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AN OVERVIEW OF LONGITUDINAL CHARACTERISTICS OF ROAD TRAFFIC FLOW

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ABSTRACT

As multi-lane roadways have been widely used in many countries for years, analysis of two-dimensional vehicular interactions acquires special importance in highway design and operation. The simultaneous consideration of both lateral and longitudinal movements of traffic flow becomes vital in many aspects of traffic engineering, like modelling. The former component was reviewed elsewhere. In this paper, previous work on longitudinal characteristics of multi-lane traffic flow is scrutinised. Non-existence of such a review in recent literature was the main motive of the work, and it is hoped that it forms a reference report for other traffic analysts. In addition, the paper underlines a number of possible areas for future research.

Key Words : Level of service, Car following, Vehicular headways

TRAFİĞİN YOL BOYKESİTİ DOĞRULTUSUNDAKİ AKIM KAREKTERİSTİKLERİ ÜZERİNE BİR LİTERATUR DERLEMESİ

ÖZET

Çok şeritli yolların birçok ülkede uzun yıllardan beri kullanılmasıyla, araç etkileşimlerinin iki boyutlu incelenmesi, yolların planlanması ve işletilmesi noktalarında özel bir önem sergilemiştir. Trafiğin yol enkesiti ve boy kesiti doğrultusundaki akım karakteristiklerinin aynı anda gözönüne alınması modelleme gibi birtakım trafik mühendisliği uygulamaları açısından hayatiyet arzetmektedir. Birinci kısım, yani enine yöndeki araç hareketleri üzerine bir literatür derlemesi daha önceden yayınlanmıştı. Bu makalede çok şeritli trafik akımı boylamasına olarak incelenecektir. Henüz böyle bir literatür taramasının yapılmamış olması bu çalışmanın ilk motivasyonunu oluşturmuştur ve diğer trafik mühendisleri için referans bir yayın olacağı ümid edilmektedir. Ayrıca makalede, gelecekte yapılabilecek araştırma alanlarının da altı çizilmiştir.

Anahtar Kelimeler : Hizmet düzeyi, Öndeki aracı takip, Araç takip aralıkları

1. INTRODUCTION

Traffic flow can be modelled at two levels; macroscopic and microscopic. According to some researchers a third level, a mesoscopic level, may also be introduced. Microscopic models explain the behaviour of individual vehicles in traffic flow. The car following theory is the most common method in this category, and will be discussed under a separate heading (Section 4), due to its special importance in other paper. The mesoscopic level deals with mainly urban networks with time dependent traffic flow (Bell, 1995). A macroscopic model is designed to represent the average characteristics of traffic flow, which has a stochastic nature. Basic parameters of the macroscopic level are speed, density and flow.

Another classification for traffic flows might be the direction of vehicle interactions, i.e. longitudinal and lateral. While the former is the subject of the present paper, the latter will be briefly summarised here. This will help the reader to comprehend the whole picture better. For more detailed and critical overview of the literature on this topic, see Günay et al. (1997).

Lateral properties of multilane traffic flow constitute a number of characteristics, like lane changing, distribution of traffic over the lanes, lane discipline and lateral friction.

Although there are variety of reasons why drivers change lanes, mainly on a multilane highway during relatively congested conditions, a common tendency of drivers is to move to the fast moving lane from the slow moving one. These switches are defined by Gazis et al. (1962) as "density oscillations between lanes".

Yousif and Hunt (1995) developed a microscopic simulation program that models lane changing behaviour on British multilane highways, establishing the relationship between lane changing frequency and flow. They also compared this relationship with that obtained from other countries. They found that with increasing flow the lane changing frequency first increases, and after reaching a maximum it decreases. The general shape usually took an inverted u shape or an inverted second order parabolic curve.

Heidemann (1994) developed a model, which was tested on the data collected from Germany, to describe the distribution of vehicles to the individual lanes. He argued that if a more balanced lane utilisation were to be achieved, for larger traffic flows, the increase in capacity and the decrease in traffic congestion would be considerable.

