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Statistical Convergence of Rough Variable

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Abstract

In this paper, we present the concept of statistically (λ-statistically) convergent sequences for rough variables. Furthermore, the relation between convergence statistically in trust and converges $\lambda-$statistically in trust is given. Also, some properties of statistically (λ-statistically) convergent sequences are discussed. In addition, we introduce statistically Cauchy sequence in rough spaces.

Keywords: *statistical convergence, λ-statistical convergence, λ-statistically Cauchy sequence, rough variable, rough space*
2010 Mathematics Subject Classification: *Use about five key words or phrases in alphabetical order, Separated by Semicolon.*

1. Introduction

In 1982, the notion of rough set theory was presented by Pawlak [5] in order to deal with vague description of objects. It helps to find new mathematical approach to handle defective data in real world. A crucial presumption in this hypothesis is that objects are seen through accessible information on their qualities, but such informations might not adequate to characterize these objects precisely. One way is changing our perspective like that approximate a set with an other set. In this way, a rough set might be defined by a combine of crisp sets, called the lower and upper approximations, that are originally produced by an equivalence relation(reflexive, symmetric, and transitive). Slowinski and Vanderpooten [14] expanded the equivalence relation to more common case and they suggested a binary relation that it is not symmetric and transitive, but reflexive. Liu [2] characterized a rough variable from rough space to the set of real numbers and he presented the description of the lower and upper approximation of the rough variable. Also, Liu[1] proposed four sorts of convergence concepts for rough variable: convergence almost surely, convergence in trust, convergence in mean and convergence in distribution.
Besides, statistical convergence for real sequence was first presented by Fast[4] in 1951. After that, in 1959 Schoenberg[9] gave some fundamental properties of statistical convergence. Furthermore, it was examined in detail by Fridy [3] in 1985. Fridy introduced the idea of statistically Cauch sequence. Statistical convergence was considered in locally convex spaces by Maddox [8] in 1987. Besides, Mursaleen[15] presented λ-statistical convergence in 2000. The concept of statistical convergence has been investigated in a number of papers [10, 11, 12, 13, 16, 17, 18] and so on.
This article is committed to display a fresh kind of convergence for rough variables sequences. Next section, the definition of rough variable is given. Together with this, the definition of rough space is given. In addition, some important definitions are given related to rough space. In section 3, the idea of statistically convergence in trust, λ-statistically convergence in trust and uniqueness of λ-statistically convergence are introduced and corresponding results are obtained. Finally, the concept of λ-statistically Cauchy in trust is given.

2. Preliminary

This section is related to important definitions for this article. If you need more information about these definitions, one can look Liu [2, 1, 6].

Definition 2.1. *[2] Let Λ be a non-empty set, $\mathscr{A}$ a σ-algebra of subset of Λ, Δ is an element in $\mathscr{A}$, and π is a set function that is satisfying the following axioms,*

- $\pi(\Lambda) < \infty$;
- $\pi(\Delta) > 0$;
- $\pi(A) \geq 0 \; \forall \, A \in \mathscr{A}$;
- *for each countable sequence of disjoint events* $\{A_i\}_{i=1}^{\infty}$, *we get*

$$\pi\left\{\bigcup_{i=1}^{\infty} A_i\right\} = \sum_{i=1}^{\infty} \pi\{A_i\}.$$

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Then we can say that $(\Lambda,\Delta,\mathscr{A},\pi)$ *is a rough space.*

Definition 2.2. *[2] A rough variable* ξ *is a measurable function from the rough space* $(\Lambda,\Delta,\mathscr{A},\pi)$ *to the set of real number* $\mathbb{R}$*, i.e., for all Borel set* B^* *of* $\mathbb{R}$*, we get*

$$\{\lambda\in\Lambda|\xi(\lambda)\in B^*\}\in\mathscr{A}.$$

Definition 2.3. *[2] Let* $(\Lambda,\Delta,\mathscr{A},\pi)$ *be a rough space. Then the lower and upper trust of an event A is defined by*

- $\overline{Tr}\{A\}=\frac{\pi\{A\}}{\pi\{\Lambda\}}$;
- $\underline{Tr}\{A\}=\frac{\pi\{A\cap\Delta\}}{\pi\{\Delta\}}$;

and the trust of event of A is defined as

$$Tr\{A\}=\frac{1}{2}(\underline{Tr}\{A\}+\overline{Tr}\{A\}).$$

Definition 2.4. *[1] Suppose that* $\{\xi_n\}$ *be a sequence of rough variables. The sequence* $\{\xi_n\}$ *converges in trust to the rough variable* ξ *if*

$$\lim_{n\to\infty} Tr\{|\xi_n-\xi|\geq\varepsilon\}=0$$

for every $\varepsilon>0$.

