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AN EXACT ELASTO-PLASTIC SOLUTION OF METAL-MATRIX COMPOSITE CANTILEVER BEAM LOADED BY A SINGLE FORCE AT ITS FREE END

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ABSTRACT

In the present study, an elastic-plastic stress analysis is carried out in a metal matrix composite cantilever beam loaded by a single force at its free end. A composite consisting of stainless-steel reinforced aluminium was produced for this work. The orientation angle of the fibers is chosen as 0°, 30°, 45°, 60° and 90°. The material is assumed to be perfectly plastic in the elasto-plastic solution. An analytical solution is performed for satisfying both the governing differential equation in the plane stress case and boundary conditions for small plastic deformations. The solution is carried out under the assumption of the Bernoulli-Navier hypotheses. The composite material is assumed as hardening linearly. The Tsai-Hill theory is used as a yield criterion.

Key Words : Exact solution, Elasto-plastic analysis, Metal-matrix composite, Perfectly plastic

SERBEST UCUNDAN TEKİL YÜKE MARUZ ANKASTRE METAL MATRİSLİ KOMPOZİT KİRİŞİN ELASTO-PLASTİK ÇÖZÜMÜ

ÖZET

Bu çalışmada, serbest ucundan tekil yüke maruz ankastre metal matrisli kompozit kirişin elasto-plastik gerilme analizi yapıldı. Paslanmaz çelik takviyeli alüminyum kompozit malzemesi kullanıldı. Fiberin oryantasyon açısı 0°, 30°, 45°, 60° ve 90° olarak seçildi. Malzeme elasto-plastik çözüm için tam plastik olarak kabul edildi. Analitik çözüm, hem düzlem gerilme durumundaki diferansiyel denklemi hem de küçük plastik deformasyonlar için sınır şartlarını sağlayacak şekilde uygulandı. Çözümde Bernoulli-Navier Teorisi kabulu yapıldı. Kompozit malzeme lineer sertleştirilmiş olarak kabul edildi. Akma teorisi olarak Tsai-Hill teorisi kullanıldı.

Anahtar Kelimeler : Tam çözüm, Elasto-plastik analiz, Metal matris kompozit, Tam plastik

1. INTRODUCTION

Metal matrix composites reinforced by fibers give high strengths, specific stiffness, ductility, yield point and good temperature performance. The technology of metal matrix composite material is being developed very rapidly. Aluminium matrix composites, which are particularly cited for their superior performance-to-weight advantage, have many applications in the aerospace and other industries Canumalla et all., (1995) have investigated discontinuously are viewed as candidate materials for elevated temperature applications because of their attractive high temperature strength properties and wear resistance. Jeronimidis and Parkyn (1998) investigated residual stresses in carbon fiber-thermoplastic matrix laminates. Karakuzu and Özcan (1996) carried out an elasto-plastic stress analysis in an aluminium matrix composite cantilever loaded by single and uniformly distributed forces by using an exact analytical solution. Sayman (1998) has investigated elasto-plastic stress analysis of aluminium metal matrix composite laminated plates under in-plane loading. Ananth and Chandra (1995) studied the application of push out test to characterize the mechanical behaviour of interfaces in metallic and intermetallic matrix composites by using finite element method. Elasto-plastic stress analysis by using finite element method the metal matrix composites were investigated (Karakuzu and Sayman, 1994; Karakuzu et all., 1997). Kang and Ku (1995) investigated the infiltration limits in the fabrication of Al_2O_3 short fiber reinforced composites for various processing conditions. Arnould et all., (1990) have studied elastic-plastic analysis of advanced composites. They have investigated the use of the compliant-layer concept in reducing residual stresses resulting from processing. Cöcen et all., (1997) produced SiC aluminium metal matrix composites to strengthen the aluminium matrix. Residual stresses in the composite materials are important because they can lead to premature failure. The vanishing fiber-diameter model, together with the thermoviscoplasticity theory based on overstress and including a recovery of state formulation, was introduced by Yeh and Krempl (1993). Akay and Özden (1994) measured the thermal residual stresses in injection moulded thermoplastics by removing thin layers from specimens. Akay and Özden (1995; 1996) investigated the influence of residual stresses on the mechanical and thermal properties of injection moulded thermoplastics.

In the present study, an elastic-plastic stress analysis is carried out in a metal matrix composite cantilever beam loaded by a single force at its free end. During the solution of the problem, the beam is assumed as linearly hardening. Bernoulli-Navier hypotheses are used in the investigation. Sample problems are given for 0°, 30°, 45°, 60° and 90°orientation angles. The Tsai-Hill theory is used as a yield criterion.

2. ELASTIC SOLUTION

A composite cantilever beam loaded by a single force at its free end, as shown in Figure1.

