a total of 15) (Yunus Emre).

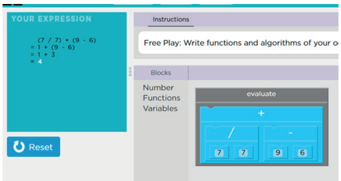
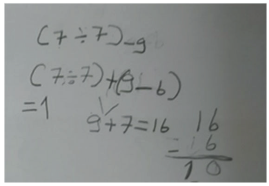
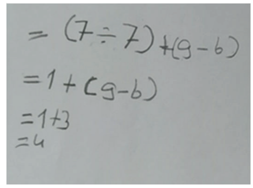
The cognitive skillssub-theme involves different actions that emerged in this mathematical learning activity: selecting, testing, modifying, calculation, thinking of possibilities, trial error, analyzing, discovery, comprehension, organizing, reasoning, searching, prediction, verifying. All these codes provided evidence of the effectiveness of this learning activity. Students selected code blocks, organized them and created expressions, tested the results of coded expressions, they modified the code blocks and numbers by calculating the results and solved the expressions by trial error. In this learning environment, they reached the results by analyzing the expressions. The examples of these codes are submitted in Table 5.

Table 5. The Codes of Cognitive Skills Theme

Theme	Codes	Quotations
Mental activities	Analyzing	Ece tried to solve the expression of (9-7)×(2+1)=? which is difficult for her. She analyzed the expression by separating it into steps. This enabled her to solve the operation. - We'll subtract 7 from 9. - What will we do later? - We will add 1 to 2. - OK, then? - We will multiply the result by the sum of 2 plus 1 (Ece).
	Calculation	Yunus Emre tried to solve the expression of (6×8)+6=? and calculated the result by counting with his fingers. - Multiply 6 by 8, then,… (He thought) - What comes out when you multiply 6 by 8? - 48. - Then? - We add 48 to 6, the result is 54. (He counted again) - Did you understand? - Yes, it is multiplication with addition. We insert brackets around the multiplication, then we put plus and add. (Yunus Emre)
	Comprehension	Ece created an expression of (7/7)+(9-6)=? with code blocks but when writing it, she confused the order of operations (see Figure 6, before examining the code blocks). - Can you write this expression? - Divide 7 by 7, insert brackets, subtract 6 from 9, no brackets here, right? - I don't know, what will you do here? - I'll subtract 9 from 7, no, it's wrong (She confused and started to examine the code blocks on the screen) - What is wrong? - I'll subtract 6 from 9, then I will put a plus, I will add it to the result I found (After a little thought, she wrote correctly; see Figure 6, after examining the code blocks). - It's going to be like this, teacher, we'll put a plus here and a minus here. - So, is there an order here? Do you need to insert brackets? - It should be, we have to do this first, we have to put brackets. Otherwise, the result is wrong. - Let's put it, how did this operation like now? - This operation is both an additive, multiplication, and division. (Ece).
	Discovery	At the last stage, Yunus Emre created an expression including nested operations with code blocks. He entered the numbers by calculating the result [(9/3)×((9+8)-2)=?]. Then he started to write it on the paper, and explained the procedures. -Divide 9 by 3, 9 plus 8, minus 2. (Yunus wrote the expression sequentially on the paper, but he confused the place of a sign). -Divide 9 by 3, insert brackets, shall I put a cross here or minus? -What are the sides of the multiplication? Which numbers are multiplied? (He pointed to the screen) -Division and addition-subtraction (He discovered that he had to put a cross between the operations and finally he wrote the expression correctly on the paper) (Yunus Emre).
	Modifying	After creating the expression of (9×9)+5=? Ece was asked to create an expression including brackets on the other side. She modified this expression and created the new one [5+ (9 × 9) =?] by putting the multiplication code block to the other side. - Can we take the multiplication to this side? - We can get it and put it here on the second side instead of 5. (Ece)
	Organizing	At the last stage, Tarık created a nested operation of 7+(12/(6-2))=? by choosing and organizing the code blocks. -I want you to create a nested operation, select multiple operations (He chose the code blocks). - Yes, there are four of them. - Now place them together (He organized the code blocks and created a nested operation). - Yes, good. (Tarık)
	Prediction	While creating a nested operation, Yunus Emre tried to predict the result by choosing different numbers accordingly. -Do you predict the result of this expression? -Let me think 6, 5, 45, 46, 48,50, 53. -How will you solve it? -Divide 53 by 2.