Main factors affecting lateral distribution of traffic were summarised by Golias and Tsamboulas (1995) as: driver behaviour and attitude; total traffic flow; type of highway; existence and types of intersections; origin-destination patterns of drivers; and road markings.

2. FLOW FUNDAMENTALS

In practice, speeds of vehicles have a distribution within some range, and have a mean value. If all vehicles had equal speeds, it would be sufficient to establish a relationship between the above three parameters of traffic as

$$q = k * u \quad (1)$$

Where,

q is flow, k is density, and u is speed.

Mean speeds can be computed in two different ways, Time Mean Speed (TMS) and Space Mean Speed (SMS), and defined by McShane and Roess (1989) as follows :

Time Mean Speed is the average speed of all vehicles passing a point on a highway over some specified time period, whereas Space Mean Speed is the average speed of all vehicles occupying a given section of highway over some specified time period. In essence, TMS is the arithmetic mean of spot speeds, while SMS is the harmonic means of speeds, relating to a length of highway or lane. The relationship between these two was first developed by Wardrop as

$$TMS = SMS + (\sigma_s^2 / SMS) \quad (2)$$

Where,

σ_s^2 is the variance about the SMS.

Density is defined as the number of vehicles occupying a given length of highway or lane. The unit of density is generally vehicles per km. If an elevated vantage point, which allows the observer to capture snapshots of traffic flow over some longitudinal section, is not available, then the density of a traffic stream is calculated from the equation

$$k = q / u \quad (3)$$

Flow is the number of vehicles passing a fixed point in a unit time interval, and is generally expressed as vehicles per hour. By measuring flow over longer periods of time, like a day or a year, volumes of traffic are obtained. Measuring flow is easiest when compared with the other two parameters, speed and density.

Various mathematical models, based on these fundamentals, have been introduced to be used in various traffic flow studies. Assuming a single-regime linear speed-density relationship, the three diagrams given in

Figure 1, demonstrate the simplest form of the interaction among these fundamental variables.

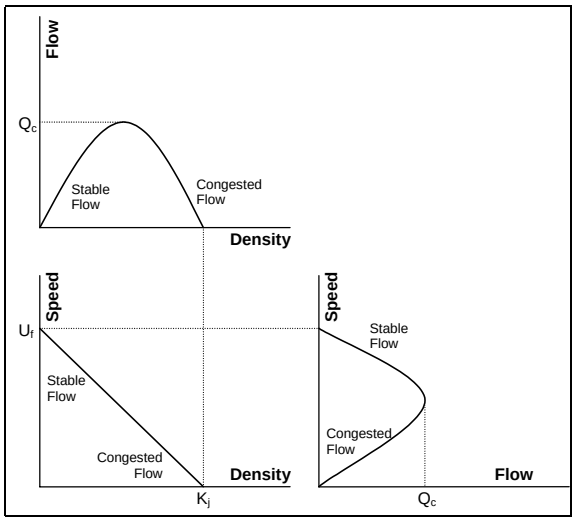

Figure 1. Relationships among speed, density, and flow (Single regime, linear u-k model in the simplest form)

This approach is called "Greenshield's Model" and is given as

$$u = U_f \left(1 - \frac{k}{K_j}\right) \quad (4)$$

Where,

u is the space mean speed of the flow at the density level of k;

U_f is the theoretical free flow speed of a single vehicle on the road; and

K_j is the jamming density.

It was mentioned above that flow of traffic (q), which is in the dimension of "vehicles per unit time", is equal to the multiplication of density by speed. Hence, the Greenshield's model may also be expressed as,

$$q = U_f \; k \; (1-k/K_j) \quad (5)$$

In any of the flow models, which is based on the various forms of the relationship between these three fundamentals, once one of the three curves is reliably established the other two can be derived. From the studies of traffic flow, some suggestions can be made as follows :

- When density is high, speed and flow are low,
- When vehicles are packed bumper to bumper, the density is jammed and speed is zero,
- Between the limits of zero and jammed density, the flow has at least one maximum.

