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Investigation of Chaoticity of Vibrations in Machining

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ABSTRACT

Vibrations that occur between the workpiece and the cutting tool during machining processes that are frequently used in the manufacturing sector, may pose serious problems in applications. These vibrations have a negative impact on the process in terms of quality, cost and efficiency. In this study, firstly the differential equations of these vibrations cited from the literature and are called as self-excited oscillations, have been reduced to the first order equations. Then, the chaoticity of the system were investigated using time series, phase portraits, Lyapunov exponents and bifurcation diagrams and the initial shear force intervals that the system exhibits chaos behavior were determined.

Keywords: Metal cutting, chaos, self-excited oscillations, bifurcation, stability analysis

1 Introduction

Machining has been one of the important methods in the study of the production. In machining, cutting forces are variable and cause vibrations between workpiece and cutting tool. Usually these vibrations are called chatter in the literature and they cause fluctuations on the workpiece [2, 3]. In this case, the surface quality, life, cost and efficiency of the product is negatively affected, as well as a serious risk in terms of tool, machine and work safety. Especially in milling machines, these types of vibrations are much more critical because the cutting forces vary considerably due to the nature of the cutting process [4, 5]. In cutting processes, undesirable increases in the amplitude of these vibrations arise and that makes the system even more unstable. As a natural consequence of that, oscillations grow steadily. Therefore, this phenomenon is called self-excited oscillations in the literature [6]. The behavior of such systems can be analyzed non-linearly, since they contain many interactive parameters in their structure [7]. When the vibrating cutting tool is examined mathematically, it can be seen that the structure of the equations of motion varies according to the degree of freedom of the model. In recent years, there has been a greater focus is placed upon one degree of freedom [8, 9, 10] and two degrees of freedom models within the literature [1, 2, 6, 11, 12, 13].

Studies on nonlinear dynamic systems in the world of science reveal the existence and importance of chaotic behaviors in many practical applications such as machining and mixing. Because of improvements in cutting tool materials and machine design, upper limits in parameters such as cutting speed, feed and cutting depth have been exceeded. These developments result in increased working efficiency of the machine tools. However, especially when high cutting depths are reached, the chatter occurs which reduces the process quality. This issue is still up to date and is the subject of the studies of many researchers [14, 15]. In this study, whether the interaction between the workpiece and the cutting

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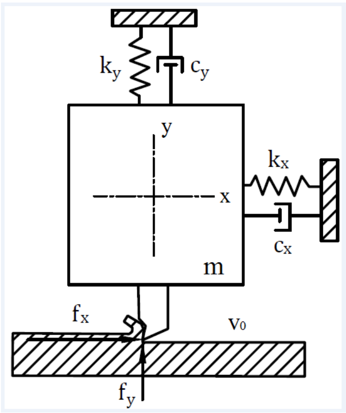
tool shows chaotic behavior is examined mathematically via time series, phase portraits, Lyapunov exponents and bifurcation diagrams, as in many studies in the field of chaos [16-19]. In the study, when the equations used in the literature are examined, the relative movement between the workpiece and the cutting tool is assumed as continuous and unidirectional for mathematical convenience. Thus, conditions such as discontinuity or non-derivative are prevented at certain points that will be caused by unit step and signum functions.

2 Expression of Equations of Cutting Process

In the contact of the cutting tool with the workpiece, the mathematical model can be constructed from simple mass-spring-damper system given below Figure 1 and expressed in sets of second degree equations [13]:



x, y: Movement directions
k_x, k_y: Spring constants in x and y directions
c_x, c_y: Damping constants in x and y directions
m: Mass of the oscillating system
f_x, f_y: x and y components of cutting force
v_0: Velocity of workpiece

Figure 1. 2 DOF Model of Workpiece and Cutting Tool system [13]