	-Why do you divide by 2? -Which number can we divide by? Let's divide by 2 (He tried to solve the expression on paper) -It is 26,5 but it is my prediction. (Yunus Emre)
Reasoning	Yunus Emre coded the expression of (5/50)-10=? and started to think how to solve it. - 5 divided by 50, if we write this first, the result is 0. (When he could not divide 5 by 50, he understood that he had to write the numbers that are perfectly divisible. He reasoned how to reach the conclusion, so he thought of changing the places of 5 and 50). - So what should you do now? - I change the places of the numbers, let's delete that. OK, I did it like this, run it. The result is 0. (He created the expression of (50/5)-10=? and pressed run). - Is it OK? - Yes. (Yunus Emre).
Searching	The researcher introduced the calc part and showed the code blocks to Ece. Ece searched the code blocks by taking, placing, inserting, and removing them to create an expression. (Ece)
Selecting	- Can we do one more expression? (Ece selected the addition code block). - Let's do multiplication, we didn't multiply. (She also took the multiplication code block) - Choose another operation here and write appropriate numbers. (Ece)
Testing	- Can something else come to this side? (The researcher pointed out the right side of the expression and wanted Yunus Emre to enter different numbers) - It may come, but let me try, if the result comes out correctly (He entered the numbers by calculating the result and tested it by pressing run). - What is the result? - The result is correct. (Yunus Emre)
Thinking of possibilities	In the first stage of the applications, Tarık created a division 5/3 =? with code blocks. - Let's write 5 here and 3 here. - Is 5 divided by 3? - No, then we have to write different numbers (He thought about different numbers that are perfectly divisible). -We can write 30 and 5. (Tarık).
Trial error	Ece created an expression with code blocks [(6+7)×5=?]. She was asked to explain the expression coded. She tried to solve it by trial and error, using the numbers in the expression. - How will this question be solved? - It adds 6 to 7. - Which numbers are multiplied? - It multiplies 13 by 7. It adds another 7 with the number found. (She tried to solve it on paper). - It adds 6 to 7, and it adds 7 to the result (She thought for a while). - It multiplies 13 by 7 (She examined the code blocks on the screen and tried to solve it on the paper again). - Sorry, it multiplies by 5, it multiplies 5 by the number it found. - Can you write this expression? - Add 6 to 7, then multiply 13 by 5, the result is 65. - Press run and see the result. - The result is 65. (Ece).
Verifying	Ece wanted to press run to check and verify her solution after writing the expression of 6+7=? on the worksheet at the first stage. - Shall we see the solution? - Let's see. - The same result that I did. (Ece)

Figure 6. The Expressions of Ece About "Coding An Expression Including Brackets on Both Sides"

	Before examining the code blocks	After examining the code blocks
		

Discussion

Learning to code would engage young children with ideas that are complex, abstract, and well beyond what is typically expected of them at their grade level and develops their mathematical competence (Gadanidis, 2015; Taylor et al., 2010; Wing, 2006). Computational thinking has also been identified among the critical 21st century skills all students should develop (Barr et al., 2011). This study aimed to investigate the codes that emerged as a result of the learning process of the order of operations with coding. As a result of data analysis, three main themes emerged: code.org, student, and mathematical process. The theme of code.org revealed that this application enables students to learn through modeling. It was seen that code blocks concretize the expression through modeling, and this provides the student to understand the priority between the operations. The solution steps provided by code.org also enabled students to comprehend the order of operations presented in the code blocks. This finding is consistent with the literature stating that coding activities enable the students to understand mathematical concepts and learn mathematics (Gadanidis, 2014; Moreno Leon, et al., 2021; Taylor et al., 2010;) and coding studies model mathematics (Hoyles and Noss, 1987; Wing, 2006, 2008). For example, Gadanidis (2015, p. 1) explained the link between mathematics and coding as "coding is a natural fit to mathematics, as it can be used to model and investigate mathematical relationships and as coding and mathematics have a shared logical structure." According to him, coding activities provide mathematics success thanks to the strong connection and young children both a meaningful context for coding and a rich mathematics learning experience (Gadanidis, 2014, 2015). Participants achieved to create the expressions with code blocks and learned the order of operations. These results provided evidence that children can learn complex and abstract ideas of mathematics thanks to learning complex and abstract coding concepts (Gadanidis, 2015). However, sometimes, students found these coding activities easy and sometimes challenging. This coding activity is also beneficial because it was found that students failed to solve simple algebraic equations and used the wrong order of operations (Linchevski and Livneh, 1999).