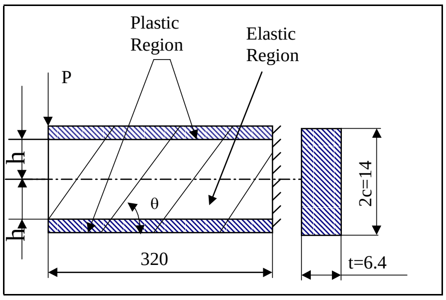

Figure 1. Composite cantilever beam

The governing differential equation for the plane stress case is given as (Lekhnitskii, 1981).

$$\bar{a}_{22}\frac{\partial^4 F}{\partial x^4} - 2\bar{a}_{26}\frac{\partial^4 F}{\partial x^3 \partial y} + \left(2\bar{a}_{12} + \bar{a}_{66}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} - 2\bar{a}_{16}\frac{\partial^4 F}{\partial x \partial y^3} + \bar{a}_{11}\frac{\partial^4 F}{\partial y^4} = 0 \quad (1)$$

Where F is a stress function and a_{ij} are the components of the compliance matrix (Jones, 1975).

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16} \\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26} \\ \bar{a}_{16} & \bar{a}_{26} & \bar{a}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (2)$$

Where ;

$$\begin{aligned}
\bar{a}_{11} &= a_{11}\cos^4\theta + \left(2a_{12} + a_{66}\right)\sin^2\theta\cos^2\theta + a_{22}\sin^4\theta \\
\bar{a}_{12} &= \left(a_{11} + a_{22} - a_{66}\right)\sin^2\theta\cos^2\theta + a_{12}\left(\sin^4\theta + \cos^4\theta\right) \\
\bar{a}_{22} &= a_{11}\sin^4\theta + \left(2a_{12} + a_{66}\right)\sin^2\theta\cos^2\theta + a_{22}\cos^4\theta \\
\bar{a}_{16} &= \left(2a_{11} - 2a_{12} - a_{66}\right)\sin\theta\cos^3\theta \\
&\quad - \left(2a_{22} - 2a_{12} - a_{66}\right)\sin^3\theta\cos\theta \\
\bar{a}_{26} &= \left(2a_{11} - 2a_{12} - a_{66}\right)\sin^3\theta\cos\theta \\
&\quad - \left(2a_{22} - 2a_{12} - a_{66}\right)\cos^3\theta\sin\theta \\
\bar{a}_{66} &= 2\left(2a_{11} + 2a_{22} - 4a_{12} - a_{66}\right)\sin^2\theta\cos^2\theta \\
&\quad + a_{66}\left(\sin^4\theta + \cos^4\theta\right)
\end{aligned} \quad (3)$$

and

$$a_{11} = \frac{1}{E_1}, a_{12} = -\frac{\upsilon_{12}}{E_1}, a_{22} = \frac{1}{E_2}, a_{66} = \frac{1}{G_{12}} \quad (4)$$

The stress function F is chosen as a polynomial to satisfy both the governing differential equation and the boundary conditions;

$$F = \frac{d_4}{6}xy^3 + \frac{e_4}{12}y^4 + \frac{a_2}{2}y^2 + b_2 xy \quad (5)$$

Putting in Equation (1) gives;

$$-2a_{16}d_4 + 2a_{11}e_4 = 0$$

$$e_4 = m\,d_4, \quad m = \frac{a_{16}}{a_{11}} \quad (6)$$

Boundary conditions for this beam are given as ;

$$\tau_{xy} = 0 \ \text{at} \ y = \mp c \tag{7}$$

$$\sigma_y = 0 \ \text{at} \ y = \mp c \tag{8}$$

$$\int_{-c}^{c} \tau_{xy} \, t \, dy = -P \ \text{at} \ x = 0 \tag{9}$$

Where t is thickness of the beam. At the free end, the resultant of σ_x and bending moment are equal to zero;

$$\int_{-c}^{c} \sigma_x \, t \, dy = 0 \tag{10}$$

$$\int_{-c}^{c} \sigma_x \, t \, y \, dy = 0 \tag{11}$$

Stress components are obtained as ;

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = d_4 xy + e_4 y^2 + a_2 = d_4 xy + m d_4 y^2 + a_2 \tag{12}$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = 0 \tag{13}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \, \partial y} = -\frac{d_4}{2} y^2 - b \tag{14}$$

The constants are determined from the boundary conditions, and the stress components become;

$$\sigma_x = -\frac{P}{I}\left(xy + my^2 - \frac{m}{3} c^2 \right) \tag{15}$$

$$\sigma_y = 0 \tag{16}$$

$$\tau_{xy} = -\frac{P}{2I}\left(c^2 - y^2 \right) \tag{17}$$

where I is the inertia moment of the cross section of the beam, and it is given as, $I = \frac{2}{3} tc^3$. Thus, both the governing differential equation and all the boundary conditions are satisfied.

