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Research Article

Bi Ideals of Nearness Semirings

Özlem Tekin[1*]

[1*] Adıyaman University, Faculty of Arts and Sciences, Department of Mathematics, Adıyaman, Turkey, (ORCID: 0000-0002-1721-1053)
umduozlem42@gmail.com

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Abstract

Bi ideals are the generalisation of quasi ideals. In this article, it is defined that the notion of bi-ideals in semirings on weak nearness approximation spaces. Afterwards, it is explained that some of the concepts and definitions related to the subject. Also, it is given that the definition of nearness m −bi ideals and nearness (m, n) −quasi ideals. Thus, we examine the relationship between nearness m-bi ideals and nearness (m,n)-quasi ideals .

Keywords: Nearness approximation space, Semirings, Nearness semiring, Quasi ideals, Bi ideals.

Yakınlık Yarı Halkalarının Bi İdealleri

Öz

Bi idealler, quasi ideallerin bir genelleştirmesidir. Bu çalışmada, zayıf yakınlık yaklaşım uzaylarında bi-idealler kavramı tanımlandı. Daha sonra, konuyla ilgili bazı tanımlar ve kavramlar açıklandı. Ayrıca, yakınlık m −bi idealleri ve yakınlık (m, n) −quasi ideallerinin tanımları verildi. Böylece, yakınlık m-bi idealleri ve yakınlık (m,n)-quasi ideallerinin aralarındaki ilişkiyi inceledik.

Anahtar Kelimeler: Yakınlık Yaklaşım Uzayları, Yarı Halkalar, Yakınlık Yarı Halkaları, Quasi idealler, Bi idealler.

* Corresponding Author: umduozlem42@gmail.com

1. Introduction

Peters studied near sets theory that is a generalisation of rough sets [1] in 2002. Peters gaved an indiscernibility relation by utilizing the features of the objects to determine the nearness of the objects [2]. Afterwards, he generalised approach theory in the study of the nearness of non-empty sets which are similar to each other [3], [4]. İnan and Öztürk introduced the notion of nearness groups [5]. Therefore, other approaches have been studied in [6], [7], [8].

The concept of semiring theory was defined by Vandier [9] in 1934 and many mathematicians proved important properties for semiring theory. Especially, semirings are very important for determinants and matrices. One of the most important notion for semirings is ideals. Henriksen and Shabir et al. [10] studied ideals for semirings. In 1952, the notion of bi ideals for semigroups was defined by Good and Hughes [11]. Afterwards, Lajos and Szasz introduced theory of bi ideals in rings and semirings [12]. Bi-ideals are a special situation of (m,n) ideal. Tekin defined quasi ideals in semirings on weak nearness approximation spaces [13].

In this article, bi ideals in semirings are defined and some of the concepts and definitions on weak nearness approximation spaces are explained. Furthermore, we study some basic properties of bi ideals.

2. Preliminaries

An object characterization is given by means of a tuple of function values $\Phi(x)$ deal with an object $x \in X$. $B \subseteq F$ is a set of probe functions and these functions stand for features of sample objects $X \subseteq \mathcal{O}$. Let $\varphi_i \in B$, that is $\varphi_i : \mathcal{O} \to \mathbb{R}$. The functions showing object features supply a basis for, $\Phi: \mathcal{O} \to \mathbb{R}^L, \Phi(x) = (\varphi_1(x), \varphi_2(x), \ldots, \varphi_L(x))$ a vector consisting of measurements deal with each functional value $\varphi_i(x)$, where the description length $|\Phi| = L$ ([2]).

The choice of functions $\varphi_i \in B$ is very important by using to determine sample objects. Each φ shows a descriptive pattern of an object. The difference φ means to a description of the indiscernibility relation " $\sim_B$ " defined by Peters in [2]. B_r is probe functions in B for $r \leq |B|$.

Definition 2.1 *[2]*

$$\sim_B = \{(x, x') \in \mathcal{O} \times \mathcal{O} \mid \triangle_{\varphi_i} = 0, \forall \varphi_i \in B, B \subseteq \mathcal{F}\}$$

means indiscernibility relation on $\mathcal{O}$, where description length $i \leq |\Phi|$. $\sim_{B_r}$ is also indiscernibility relation determined by utilizing B_r.