3. Main Results

In this section, the definition of statistically(λ-statistically) convergence is given and some conclusions are presented.

Definition 3.1. *Let* $\xi,\xi_1,\xi_2,\cdots$ *be a rough variables. Then* ξ_n *is said to be statistically convergent in trust to the rough variable* ξ *if* $\forall\ \varepsilon>0$ *and* $\delta>0$,

$$\lim_{n\to\infty}\frac{1}{n}\left|\{k\leq n: Tr\{|\xi_k-\xi|\geq\varepsilon\}\geq\delta\}\right|=0. \tag{3.1}$$

In that case, we can say $\xi_n \xrightarrow[S_n(Tr)]{} \xi$.

Now, let $\{\lambda_n\}$ be a non decreasing sequence of positive numbers such that

$$\lambda_{n+1}\leq\lambda_n+1,\lambda_1=1,\lim_{n\to\infty}\lambda_n=\infty.$$

Let us define,

$$T_n(Tr)=\frac{1}{\lambda_n}\sum_{k=m}^{n}Tr\{|\xi_k-\xi|\geq\varepsilon\},$$

where $m\in B_n$, $B_n=[n-\lambda_n+1,n]$.
Some convergence concepts can be adopted via these concepts.

Definition 3.2. *Let* $\xi,\xi_1,\xi_2,\cdots$ *be a rough variables. Then* ξ_n *is said to be statistical* $T_n(Tr)$*-summable to* ξ *if* $\forall\ \varepsilon>0$ *and* $\forall\delta>0$,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left|\{m\in B_n:\sum_{k=m}^{n}Tr\{|\xi_k-\xi|\geq\varepsilon\}\geq\delta\}\right|=0. \tag{3.2}$$

In that case, we can say $\xi_n \xrightarrow[T_n(Tr)]{} \xi$.

Definition 3.3. *Let* $\xi,\xi_1,\xi_2,\cdots$ *be a rough variables. Then* ξ_n *is said to be* λ*-statistically convergent in trust to the rought variable* ξ *if* $\forall$ $\varepsilon>0$ *and* $\delta>0$,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left|\{k\in B_n: Tr\{|\xi_k-\xi|\geq\varepsilon\}\geq\delta\}\right|=0. \tag{3.3}$$

In that case, we can say $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$.

Theorem 3.4. *Let* $\xi,\xi_1,\xi_2,\cdots$ *be rough variables.* $\{\xi_n\}$ *is statistically convergent to the rough variable* ξ *if* $\{\xi_n\}$ *convergent in trust to the rough variable* ξ.

Proof. Since $\{\xi_n\}$ converges in trust to the rough variable ξ, by using definition(2.4) we can say that,

$$\lim_{n\to\infty} Tr\{|\xi_n-\xi|\geq\varepsilon\}=0$$

for any $\varepsilon>0$. That is, for any $\delta>0$ $\exists U\in\mathbb{N}$

$$Tr\{|\xi_k-\xi|\geq\varepsilon\}\leq\delta$$

for all $k>U$.
Therefore,

$$|\{k\leq n: Tr\{|\xi_k-\xi|\geq\varepsilon\}\geq\delta\}|\leq U$$

for all $n\in\mathbb{N}$. Thus,

$$\lim_{n\to\infty}\frac{1}{n}\left|\{k\leq n: Tr\{|\xi_k-\xi|\geq\varepsilon\}\geq\delta\}\right|=0.$$