Here, the cutting tool is considered as a simple mechanical model consisting of spring and dampers in the x and y directions and the mass of the cutting tool. The relative speed and the contact forces in the x and y directions resulting from the movement of the cutting tool on the workpiece are expressed visually. In the studies carried out in the literature, a system of equations such as the geometry of the removed chip, contact angle and material properties has been introduced including the following [1, 2, 12]. The equations used in the study are shown in Equation 1-6 [7]:

$$F_x = F_0 H[C_1(V_r - 1)^2 + 1] \qquad F_y = \mu F_x \tag{1}$$

$$\mu = \mu_0 \left[C_2\left(V_p - 1\right)^2 + 1\right] [C_3(H - 1)^2 + 1] \tag{2}$$

$$H = h_0 - \mathrm{Y} \qquad V_r = v_0 - \dot{\mathrm{X}} \qquad V_p = V_r - R\dot{\mathrm{Y}} \tag{3}$$

$$R = R_0[C_4(V_r - 1)^2 + 1] \tag{4}$$

These equations can be shown as a second order differential equation system as follows:

$$\ddot{\mathrm{X}} = F - A\mathrm{X} \qquad Amw_0^2 = k_x \tag{5}$$

$$\ddot{\mathrm{Y}} = \mu F - B\mathrm{Y} \qquad Bmw_0^2 = k_y \tag{6}$$

Denote that C_{1-4}: cutting process constants, R_0: shear plastic deformation constant, R: variable shear plastic deformation constant, μ_0: static friction coefficient, F_0: initial cutting force, V_p: relative speed between cutting tool and the workpiece in y direction, V_r: relative speed between cutting tool and the workpiece in x direction, μ : the relative friction coefficient of the material removed from the surface, h_0: initial cutting depth and w_0: natural frequency of cutting process. Considering the studies carried out in the literature, natural frequency is chosen as $w_0 = 27.10^3 s^{-1}$. There are a large number of published studies [9, 12] that implement values of cutting process parameters determined by material properties and cutting speed are A=1, B=0.25, C_1=0.3, C_2=0.7, C_3=1.5, C_4=1.2, μ_0=0.35, R_0=2.2, v_0=0.5, m=15 and h_0=0.25 respectively. Chaoticity of the system can be investigated by replacing these values in the equations (1-6). The new form of these second order equations given below:

$$
\begin{aligned}
&\ddot{X} = -X + F_0(0.075\dot{X}^2 + 0.075\dot{X} + 0.26875 - 0.3Y\dot{X}^2 - 0.3Y\dot{X} - 1.075Y)\\
&\ddot{Y} = -0.25Y + F_0(0.075\dot{X}^2 + 0.075\dot{X} + 0.26875 - 0.3Y\dot{X}^2 - 0.3Y\dot{X} - 1.075Y) \qquad (7)\\
&(1.5Y^2 + 2.25Y + 1.84375)[0.245(2.64\dot{X}^2Y + 2.64\dot{X}\dot{Y} + 2.86\dot{Y} + \dot{X} + 0.5)^2 + 0.35]
\end{aligned}
$$

3 Times Series, Phase Portraits and Lyapunov Exponents of 2-DOF Metal Cutting

In order to examine the chaoticity of the system graphically, two previously obtained second order equations should be arranged as four first order equations. With this approach, the state variables of the system can be examined according to time and each other. In the equations given above, variable transformations are made such that x_1=X, x_2=$\dot{X}$, x_3=Y, x_4=$\dot{Y}$. The equations are rearranged as,

$$
\begin{aligned}
&x = \dot{x}_1 = x_2\\
&y = \dot{x}_2 = -x_1 + F_0(0.075{x_2}^2 + 0.075x_2 + 0.26875 - 0.3x_3{x_2}^2 - 0.3x_3x_2 - 1.075x_3)\\
&z = \dot{x}_3 = x_4\\
&w = \dot{x}_4 = 0.25x_3 + F_0(0.075{x_2}^2 + 0.075x_2 + 0.26875 - 0.3x_3{x_2}^2 - 0.3x_3x_2 - 1.075x_3)\\
&(1.5{x_3}^2 + 2.25x_3 + 1.84375)[0.245(2.64{x_2}^2x_3 + 2.64x_4x_2 + 2.86x_4 + x_2 + 0.5)^2 + 0.35]
\end{aligned}
$$

in the form of 4 dimensional non-linear equation system is obtained. For initial cutting force amplitude value F_0=0.17, this equation system is solved numerically. The initial conditions are selected as $[x\ y\ z\ w]^T = [0\ 0\ 0\ 0]^T$. The time series, the phase portraits and the 3-D plot graphs of signals are shown in Figures 2, 3, 4 and 5 respectively.