Near equivalence class is stated as $[x]_{B_r} = \{x' \in \mathcal{O} \mid x \sim_{B_r} x'\}$. After attaining near equivalence classes, quotient set $\mathcal{O}/\sim_{B_r} = \{[x]_{B_r} \mid x \in \mathcal{O}\} = \xi_{\mathcal{O},B_r}$ and set of partitions $N_r(B) = \{\xi_{\mathcal{O},B_r} \mid B_r \subseteq B\}$ can be found. By using near equivalence classes, $N_r(B)^*X = \bigcup_{[x]_{B_r} \cap X \neq \emptyset} [x]_{B_r}$ upper approximation set can be got.

Definition 2.2 *[14]*

Let $\mathcal{O}$ be a set of sample objects, $\mathcal{F}$ a set of the probe functions, $\sim_{B_r}$ an indiscernibility relation, and $N_r(B)$ a collection of partitions. Then, $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ is called a weak nearness approximation space.

Theorem 2.1 *[14]*

Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ be a weak nearness approximation space and $X, Y \subset \mathcal{O}$. Then the followings hold:

i. $X \subseteq N_r(B)^*X$,
ii. $N_r(B)^*(X \cup Y) = N_r(B)^*X \cup N_r(B)^*Y$,
iii. $X \subseteq Y$ implies $N_r(B)^*X \subseteq N_r(B)^*Y$,
iv. $N_r(B)^*(X \cap Y) \subseteq N_r(B)^*X \cap N_r(B)^*Y$.

Afterward, $\mathcal{O}$ means a $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ is weak near approximation spaces unless otherwise said.

Definition 2.3 *[6]*

Let S be a nearness semigroup. For all $x \in S$,

there exists an element $e \in N_r(B)^*S$ such that

$x \cdot e = e \cdot x = x$ hold, then $(S,\cdot)$ is called a nearness monoid.

Definition 2.4 *[6]*

Let $S \subseteq \mathcal{O}$. Then, S is called a semiring on weak near approximation spaces $\mathcal{O}$ if the following properties hold:

NSR_1) $(S, +)$ is an abelian monoid on $\mathcal{O}$ with identity element 0,

NSR_2) $(S,\cdot)$ is a monoid on $\mathcal{O}$ with identity element 1_S,

NSR_3) for all $x, y, z \in S$ such that

$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ and $(x + y) \cdot z = (x \cdot z) + (y \cdot z)$ hold in $N_r(B)^*S$,

NSR_4) for all $x \in S$ such that

$$0 \cdot x = 0 = x \cdot 0$$

hold in $N_r(B)^*S$,

NSR_5) $1 \neq 0$.

Theorem 2.2 *[6]*

Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ be a weak nearness approximation space and $X, Y \subset \mathcal{O}$. Then the followings hold:

i. $(N_r(B)^*X) + (N_r(B)^*Y) \subseteq N_r(B)^*(X + Y)$
ii. $(N_r(B)^*X) \cdot (N_r(B)^*Y) \subseteq N_r(B)^*(X \cdot Y)$.

Definition 2.5 *[6]*

Let S be a nearness semiring, and A is a non-empty subset of S.

i. A is called a subsemiring of S, if $A + A \subseteq N_r(B)^*A$ and $A \cdot A \subseteq N_r(B)^*A$.
ii. A is called an upper-near subsemiring of S, if $(N_r(B)^*A) + (N_r(B)^*A) \subseteq N_r(B)^*A$ and $(N_r(B)^*A) \cdot (N_r(B)^*A) \subseteq N_r(B)^*A$.

Definition 2.6 *[6]*

Let S be a nearness semiring, and A be a subsemigroup of S, where $A \neq S$.

i. A is called a right (left) ideals of S, if $A \cdot S \subseteq N_r(B)^*A$ $(S \cdot A \subseteq N_r(B)^*A)$.
ii. A is called an upper-near right (left) ideals of S, if $(N_r(B)^*A) \cdot S \subseteq N_r(B)^*A$ $(S \cdot (N_r(B)^*A) \subseteq N_r(B)^*A)$.

Definition 2.7 *[15]*

Let M be a semiring and A be a non-empty subset of semiring M.