□

Example 3.5. *To show that the inverse of theorem doesn't need to be true, we have the following example, which means convergence statistically of $\{\xi_n\}$ to the rough variable ξ does not require convergence in trust of $\{\xi\}$ to the rough variable ξ.*
For instance, define $\Lambda=\{A_1,A_2\}$, $\mathscr{A}=\mathscr{P}(\Lambda)$ where $\mathscr{P}$ is the power set, $\Delta=\Lambda$ and $\pi\{A_1\}=8=\pi\{A_2\}$. So it can be easily seen $(\Lambda,\Delta,\mathscr{A},\pi)$ is a rough space. Define $Tr\{A_1\}=\frac{1}{2}=Tr\{A_2\}$ and

$$\xi_k\{A\}=\begin{cases}-1, & k=m^2\wedge A=A_1\\ 1, & k=m^2\wedge A=A_2\\ 0, & k\neq m^2\wedge A=A_1\\ 1, & k\neq m^2\wedge A=A_2\end{cases}$$

where $m\in\mathbb{N}$. Now, let $\xi=0$, $0<\varepsilon<1$ and $\frac{1}{2}<\delta<1$. Then we have,

$$Tr\{|\xi_k-0|\geq\varepsilon\}=\begin{cases}1, & k=m^2\\ \frac{1}{2}, & k\neq m^2\end{cases}$$

Thus,

$$\lim_{k\to\infty} Tr\{|\xi_k-0|\geq\varepsilon\}\neq 0.$$

However,

$$\lim_{n\to\infty}\frac{1}{n}|\{k\leq n: Tr\{|\xi_k-0|\geq\varepsilon\}\geq\delta\}|=0.$$

Hence, $\xi_n\xrightarrow[S_n(Tr)]{}\xi$ but ξ_n is not convergent in trust.

Therefore we show that a sequence which is statistically convergent doesn't need to be convergent in trust. Next theorem, we discuss uniqueness of λ-statistical convergence.

Theorem 3.6. *Let $\xi,\xi_1,\xi_2,\cdots$ be a rough variables. If $\{\xi_n\}$ is a λ-statistically convergent to the rough variable ξ in trust, then $\xi_n\xrightarrow[S_\lambda(Tr)]{}\xi$ is unique in trust.*

Proof. Assume that $\xi_n\xrightarrow[S_\lambda(Tr)]{}\xi$ and $\xi_n\xrightarrow[S_\lambda(Tr)]{}y$. So we can say that $\forall\varepsilon>0,\delta>0$,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left|\{k\in B_n: Tr\{|\xi_k-\xi|\geq\frac{\varepsilon}{2}\}\geq\frac{\delta}{2}\}\right|=0.$$

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left|\{k\in B_n: Tr\{|\xi_k-y|\geq\frac{\varepsilon}{2}\}\geq\frac{\delta}{2}\}\right|=0$$

, respectively. Let us define,

$$X=\left\{k\in B_n: Tr\{|\xi_k-y|\geq\frac{\varepsilon}{2}\}\geq\frac{\delta}{2}\right\}$$

$$Y=\left\{k\in B_n: Tr\{|\xi_k-\xi|\geq\frac{\varepsilon}{2}\}\geq\frac{\delta}{2}\right\}.$$

Let take an element m $\in X^c\cap Y^c$. Then we can get,

$$Tr\{|\xi_m-\xi|\geq\frac{\varepsilon}{2}\}<\frac{\delta}{2}\wedge Tr\{|\xi_m-y|\geq\frac{\varepsilon}{2}\}<\frac{\delta}{2}.$$

Thus,

$$Tr\{|\xi - y| \geq \varepsilon\} = Tr\{|\xi + \xi_m - \xi_m - y| \geq \varepsilon\}$$

$$\leq Tr\{|\xi_m - \xi| \geq \frac{\varepsilon}{2}\} + Tr\{|\xi_m - y| \geq \frac{\varepsilon}{2}\}$$

$$< \frac{\delta}{2} + \frac{\delta}{2} = \delta.$$

Since $\delta > 0$ is an arbitrary, we might obtain $Tr\{|\xi - y| \geq \varepsilon\} = 0$ that satisfies $\xi = y$ in trust. □

Together with this proof, we show that uniqueness of λ-statistically convergence.