The other theme of student showed that this coding activity attracted the students' attention and motivated them, they were also surprised about the specialties of the program, but they also experienced confusion. More importantly, teaching this subject with coding enabled students to develop their mathematical competencies, especially problem solving, by engaging in many cognitive skills. Other researchers expressed similar results; learning mathematics through coding promotes mathematical learning, including problem-solving (Fessakis et al., 2013; Savard and Highfeld, 2015). To summarize these studies briefly; Taylor et al. (2010) also used a case study approach to describe how engagement with Scratch and interactive whiteboard enabled children to solve design challenges and found that children used problem-solving processes such as goal setting, generating, and testing ideas. Fessakis et al. (2013) conducted an exploratory case study with 5–6 years old kindergarten children. A series of similar computer programming problems using a Logo-based environment on an Interactive White Board were applied and the results supported the view that children had opportunities to develop mathematical concepts, problem-solving, and social skills. Similarly, Holmes et al. (2018) also found that participating teachers' self-efficacy in integrating math and coding increased significantly after a relatively short engagement with ScratchMaths. Moreno Leon et al. (2021) investigated whether it is possible to develop mathematical competence through programming activities in primary 5th grade, and the results showed that students in the experimental group developed mathematical competence significantly more than students in the control group. The effect size revealed that the study achieved the intended effect on mathematical competence. In addition, it was determined that when 9 and 10-year-old children use ScratchMath, they use problem solving and collaboration processes using mathematical and coding language, and they are cognitively engaged (Calder and Rhodes, 2021). So as a result of the studies, consistent with the results of this study, it can be concluded that coding activities provide students with mathematical gains.

This application also showed students to use higher-level mathematical thinking skills such as generalizing in the coding learning environment. Students generalized the rules of order of operations during the coding activities. It was observed that they mostly used generalizing expressions after learning the types of expressions. Miller (2019) also found that coding instruction can lead to higher mathematical thinking levels, such as generalization. Popat and Starkey (2019) state that an educational outcome of coding researches outside of programming, problem-solving through mathematical concepts was a more generic skill. This study contributes to this field by developing a different mathematical thinking skill, generalizing.

So many cognitive skillsthat emerged in this mathematical activity showed that mathematics education with coding develops students' cognitive skills by enabling them to understand the subject engaging in different activities. These codes are compatible with the activities like discovering, innovating, predicting, planning, exploring, choosing, thinking, testing, evaluating, modifying provided by the literature about coding studies (Geist, 2016; Popat and Starkey, 2019; Taylor et al., 2010). Besides, codes such as trial error, organizing, testing found in this study revealed that students use the components of computational thinking consisting of experimenting and iterating, testing and debugging, reusing and remixing, abstracting, and modularizing by Brennan and Resnick (2012). These coding activities allow children to discover, innovate, predict, plan, explore, choose, think, and support both literacy and mathematics learning (Geist, 2016, Taylor et al., 2010). Therefore, coding has to be added as a context for mathematics learning, rather than a separate content area as in the countries like Finland, France (Bocconi et al., 2016). This suggestion has also been expressed by Gadanidis (2014), who deems it necessary because mathematics education with coding reduces the stress on the crowded curriculum and offers new ways of experiencing, representing, and investigating mathematics concepts and relationships. Moreover, current institutional recommendations published throughout Europe, Canada, and the USA recommend that coding have to be introduced in primary school as early as possible and furthered in secondary school (Falloon, 2016; Florez et al., 2017). The other result that has to be emphasized is following a constructivist approach in the applications. The importance of using coding concepts through an active approach was also stated by Saez Lopez et al. (2016).

Conclusion and Recommendations

As a result of the study, it was revealed that coding activities developed students' understanding by concretizing and modeling the order of operations topic and lead to the development of problem-solving and generalizing skills. Therefore, coding activities integrated into the mathematics program can increase students' mathematics achievement by understanding abstract mathematical concepts and acquiring mathematical gains. Therefore, coding should be included in the mathematics program as a method used in mathematics education. However, the reapplication of this study at a different grade level of primary school, secondary school, or advanced levels, and the examination of the effects of teaching different mathematics subjects with coding using both empirical and in-depth qualitative analysis will guide this field.

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