There are other models which might be more sophisticated and realistic than the Greenshield's model, but since studying all the traffic flow models is not in the scope of this paper, they will not be discussed here.

3. LEVEL OF SERVICE

A network is a set of links and a set of nodes, where nodes connect links and links connect nodes. Each network link has an impedance magnitude, which directly affects the flow on that particular link. The higher the impedance, the lower the flow. On a transportation network low levels of service refer to high impedance. Therefore, the aim of a traffic engineer is to achieve higher level of service values on each link by reducing the resistance of the network. The term resistance can be expressed as a function of the geometry of roads, degree of queue formation, bendiness or degree of delay. Starting from the highest level of service, six categories can be defined as follows (Anon., 1985).

- **Level of Service A :** Free flow conditions with small density values, where speed is controlled by drivers' desires, speed limits or geometrical restrictions;
- **Level of Service B :** Stable flow conditions with lower speed values, but still reasonable freedom in vehicles' movements;
- **Level of Service C :** Still stable flow but vehicle movements are affected by the level of the traffic, overtaking is sometimes restricted;
- **Level of Service D :** Unstable flow with lower speeds and lower comfort, the free movements of vehicles are considerably restricted;
- **Level of Service E :** Unstable flow conditions, stoppings with short durations; and
- **Level of Service F :** Low throughput with forced flow and long stoppings and queues.

According to the same source (ibid), factors affecting capacity and level of service could be roadway, traffic, control conditions, and whether the ideal conditions are met or not, where the definition of an ideal condition would be the one for which further improvements will not achieve any increase in capacity. Ideal conditions for uninterrupted traffic flow are 3.65 metre lane width, 1.8 metre clearance to the nearest obstruction, 113 km/h design speed, and the traffic composition with all passenger cars.

4. CAR FOLLOWING

The car following theory (also known as Follow-the-Leader) is one of the deterministic approaches to traffic flow theory at the microscopic level. It describes how one vehicle follows another. Macro flow theories can also be produced from individual vehicle-driver behaviour, provided that a mathematical bridge is carefully constructed between vehicular spacings and density of traffic. The longitudinal spacing of vehicles as well as the longitudinal space occupied by individual vehicles are of particular importance as far as capacity, safety and level of service are concerned. Thus, car following models try to explain the internal as well as the overall behaviour of a traffic stream.

Research on car following theory dates back to the 1950s. Three parallel efforts undertaken in those years (i.e. Kometani and Sasaki in Japan, Forbes, and the General Motors team in the US) were the first establishers of the theory. Many other researchers studied various aspects of the theory (Steady state theory and stability by Edie (1961), Herman et al. (1959), Gazis et al. (1959) and Kometani and Sasaki (1961). Close following behaviour by Postans and Wilson (1983); Car following headways by (Wasielewski 1979; Chishaki and Tamura, 1984); Non-integer car following models by May and Keller (1967). Reaction and anticipation in the car following behaviour by (Hoffman and Mortimer 1994; Ozaki 1993; Pipes, 1966; Lee and Jones, 1967 and Colbourn et al. 1978). Composite car following models by Ceder (1976) and Tolle (1974); and Stopping distance based models by Benekohal and Treiterer (1988). and various mechanisms were introduced. For more extensive reviews of the models see (Chandler et al. 1958 and Holland 1998). Only the well known ones will be reviewed here.

Pipes (1966) characterised the suggestion of the California Motor Vehicle code as "a good rule for following another vehicle at a safe distance is to allow yourself at least the length of a car between your vehicle and the vehicle ahead for every ten miles per hour of speed at which you are travelling". It is clear that Pipes' theory gives linear increase of the minimum safe distance headway with increasing speed.