If A is a subsemigroup of M and $BMB \subseteq B$, then B is called a bi ideal of M.

Definition 2.8 *[13]*

Let S be a nearness semiring and Q be non-empty subset of S, where $Q \neq S$. Q is called quasi-ideal of S if Q is a subnearness semigroup of S and $QS \cap SQ \subseteq N_r(B)^*Q$.

Lemma 2.1 *[13]*

Let S be a nearness semiring. If S is commutative, then each quasi-ideal of S is two-sided ideal of S.

3. Bi Ideals of Nearness Semirings

Definition 3.9

Let S be a nearness semiring and A is a subsemigroup of S, where $A \subseteq S$.

i. A is called bi ideal of S if $AMA \subseteq N_r(B)^*A$.
ii. A is called an upper-near bi ideal of S if $(N_r(B)^*A)S(N_r(B)^*A) \subseteq N_r(B)^*A$.

Example 3.1

Let $\mathcal{O} = \{a,b,c,d,e,f,g,h,i,j\}$ be a set of perceptual objects where $r = 1$, $B = \{\varphi_1, \varphi_2, \varphi_3\} \subseteq \mathcal{F}$ be a set of probe functions. Let $S = \{c,d,e,f\} \subset \mathcal{O}$. Probe functions' values

$$\varphi_1 : \mathcal{O} \to V_1 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\},$$

$$\varphi_2 : \mathcal{O} \to V_2 = \{\alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\},$$

$$\varphi_3 : \mathcal{O} \to V_3 = \{\alpha_2, \alpha_4, \alpha_6, \alpha_7, \alpha_8\}$$

are presented in the following table:

	a	b	c	d	e	f	g	h	i	j
φ_1	α_1	α_2	α_3	α_2	α_3	α_4	α_2	α_1	α_1	α_5
φ_2	α_2	α_3	α_2	α_4	α_3	α_4	α_5	α_2	α_6	α_5
φ_3	α_2	α_4	α_6	α_6	α_7	α_7	α_2	α_4	α_8	α_8

Now, we find the near equivalence classes according to the indiscernibility relation $\sim_{B_r}$ of elements in $\mathcal{O}$:

$$[a]_{\varphi_1} = \{x \in \mathcal{O} | \varphi_1(x) = \varphi_1(a) = \alpha_1\} = \{a,h,i\} = [h]_{\varphi_1} = [i]_{\varphi_1},$$

$$[b]_{\varphi_1} = \{x \in \mathcal{O} | \varphi_1(x) = \varphi_1(b) = \alpha_2\} = \{b,d,g\} = [d]_{\varphi_1} = [g]_{\varphi_1},$$

$$[c]_{\varphi_1} = \{x \in \mathcal{O} | \varphi_1(x) = \varphi_1(c) = \alpha_3\} = \{c,e\} = [e]_{\varphi_1},$$

$$[f]_{\varphi_1} = \{x \in \mathcal{O} | \varphi_1(x) = \varphi_1(f) = \alpha_4\} = \{f\},$$

$$[j]_{\varphi_1} = \{x \in \mathcal{O} | \varphi_1(x) = \varphi_1(j) = \alpha_5\} = \{j\}.$$

Then, we have that $\xi_{\varphi_1} = \{[a]_{\varphi_1}, [b]_{\varphi_1}, [c]_{\varphi_1}, [f]_{\varphi_1}, [j]_{\varphi_1}\}$.

$$[a]_{\varphi_2} = \{x \in \mathcal{O} | \varphi_2(x) = \varphi_2(a) = \alpha_1\} = \{a,c,h\} = [c]_{\varphi_2} = [h]_{\varphi_2},$$

$$[b]_{\varphi_2} = \{x \in \mathcal{O} | \varphi_2(x) = \varphi_2(b) = \alpha_3\} = \{b,e\} = [e]_{\varphi_2},$$

$$[d]_{\varphi_2} = \{x \in \mathcal{O} | \varphi_2(x) = \varphi_2(d) = \alpha_4\} = \{d,f\} = [f]_{\varphi_2},$$

$$[g]_{\varphi_2} = \{x \in \mathcal{O} | \varphi_2(x) = \varphi_2(g) = \alpha_5\} = \{g,j\} = [j]_{\varphi_2},$$

$$[i]_{\varphi_2} = \{x \in \mathcal{O} | \varphi_2(x) = \varphi_2(i) = \alpha_6\} = \{i\}.$$