Theorem 3.7. *Let $h : \mathbb{R} \to \mathbb{R}$ be a continuous function. If $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$, then $h(\xi_n) \xrightarrow[S_\lambda(Tr)]{} h(\xi)$.*

Proof. Since h is a continuous function, $\forall\varepsilon > 0$, $\exists M > 0$ such that,

$$|h(\xi_k) - h(\xi)| < \varepsilon \qquad \text{whenever} \quad |\xi_k - \xi| < M.$$

That is, $|h(\xi_k) - h(\xi)| \geq \varepsilon$ implies $|\xi_k - \xi| \geq M$.
For that reason one can say,

$$\{|h(\xi_k) - h(\xi)| \geq \varepsilon\} \subseteq \{|\xi_k - \xi| \geq M\}.$$

Take trust both sides,

$$Tr\{|h(\xi_k) - h(\xi)| \geq \varepsilon\} \leq Tr\{|\xi_k - \xi| \geq M\}.$$

That gives,

$$\{k \in B_n : Tr\{|h(\xi_k) - h(\xi)| \geq \varepsilon\} \geq \delta\} \subseteq \{k \in B_n : Tr\{|\xi_k - \xi| \geq M\} \geq \delta\}.$$

Take absolute value both sides and divide both sides to $\frac{1}{\lambda_n}$,

$$\frac{1}{\lambda_n}|\{k \in B_n : Tr\{|h(\xi_k) - h(\xi)| \geq \varepsilon\} \geq \delta\}| \leq \frac{1}{\lambda_n}|\{k \in B_n : Tr\{|\xi_k - \xi| \geq M\} \geq \delta\}|.$$

Since $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$, we can say

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\left|\{k \in B_n : Tr\{|\xi_k - \xi| \geq M\} \geq \delta\}\right| = 0.$$

So by using squeeze theorem, we get

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\left|\{k \in B_n : Tr\{|h(\xi_k) - h(\xi)| \geq \varepsilon \geq \delta\}\right| = 0$$

which is mean $h(\xi_n) \xrightarrow[S_\lambda(Tr)]{} h(\xi)$. □

Next theorem we are going to show that statistical summable implies λ-statistical convergence.

Theorem 3.8. *Let $\xi, \xi_1, \xi_2, \cdots$ be rough variables. If $\xi_n \xrightarrow[T_n(Tr)]{} \xi$ then $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$.*

Proof. Since $\xi_n \xrightarrow[T_n(Tr)]{} \xi$, $\forall\varepsilon > 0$ and $\delta > 0$. For any $t \in B_n$, we have

$$\sum_{k=t}^{n} Tr\{|\xi_k - \xi| \geq \varepsilon\} \geq Tr\{|\xi_t - \xi| \geq \varepsilon\}$$

Then we can say,

$$\{t \in B_n : \sum_{k=t}^{n} Tr\{|\xi_k - \xi| \geq \varepsilon\} \geq \delta\} \supseteq \{t \in B_n : Tr\{|\xi_t - \xi| \geq \varepsilon\} \geq \delta\}$$

Take absolute value both sides,

$$|\{t \in B_n : \sum_{k=t}^{n} Tr\{|\xi_k - \xi| \geq \varepsilon\} \geq \delta\}| \geq |\{t \in B_n : Tr\{|\xi_t - \xi| \geq \varepsilon\} \geq \delta\}|.$$

Since $\xi_n \xrightarrow[T_n(Tr)]{} \xi$, we have $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$. □

Next theorems [5,6], we discuss relation between statistical convergence and λ-statistical convergence.

Theorem 3.9. *Let $\xi, \xi_1, \xi_2, \cdots$ be rough variables. If $\lim_{n\to\infty} \frac{\lambda_n}{n} > 0$, then $\xi_n \xrightarrow[S_n(Tr)]{} \xi$ implies that $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$.*

Proof. $\forall \varepsilon > 0$, $\delta > 0$,
$\{k \le n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\} \supseteq \{k \in B_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}$.
Take absolute value and divide both sides by $\frac{1}{n}$ we have,

$$\frac{1}{n}|\{k \le n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}| \ge \frac{1}{n}|\{k \in B_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}|$$
$$= \frac{\lambda_n}{n}\frac{1}{\lambda_n}|\{k \in I_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}|.$$

Since $\xi_n \xrightarrow[S_n(Tr)]{} \xi$, we can say

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\left|\{k \in B_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}\right| = 0.$$

□

Theorem 3.10. *Let $\xi, \xi_1, \xi_2, \cdots$ be rough variables. If $\lim_{n\to\infty} \frac{n-\lambda_n}{n} = 0$, then $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$ implies that $\xi_n \xrightarrow[S_n(Tr)]{} \xi$.*