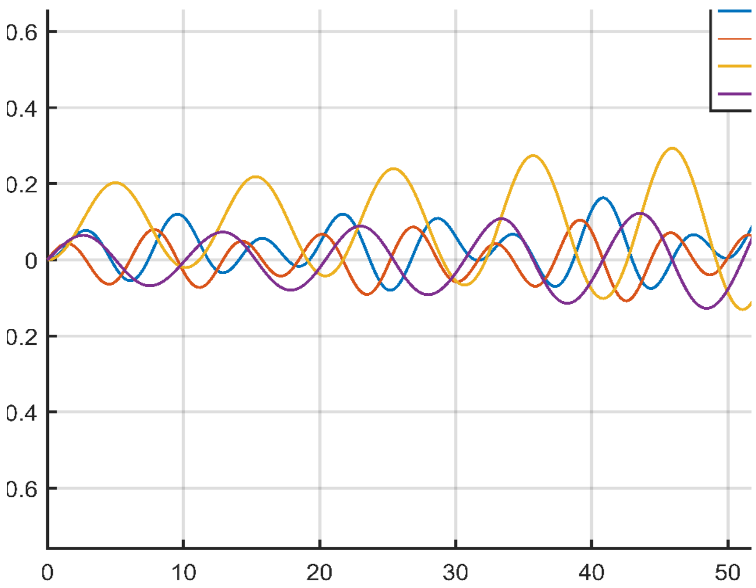


Figure 2. Time series graph of signals x, y, z and w

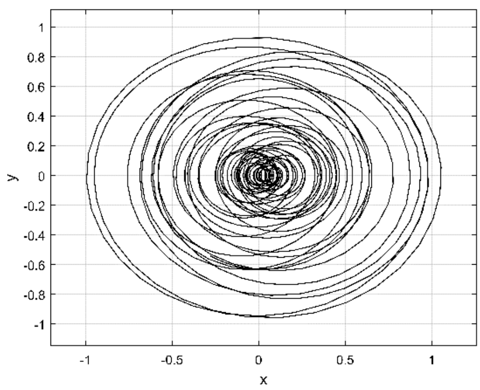


Figure 3. Phase portrait of x and y signals

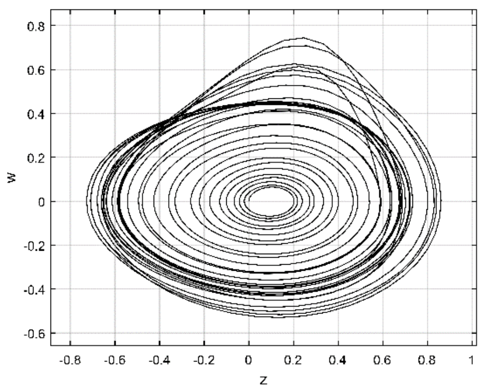


Figure 4. Phase portrait of z and w signals

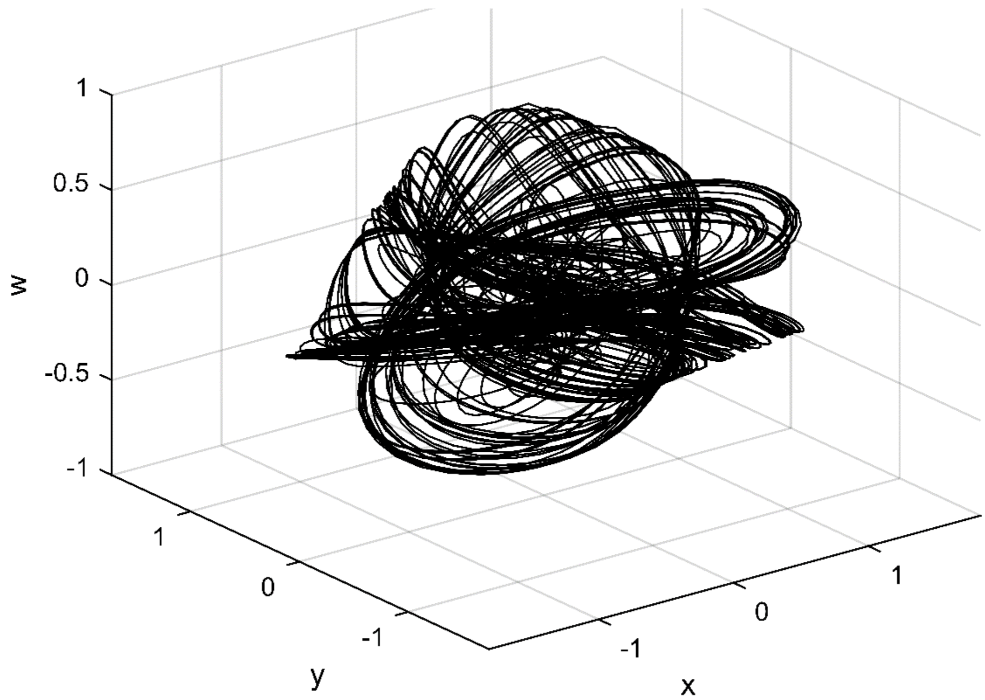


Figure 5. 3-D plot of x, y and w signals

In the Figure 3 and 4, these 2-dimensional graphs in which the changes of x, y, z and w signals are expressed visually are called phase portraits. When the phase portraits are checked, it is observed that the signals do not change randomly and there is a complex but regular relationship between them that cannot be explained in the time series graphs in Figure 2. These graphs, which give an idea that the system is chaotic, are confirmed by the calculation of Lyapunov exponents (λ). The graph of Lyapunov exponents is given in Figure 6.