We attain that $\xi_{\varphi_2} = \{[a]_{\varphi_2}, [b]_{\varphi_2}, [d]_{\varphi_2}, [g]_{\varphi_2}, [i]_{\varphi_2}\}$.

$$[a]_{\varphi_3} = \{x \in \mathcal{O} | \varphi_3(x) = \varphi_3(a) = \alpha_2\} = \{a,g\}$$

$$= [g]_{\varphi_3},$$

$$[b]_{\varphi_3} = \{x \in \mathcal{O} | \varphi_3(x) = \varphi_3(b) = \alpha_4\} = \{b,h\} = [h]_{\varphi_3},$$

$$[c]_{\varphi_3} = \{x \in \mathcal{O} | \varphi_3(x) = \varphi_3(c) = \alpha_6\} = \{c,d\} = [d]_{\varphi_3},$$

$$[e]_{\varphi_3} = \{x \in \mathcal{O} | \varphi_3(x) = \varphi_3(e) = \alpha_7\} = \{e,f\} = [f]_{\varphi_3},$$

$$[i]_{\varphi_3} = \{x \in \mathcal{O} | \varphi_3(x) = \varphi_3(i) = \alpha_8\} = \{i,j\} = [j]_{\varphi_3}.$$

From here, we get that $\xi_{\varphi_3} = \{[a]_{\varphi_3}, [b]_{\varphi_3}, [c]_{\varphi_3}, [e]_{\varphi_3}, [i]_{\varphi_3}\}$. Consequently, a set of partitions of $\mathcal{O}$ is $N_r(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$ for $r = 1$. Hence,

$$N_1(B)^*S = \bigcup_{[x]_{\varphi_i} \cap S \neq \emptyset} [x]_{\varphi_i}$$

$$= [b]_{\varphi_1} \cup [c]_{\varphi_1} \cup [f]_{\varphi_1} \cup [a]_{\varphi_2} \cup [b]_{\varphi_2} \cup [d]_{\varphi_2} \cup [c]_{\varphi_3} \cup [e]_{\varphi_3}$$

$$= \{a,b,c,d,e,f,g,h\}.$$

Taking operation tables for S in the following tables:

$+$	c	d	e	f
c	d	e	f	g
d	e	f	g	b
e	f	g	b	c
f	g	b	c	d

$\cdot$	c	d	e	f
c	c	d	e	f
d	d	f	b	d
e	e	b	e	b
f	f	d	b	f

In this case, $(S,+,\cdot)$ is a nearness semiring. Let take $A=\{d,e,f\}$ is subset of S.

$$N_1(B)^*A=\bigcup_{[x]_{\varphi_i}\cap Q\neq\varnothing}[x]_{\varphi_i}$$

$$=[b]_{\varphi_1}\cup[c]_{\varphi_1}\cup[f]_{\varphi_1}\cup[b]_{\varphi_2}\cup[d]_{\varphi_2}\cup [c]_{\varphi_3}\cup[e]_{\varphi_3}=\{b,c,d,e,f,g\}.$$

Since, A is a subsemigroup of S and $ASA\subseteq N_r(B)^*A$, A is a bi ideal of nearness semiring S.

Lemma 3.2

Let S be a nearness semiring and A be a non-empty subset of S. If S is commutative and $N_r(B)^*(N_r(B)^*A)=N_r(B)^*A$, then each quasi ideal of S is bi ideal of S.

Proof. Let S be a commutative nearness semiring and A be a quasi ideal of S. $ASA=(ASA)\cap(ASA)=A(SA)\cap(AS)A\subseteq S(SA)\cap(AS)S\subseteq(SS)A\cap A(SS)$ since S is a nearness semiring. Afterward, $(SS)A\cap A(SS)\subseteq N_r(B)^*SN_r(B)^*A\cap N_r(B)^*AN_r(B)^*S\subseteq N_r(B)^*(SA)\cap N_r(B)^*(AS)$ by Theorem 1 and Theorem 2.(ii). In this case, $N_r(B)^*(SA)\cap N_r(B)^*(AS)\subseteq N_r(B)^*(N_r(B)^*A)\cap N_r(B)^*(N_r(B)^*A)=N_r(B)^*(N_r(B)^*A)=N_r(B)^*A$ from Lemma 1. Hence, $ASA\subseteq N_r(B)^*A$ and A is a nearness bi ideal of S.