The key factor of Forbes' theory, Forbes (1963), is the reaction time needed for the following driver to perceive the need to decelerate and apply the brakes. Thus the minimum safe time headway is the sum of the reaction time and the time required for the leading vehicle to traverse a distance which is equal to its length. The relationship between the speed and the time headway is a linear one like the previous theory.

The research team of the General Motors Company, May (1990), produced five generations of their car following models which were all based on the analogy that the response of the following driver (either acceleration or deceleration) is a function of the sensitivity of this driver and the stimulus. This stimulus is generally expressed by the relative speed of these two vehicles. Their models took the general form as

$$\ddot{x}_j(t+\tau) = \ell_j \frac{\left[v_i(t)-v_j(t)\right]^k}{\left[x_i(t)-x_j(t)\right]^m} \qquad (6)$$

Where,

i and j are the leading and following vehicles, respectively;
$\ddot{x}_j(t+\tau)$ is the acceleration or deceleration of the following vehicle at the end of the reaction time;
$v_i(t)$ and $v_j(t)$ are the speeds of the leader and the follower at time t, respectively;
$x_i(t)$ and $x_j(t)$ are the longitudinal locations of the leader and the follower at time t, respectively; and
l_j is the sensitivity factor of the following driver.

Gipps (1981) stated that most of the current models are the variations of the General Motors' study, where some parameters, e.g. k and m, need to be estimated and calibrated by some considerable amount of data. Gipps' model, thus, is designed to possess the following properties :

- The model should mimic the behaviour of real traffic; and the parameters in the model should correspond to obvious characteristics of drivers and vehicles so that most can be assigned values without resorting to elaborate calibration procedures; and
- The model should be well behaved when the interval between successive recalculations of speed and position is the same as the reaction time.

His model basically sets limits on the performance of driver and vehicle in order to calculate a safe speed with respect to the preceding vehicle. First constraint is that the speed of the following vehicle, v_j, will not exceed the following driver's desired speed, V_j, and the acceleration of the following vehicle, a_j, first increases with speed as engine torque increases and then decreases to zero as the vehicle approaches the desired speed, (Eq. 7). This inequality was the result of fitting an envelope to a plot of instantaneous

speeds and accelerations obtained from an instrumented car travelling down an arterial road in moderate traffic.

$$v_j(t+\tau) \le v_j(t) + 2.5a_j\tau(1 - v_j(t)/V_j)(0.025 + v_j(t)/V_j)^{1/2} \quad (7)$$

Where,

$v_i(t+\tau)$ is the speed of vehicle *i* at time $(t+\tau)$;
a_j is the maximum acceleration rate of vehicle *j*;
τ is the reaction time;
$v_j(t)$ is the speed of vehicle *j* at time *t*; and
V_j is the desired speed of vehicle *j*.
His second limitation states that if the leading vehicle *i* commences braking as hard as desirable at time *t*, it will come to rest at point y_i *(rest)* given by

$$y_i(t) - \frac{[v_i(t)]^2}{2b_i} \quad (bi < 0) \quad (8)$$

Where,

$y_i(t)$ is the longitudinal location of vehicle *i* at time *t*;
$v_i(t)$ is the speed of vehicle *i* at time *t;* and
b_i is the maximum deceleration rate of vehicle *i*.
Similarly, vehicle *j* will not react until time $(t+\tau)$ and will not come to rest before y_j*(rest),* given by

$$y_j(t) + [v_j(t) + v_j(t+\tau)]\frac{\tau}{2} - \frac{[v_j(t+\tau)]^2}{2b_j}, \quad (9)$$

Where,

bj < 0;
$y_j(t)$ is the longitudinal location of vehicle *j* at time *t*;
$v_j(t+\tau)$ is the speed of vehicle *j* at time $(t+\tau)$; and
b_j is the maximum deceleration rate of vehicle *j*.
Thus, the driver of vehicle j must ensure that
y_i *(rest)* - $s_i > y_j$ *(rest)* Eq. 6.