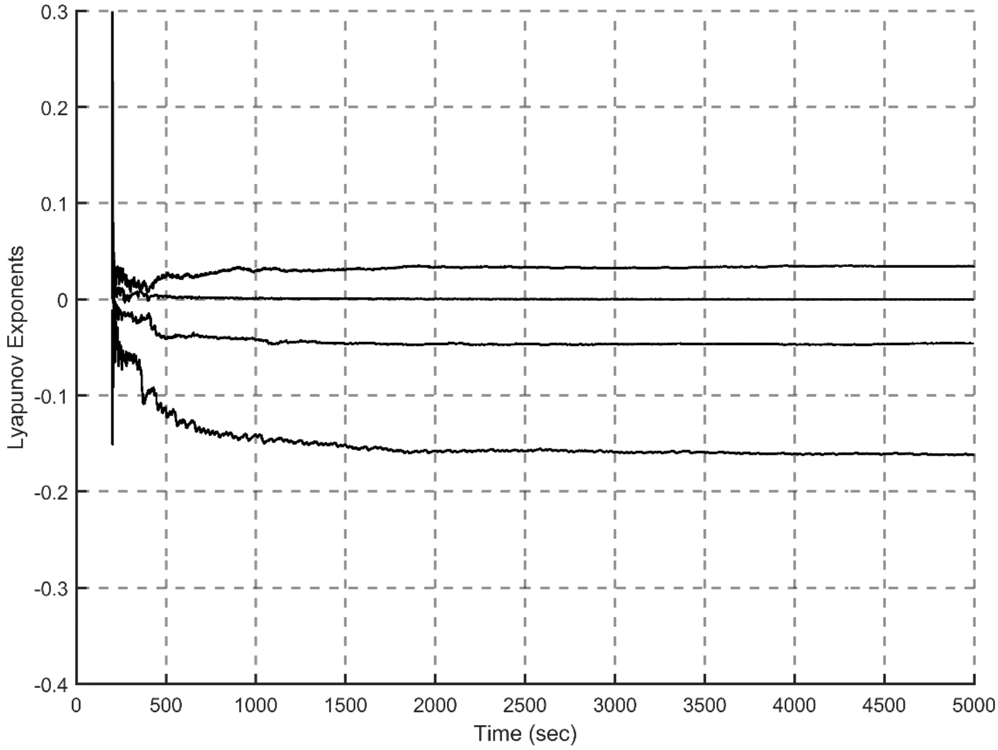


Figure 6. Lyapunov exponents graph of cutting process system

Lyapunov exponents describe the tendency of the trajectories of dynamic system variables to move away from each other, and any positive Lyapunov exponent to be calculated is considered to be one of the most concrete evidence that the system is chaotic. In theory, this process can only give the final result in infinite time or iteration. In the study, λ_1= 0.03484, λ_2= -0.04575, λ_3=0 and λ_4= -0.16116 results in 5000 iterations. The positive Lyapunov exponent found proves that the system exhibits chaotic behavior.

4 Chaoticity Analysis of Variation in Amplitude of Initial Shear Force in Cutting Process

Changes in the parameters in the non-linear equation set directly affect the behavior of the system. In such a case, the system may exhibit features such as point stability, limit cycle stability, semi-periodic and semi-chaotic stability, chaotic state and instability. The effect of changes in the amplitude of the initial shear force on the chaotic behavior of the system can be found in the bifurcation diagrams. In the study, the bifurcation graph plotted for -0.4<F_0 <0.25 values is given in Figure 7.

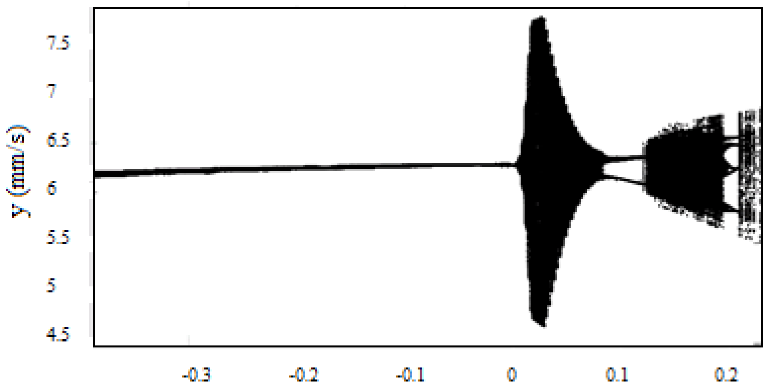


Figure 7. Bifurcation diagram plotted for $-0.4<F_0<0.25$ values

When the graph given above is examined, it can be easily seen that the system loses its chaotic property and shows stable behavior at shear force values less than 0.02. Likewise, it is seen that the system shows double-period behavior between 0.07-0.13 and exhibits limit cycle stability. Negative values of the force verify the calculation of the termination of the chaotic behavior as this means losing the contact between the workpiece and the cutting tool. Although 0.19-0.21 can be interpreted as 4-period behavior of the system, the graph can be drawn in more detail (by arranging the sample numbers taken during the iteration) and a final judgement can be reached.

5 Results and Discussion

In the study, equations related to vibrations arising from dynamic interactions between cutting tool and workpiece were obtained from literature and transformed into 4-dimensional equation systems by using variable transformations and chaotic properties of the process were presented as time series, phase portraits, Lyapunov exponents and bifurcation diagrams.

As a result of the analyzes, it was shown that possible changes in the amplitude of the initial shear force lead to periodic, semi-periodic, chaotic and unstable behaviors of the system and it was shown that the chaotic and unstable behaviors encountered as undesirable situation can be prevented by selecting the appropriate value ranges to be obtained from the bifurcation diagrams.

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