Theorem 3. 3

Let S be a nearness semiring and A be a non-empty subset of S. Each right or left nearness ideal of S is a nearness bi ideal of S if $N_r(B)^*A$ is grupoid.

Proof. Let A be left nearness ideal of S. In this case, $SA\subseteq N_r(B)^*A$. Then, $ASA\subseteq A(SA)\subseteq(N_r(B)^*A)(N_r(B)^*A)$ by Theorem 1. Then, $(N_r(B)^*A)(N_r(B)^*A)\subseteq N_r(B)^*A$ since $N_r(B)^*A$ is grupoid. Hence, $ASA\subseteq N_r(B)^*A$ and A is a nearness bi ideal of S.

Similarly, A be right nearness ideal. In this way, $AS\subseteq N_r(B)^*A$. Then, $ASA\subseteq(AS)A\subseteq(N_r(B)^*A)(N_r(B)^*A)$ by Theorem 1. Thus, $(N_r(B)^*A)(N_r(B)^*A)\subseteq N_r(B)^*A$ since $N_r(B)^*A$ is grupoid. Afterward, $ASA\subseteq N_r(B)^*A$ and A is a nearness bi ideal of S.

Lemma 3.3

Let S be a nearness semiring and A be a non-empty subset of S. Every bi ideal of S is an upper-near bi ideal of S if $N_r(B)^*(N_r(B)^*A)=N_r(B)^*A$.

Proof. Let S be a nearness semiring and A is a bi ideal of S. $(N_r(B)^*A)S(N_r(B)^*A)\subseteq(N_r(B)^*A)(N_r(B)^*S)(N_r(B)^*A)$ by Theorem 1. Then, $(N_r(B)^*A)(N_r(B)^*S)(N_r(B)^*A)\subseteq N_r(B)^*(AS)N_r(B)^*A$ from Theorem 2.(ii). Again, by Theorem 2.(ii), $N_r(B)^*(AS)N_r(B)^*A\subseteq N_r(B)^*(ASA)$. Since A is a bi ideal of S, we get that $N_r(B)^*(ASA)\subseteq N_r(B)^*(N_r(B)^*A)=N_r(B)^*A$. Hence, $(N_r(B)^*A)S(N_r(B)^*A)\subseteq N_r(B)^*A$ and A is an upper-near bi ideal of S.

Corollary 3.1

Let S be a nearness semiring. If S is commutative and $N_r(B)^*(N_r(B)^*A)=N_r(B)^*A$, then each quasi ideal is an upper-near bi ideal.

Example 3.2

Let $\mathcal{O}=\{a,b,c,d,e,f,g,h,i,j\}$ be a set of perceptual objects where $r=1$, $B=\{\varphi_1,\varphi_2,\varphi_3\}\subseteq\mathcal{F}$ be a set of probe functions. Let $S=\{c,d,e,f\}\subset\mathcal{O}$. Probe functions' values

$$\varphi_1:\mathcal{O}\to V_1=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5\},$$

$$\varphi_2:\mathcal{O}\to V_2=\{\alpha_1,\alpha_3,\alpha_4,\alpha_5,\alpha_6\},$$

$$\varphi_3:\mathcal{O}\to V_3=\{\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6\}$$

are presented in the table below:

	a	b	c	d	e	f	g	h	i	j
φ_1	α_1	α_2	α_3	α_3	α_2	α_4	α_4	α_1	α_4	α_5
φ_2	α_4	α_1	α_4	α_4	α_3	α_3	α_1	α_5	α_6	α_5
φ_3	α_2	α_3	α_2	α_3	α_4	α_5	α_4	α_6	α_6	α_6

Now, we find the near equivalence classes according to the indiscernibility relation $\sim_{B_r}$ of elements in $\mathcal{O}$:

$[a]_{\varphi_1}=\{x\in\mathcal{O}|\varphi_1(x)=\varphi_1(a)=\alpha_1\}=\{a,h\}=[h]_{\varphi_1}$,

$[b]_{\varphi_1}=\{x\in\mathcal{O}|\varphi_1(x)=\varphi_1(b)=\alpha_2\}=\{b,e\}=[e]_{\varphi_1}$,

$[c]_{\varphi_1}=\{x\in\mathcal{O}|\varphi_1(x)=\varphi_1(c)=\alpha_3\}=\{c,d\}=[d]_{\varphi_1}$,

$[f]_{\varphi_1}=\{x\in\mathcal{O}|\varphi_1(x)=\varphi_1(f)=\alpha_4\}=\{f,g,i\}=[g]_{\varphi_1}=[i]_{\varphi_1}$,

$[j]_{\varphi_1}=\{x\in\mathcal{O}|\varphi_1(x)=\varphi_1(j)=\alpha_5\}=\{j\}$.

Then, we have that $\xi_{\varphi_1}=\{[a]_{\varphi_1},[b]_{\varphi_1},[c]_{\varphi_1},[f]_{\varphi_1},[j]_{\varphi_1}\}$.

$[a]_{\varphi_2}=\{x\in\mathcal{O}|\varphi_2(x)=\varphi_2(a)=\alpha_4\}=\{a,c,d\}=[c]_{\varphi_2}=[d]_{\varphi_2}$,

$[b]_{\varphi_2}=\{x\in\mathcal{O}|\varphi_2(x)=\varphi_2(b)=\alpha_1\}=\{b,g\}=[g]_{\varphi_2}$,

$[e]_{\varphi_2}=\{x\in\mathcal{O}|\varphi_2(x)=\varphi_2(e)=\alpha_3\}=\{e,f\}=[f]_{\varphi_2}$,

$[h]_{\varphi_2}=\{x\in\mathcal{O}|\varphi_2(x)=\varphi_2(h)=\alpha_5\}=\{h,j\}=[j]_{\varphi_2}$,

$[i]_{\varphi_2}=\{x\in\mathcal{O}|\varphi_2(x)=\varphi_2(i)=\alpha_6\}=\{i\}$.

We attain that $\xi_{\varphi_2}=\{[a]_{\varphi_2},[b]_{\varphi_2},[e]_{\varphi_2},[h]_{\varphi_2},[i]_{\varphi_2}\}$.

$[a]_{\varphi_3}=\{x\in\mathcal{O}|\varphi_3(x)=\varphi_3(a)=\alpha_2\}=\{a,c\}=[c]_{\varphi_3}$,

$[b]_{\varphi_3}=\{x\in\mathcal{O}|\varphi_3(x)=\varphi_3(b)=\alpha_3\}=\{b,d\}=[d]_{\varphi_3}$,

$[e]_{\varphi_3}=\{x\in\mathcal{O}|\varphi_3(x)=\varphi_3(e)=\alpha_4\}=\{e,g\}=[g]_{\varphi_3}$,

$[f]_{\varphi_3}=\{x\in\mathcal{O}|\varphi_3(x)=\varphi_3(f)=\alpha_5\}=\{f\}$,

$[h]_{\varphi_3}=\{x\in\mathcal{O}|\varphi_3(x)=\varphi_3(h)=\alpha_6\}=\{h,i,j\}=[i]_{\varphi_3}=[j]_{\varphi_3}$.

From here, we get that $\xi_{\varphi_3}=\{[a]_{\varphi_3},[b]_{\varphi_3},[e]_{\varphi_3},[f]_{\varphi_3},[h]_{\varphi_3}\}$. Consequently, a set of partitions of $\mathcal{O}$ is $N_r(B)=\{\xi_{\varphi_1},\xi_{\varphi_2},\xi_{\varphi_3}\}$ for $r=1$. Hence,

$$N_1(B)^*S = \bigcup_{[x]_{\varphi_i}\cap S\neq\emptyset} [x]_{\varphi_i}$$
$= [b]_{\varphi_1} \cup [c]_{\varphi_1} \cup [a]_{\varphi_2} \cup [b]_{\varphi_2} \cup [e]_{\varphi_2} \cup [a]_{\varphi_3} \cup [b]_{\varphi_3} \cup [b]_{\varphi_3} = \{a, b, c, d, e, f, g\}$.