Where,

s_i is the effective length of vehicle *i*; and
y(rest) is the longitudinal location of the vehicle when it stops.
However, if this were the governing inequality, the driver *j* would have no margin for error. Therefore the safety expression $v_j(t+\tau)\theta$, where θ is assumed to be $\tau/2$, needs to be added to Eq. 9.
He then indicated that in real traffic, it is possible for driver *j* to estimate all values in Eq. 8, except b_i, by direct observation. Thus, b_i should be replaced by Some estimate $\hat{b}$. If the willingness of the previous driver to brake hard has not been underestimated, a vehicle travelling at a safe speed and distance will be able to maintain a state of safety indefinitely. Hence, by substituting θ by $\tau/2$, he obtained Eq. 10. The speed of vehicle *j* at time $(t+\tau)$, therefore, will be the minimum of Eq. 7 and Eq. 10.

$$v_j(t+\tau) \le v_j\tau + \sqrt{(b_j^2\tau^2 - b_j(2(x_i(t) - s_i - x_j(t)) - v_j(t)\tau - [v_i(t)]^2/\hat{b}))} \quad (10)$$

The car following theory has also attracted other researchers who were interested in treating the follower's response as a result of the physiological functions of his/her eyes when detecting the changes produced by the leading vehicle. Their approach was as follows; the speed difference between the leading and the following vehicles cause the continuous change in the spacing between them. This variation is detected by the eyes of the following driver, based on the visual angle subtended by the leading vehicle. The hypothesis then states that the rate of change of this angle is the main stimulus to the driver of the following vehicle. However research revealed that this stimulus could not be detected all the time. The following driver's eyes are only able to detect the rate at which the visual angle changes, if this rate is above a threshold value. This threshold is given by Lee and Jones (1967) as 6 x 10^{-4} radians/second. More recent work by Hoffman and Mortimer (1994) has specified this value as 0.003 to 0.004 radians/second, which was more consistent with the experimental data.

The way the drivers behave in the car following theories is various. Ratio of experienced drivers to inexperienced drivers in the same population, their ages, and sexes are also the factors affecting the results. For example: the British accident data revealed that 18-28 year old male drivers are involved in almost twice as many car following accidents as 35-54 year-old male drivers (Colbourn et al., 1978). Therefore, traffic flow models where the car following theory is facilitated, the average age of the driver population may be included.

Another important point that should be taken into account in car following studies is the range in which the theory is applicable. It was stated that car following theories assume a line of vehicles, in which all vehicles follow each other without being able to pass. In real traffic, however, not every vehicle is a follower. The range in which car following theories are applicable, as the name suggests, depends on the flow situation. There are a number studies on when car following starts and when vehicles move independently of each other. The vehicle population in traffic flow can be divided into two groups as followers and free movers. Consequently the headways may also be categorised in these two groups.

The most common and vital assumption made by all the car following studies was that drivers follow each other in a platoon of vehicles which move along one lane without being able to change lanes. Although car following theory was found to be rather impractical in many traffic situations, there have been some successful practical applications (See Traffic Engineering and Practice by E. Davies (1968) pp. 79-81 for details.)

- Flow control experiments in the Holland and Lincoln Tunnels in New York, Detroit and Chicago motorways,
- Traffic funnelling attempts in Düsseldorf,
- Area traffic control through signals in Glasgow and West London.