Proof. If $\lim_{n\to\infty} \frac{n-\lambda_n}{n} = 0$, then for any and $M > 0$, $\exists N \in \mathbb{N}$ s.t. $\frac{n-\lambda_n}{n} < \frac{M}{2}$ where $n \ge N$. Then for any $\varepsilon > 0, \delta > 0$ and $n \ge M$, we get

$$\frac{1}{n}|\{k \le n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}|$$
$$= \frac{1}{n}|\{k \le n-\lambda_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}| + \frac{1}{n}|\{k \in B_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}|$$
$$\le \frac{n-\lambda_n}{n} + \frac{1}{n}|\{k \in B_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}|$$
$$\le \frac{M}{2} + \frac{1}{\lambda_n}|\{k \in B_n : Tr\{|\xi_k - \xi| \ge \varepsilon\} \ge \delta\}|.$$

Take the limit both sides as $n \to \infty$ and since $\xi_n \xrightarrow[S_\lambda(Tr)]{} \xi$, we have $\xi_n \xrightarrow[S_n(Tr)]{} \xi$. □

Definition 3.11. *Let $\xi, \xi_1, \xi_2, \cdots$ be rough variables. A sequence $\{\xi_n\}$ is a λ-statistically Cauchy sequence in trust if $\forall\ \varepsilon > 0, \delta > 0$ then $\exists M = M(\varepsilon)$ s.t.*

$$\lim_{n\to\infty} \frac{1}{\lambda_n}|\{k \in B_n : Tr\{|\xi_k - \xi_M| \ge \varepsilon\} \ge \delta\}| = 0.$$

Theorem 3.12. *Let $\xi, \xi_1, \xi_2, \cdots$ be rough variables. If $\{\xi_n\}$ is λ-statistically convergent to the rough variable ξ in trust, then it is λ-statistically Cauchy in trust.*

Proof. Since $\{\xi\}$ converges λ-statistically to the rough variable ξ in trust, for any $\varepsilon > 0$ and $\delta > 0$ we have,

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\left|\left\{k \in B_n : Tr\{|\xi_k - \xi| \ge \frac{\varepsilon}{2}\} \ge \frac{\delta}{2}\right\}\right| = 0.$$

Let us define,

$$X = \left\{k \in B_n : Tr\{|\xi_k - \xi| \ge \frac{\varepsilon}{2}\} \ge \frac{\delta}{2}\right\}$$

and,

$$Y = \{k \in B_n : Tr\{|\xi_k - \xi_N| \ge \varepsilon\} \ge \delta\}.$$

To prove $Y \subseteq X$, we need to prove $X^c \subseteq Y^c$.

$$X = \left\{k \in B_n : Tr\{|\xi_k - \xi| \ge \frac{\varepsilon}{2}\} < \frac{\delta}{2}\right\}$$

and,

$$Y = \{k \in B_n : Tr\{|\xi_k - \xi_N| \ge \varepsilon\} < \delta\}.$$

Let us take $N \in A^c$, $Tr\{\xi_N - \xi \ge \frac{\varepsilon}{2}\} < \frac{\delta}{2}$
and take $k \in A^c$, $Tr\{\xi_k - \xi \ge \frac{\varepsilon}{2}\} < \frac{\delta}{2}$.
Thus, we can say,

$$Tr\{|\xi_k - \xi_N| \ge \varepsilon\} \le Tr\{|\xi_k - \xi| \ge \frac{\varepsilon}{2}\} + Tr\{|\xi_N - \xi| \ge \frac{\varepsilon}{2}\}$$
$$< \frac{\delta}{2} + \frac{\delta}{2} < \delta$$

Hence, we have $Y \subset X$ implies that

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\left|\left\{k \in B_n : Tr\{|\xi_k - \xi_N| \ge \frac{\varepsilon}{2}\} \ge \frac{\delta}{2}\right\}\right| = 0.$$

Thus, we can say that $\{\xi_n\}$ is λ-statistically Cauchy in trust. □

4. Conclusion

This paper contributed to rough set theory with two new convergence concepts which are called statistically and λ-statistically convergence. Some conclusions were presented about these concepts. Finally, the definition of λ-statistically Cauchy in trust was defined.

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