Taking operation tables for S in the tables below:

$+$	b	c	d	e
b	c	d	e	f
c	d	e	f	g
d	e	f	g	a
e	f	g	a	b

$\cdot$	b	c	d	e
b	b	c	d	e
c	c	e	g	b
d	d	g	c	f
e	e	b	f	c

In this case, $(S, +,\cdot)$ is a nearness semiring. Let take $M = \{b, c, d\}$ is subset of S.

$$N_1(B)^*M = \bigcup_{[x]_{\varphi_i}\cap Q\neq\emptyset} [x]_{\varphi_i}$$
$= [b]_{\varphi_1} \cup [c]_{\varphi_1} \cup [a]_{\varphi_2} \cup [b]_{\varphi_2} \cup [a]_{\varphi_3} \cup [b]_{\varphi_3} = \{a, b, c, d, e, g\}$.
Since $f \in MSM$ and $f \notin N_r(B)^*M$, $MSM \nsubseteq N_r(B)^*M$. Thus, M is not a bi ideal of nearness semiring S.

Theorem 3.4

Let S be a nearness semiring. If S is commutative and $N_r(B)^*(N_r(B)^*A) = N_r(B)^*A$, then the product of two quasi ideals of S is a bi ideal of S.

Proof. Let A_1 and A_2 be quasi ideals of S. We show that $(A_1A_2)S(A_1A_2) \subseteq N_r(B)^*(A_1A_2)$.

We attain that $(A_1A_2)S(A_1A_2) \subseteq (A_1A_2)S(SA_2) \subseteq (A_1A_2)S(N_r(B)^*A_2)$ from Lemma 1. Afterward, $(A_1A_2)S(N_r(B)^*A_2) = A_1A_2S(N_r(B)^*A_2) \subseteq (N_r(B)^*A_1)(N_r(B)^*A_2)S(N_r(B)^*A_2) \subseteq (N_r(B)^*A_1)(N_r(B)^*A_2)$ by Corollary 1.

In this case, $(N_r(B)^*A_1)(N_r(B)^*A_2) \subseteq N_r(B)^*(A_1A_2)$ by Theorem 2.(ii). Hence, $(A_1A_2)S(A_1A_2) \subseteq N_r(B)^*(A_1A_2)$ and the product of two quasi ideals of S is a bi ideal of S.

Definition 3.10

Let S be a nearness semiring. A is called nearness m-bi ideal of S if A is subsemigroup of S and $AS^mA \subseteq N_r(B)^*A$, where m is pozitive integer and not necessarily 1.

Definition 3.11

Let S be a nearness semiring. Q is called nearness (m, n)-quasi ideal of S if Q is subsemigroup of S and $QS^m \cap S^nQ \subseteq N_r(B)^*Q$, where m, n are pozitive integers.

Theorem 3.5

Let S be a nearness semiring. Each nearness $(m + 1, m + 1)$-quasi ideal of S is m-bi ideal of S.

Proof. Let S be a nearness semiring and A be a $(m + 1, m + 1)$-quasi ideal of S. In this case, it is attainned that $AS^mA \subseteq AS^mS = AS^{m+1}$ and $AS^mA \subseteq SS^mA = S^{m+1}A$. Thus, $AS^mA \subseteq AS^{m+1} \cap S^{m+1}A$. Since A is a $(m + 1, m + 1)$-quasi ideal of S, we get that $AS^mA \subseteq AS^{m+1} \cap S^{m+1}A \subseteq N_r(B)^*A$. Hence $AS^mA \subseteq N_r(B)^*A$ and A is m-bi ideal of S.

4. Conclusion

As a recent study of nearness semirings, it is defined that the notion of bi ideals in nearness semirings. Afterward, it is explained that some of the concepts and definitions and an example is given with related to the subject. Furthermore, it is given that the definition of nearness $\boldsymbol{m}$-bi ideals and nearness $(\boldsymbol{m}, \boldsymbol{n})$-quasi ideals. And, it is examined that the relationship between them. We believe that these properties will be more useful theoretical development for nearness semiring theory.

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