5. VEHICULAR HEADWAYS

Headways in traffic engineering are categorised in two groups: time headways and distance headways. Time headway is the time between the front of successive vehicles passing a given point. The average time headway is thus the reciprocal of the average flow. Distance headway is the distance between the front of successive vehicles at a given instance of time. The average distance headway is thus the reciprocal of the average density. Both of these descriptions are based on a line of traffic. The headways between vehicles is one of the factors affecting the flow characteristics such as level of service, safety and capacity. In particular :

- The percentage of time in which movements of two consecutive vehicles are expressed by the car following theory is one indication of level of service.
- Time headway distributions may be employed when studying passing, merging, weaving and lane changing.
- Determination of the capacity of a roadway needs the minimum time headway and time headway distribution information.
- Branston (1977) stated that cars and commercial vehicles attained higher speeds when the leading vehicle was a car than a commercial vehicle.
- The length of the hood of the following vehicle influences the following distance. Evans and Rothery (1976) found that the longer the hood, the greater the spacing between the two consecutive vehicles.
- Bunker and Troutbeck (1995) claimed that the shape of headway distribution tends to be different for shoulder, median and middle lanes. He also concluded that drivers in the median lane are more likely to be dissatisfied with their speeds, travelling at closer headways.
- It seems clear that a large population of accidents on highways are rear end collisions. According to the report by Postans and Wilson (1983), 12 % of drivers on the M4 motorway had following gaps with less than 1 second; 29 % of all vehicles travelling at 40 m/h or more had following gaps less than 2 seconds; 15% of drivers had following gaps less than 1 second; and 19% of lorries on M1 in the act of overtaking were tailgating. Tailgating was defined as close following with gaps of 1 second or less. This is in fact half of DoT's recommendation (Anon., 1975), which is two seconds. The question is what are the boundaries between these three categories of driving. In other words when the free moving state ends and car following interactions start, and when car following becomes unsafe, since the traffic flow needs to be treated differently in these states, are the issues to be examined. Further work on this issue might be worth undertaking.
- Various studies proved that as the traffic flow rate increased, the shape of the time headway distribution varied considerably. This relationship is caused by the vehicular interactions which vary with changing flow rates. Low flow conditions result in less interaction and hence more random time headway distributions. In other words, all vehicles may be thought of as independent of one another. On the other hand, near capacity almost all vehicles are in a car following process and give rise to almost constant headways.
- Based on the above two extreme cases, two categories of the time headway state, quite similar to the May (1990) classification, can be introduced namely, the "random headway state"

and the "constant headway state". The negative exponential distribution for the random headway state, and the normal distribution for the constant headway state may then be suggested.

- Based on an extensive data set of some 14570 individually measured time headways, May (1990) found that individual time headways are rarely less than 0.5 second and individual time headways are rarely over 10 seconds.

6. CONCLUDING REMARKS

The paper has reviewed the most relevant literature on the longitudinal characteristics of traffic flow. The importance of the study becomes vital when looked at multilane traffic flows, where the lateral characteristics, too, are taken into account simultaneously. Secondly, most of the previous work considered steady state situations. Investigation of forced flow or jammed circumstances is still a weak point. In the light of this review, the main features of multi-lane traffic flow can be summarised as follows. In the car following theory, it is essentially assumed that each vehicle is influenced directly by the one in front, as often happens in real traffic flow where lane discipline is good. The most well known car following models given in Section 4 can be categorised in two groups:

a) Stimulus based models (Pipes, 1966, and General Motors, reviewed by May, 1990),
b) Stopping-distance based models (Forbes, 1963 and Gipps, 1981).

Existing car following theories, due to their typical assumptions, may not be applicable in many situations. Best examples to be represented easily by the car following models are tunnel traffic or a funeral convoy. The situation is even more complex in untidy traffic and hence further research may be worth undertaking especially for developing countries.

Besides, according to Ovuworie et al. (1980) a vehicle is said to be a free mover if its speed is not restricted by other vehicles. A follower is the second or subsequent member of a platoon. But, the author of the paper believes that a third population may also be considered as a separate group in traffic flow models. That is the drivers who follow too closely so that in the case of a sudden deceleration of the leading vehicle, the gap between the two vehicles would not be enough to avoid a rear end shunt. Investigation of this was left for future work.

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