2. 1. Displacement Components

By using the stress-strain relation, the strain components can be written as follows;

$$\varepsilon_x = \bar{a}_{11}\sigma_x = \bar{a}_{11}\left(d_4 xy + m d_4 y^2 - \frac{mc^2}{3} d_4 \right) \tag{18}$$

$$\varepsilon_y = \bar{a}_{12}\sigma_x = \bar{a}_{12}\left(d_4 xy + m d_4 y^2 - \frac{mc^2}{3} d_4 \right) \tag{19}$$

$$\gamma_{xy} = \bar{a}_{16}\sigma_x = \frac{1}{2}\bar{a}_{16}\left(d_4 xy + m d_4 y^2 - \frac{mc^2}{3} d_4 \right) \tag{20}$$

The integration of the above two equations gives the displacement components as ;

$$u = \bar{a}_{11}\left(d_4 \frac{x^2 y}{2} + m d_4 x y^2 - \frac{mc^2}{3} x d_4 \right) + c_1(y) \tag{21}$$

$$v = \bar{a}_{12}\left(d_4 \frac{x y^2}{2} + m d_4 \frac{y^3}{3} - \frac{mc^2}{3} y d_4 \right) + c_2(x) \tag{22}$$

Substituting them in ε_{xy} gives following relations;

$$\bar{a}_{12} \frac{d_4}{4} y^2 + \frac{1}{2}\frac{\partial c_1(y)}{\partial y} - m d_4 y^2 \bar{a}_{16} = K_1 \tag{23}$$

$$\bar{a}_{11} \frac{d_4}{4} x^2 + \frac{1}{2}\frac{\partial c_2(x)}{\partial x} = G_1 \tag{24}$$

$$K_1 + G_1 = \frac{mc^2}{3}\bar{a}_{16} d_4 \Rightarrow G_1 = -K_1 + \frac{mc^2}{3}\bar{a}_{16} d_4 \tag{25}$$

From the solutions of these ordinary differential equations, the displacement components are found as ;

$$\begin{aligned} u &= \bar{a}_{11}\left(d_4 \frac{x^2 y}{2} + m d_4 x y^2 - \frac{mc^2}{3} x d_4 \right) \\ &\quad + 2K_1 y - \bar{a}_{12}\frac{y^3}{6} d_4 + \frac{2}{3} m d_4 y^3 \bar{a}_{16} + c_4 \\ v &= \bar{a}_{12}\left(d_4 \frac{x y^2}{2} + m d_4 \frac{y^3}{3} - \frac{mc^2}{3} y d_4 \right) \\ &\quad + 2G_1 x - \frac{1}{6}\bar{a}_{11} d_4 x^3 + c_3 \end{aligned} \tag{26}$$

c_3, c_4 and K_1 are determined by using the boundary conditions at the fixed end, as ;

$$u = v = \frac{\partial v}{\partial x} = 0 \ \text{at} \ x = L, \, y = 0 \tag{27}$$

Putting them at Eqn. (26) gives the displacement components in the elastic region as follows ;

$$u = \bar{a}_{11}\left(d_4 \frac{x^2 y}{2} + md_4 xy^2 - \frac{mc^2}{3} xd_4 \right) + \frac{2}{3} mc^2 d_4 y - \frac{1}{2}\bar{a}_{11} L^2 yd_4 - \bar{a}_{12}\frac{y^3}{6} d_4 + \frac{2}{3} md_4 y^3 \bar{a}_{16} + \frac{1}{3}\bar{a}_{11} mc^2 Ld_4 \quad (28)$$

$$v = \bar{a}_{12}\left(d_4 \frac{xy^2}{2} + md_4 \frac{y^3}{3} - \frac{mc^2}{3} yd_4 \right) + \frac{1}{2}\bar{a}_{11} L^2 xd_4 - \frac{1}{6}\bar{a}_{11} d_4 x^3 + \frac{1}{3}\bar{a}_{11} d_4 L^3 \quad (29)$$

3. ELASTIC-PLASTIC SOLUTION

In this solution Bernoulli-Navier assumptions are used. By using these assumptions, the unit strain for the elastic and elastic-plastic cases can be written as;

$$\varepsilon_x = \frac{y}{\rho} \quad (30)$$

Where ρ is the radius of the curvature. When the Tsai-Hill theory is used as a yield criterion, the equivalent stress is given as (Jones, 1975).

$$\bar{\sigma} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \frac{\sigma_2^2 X^2}{Y^2} + \frac{\tau_{12}^2 X^2}{S^2}} = X \quad (31)$$

Where X and Y are the yield points in the material direction 1st, 2nd respectively, S is the yield point in the 1-2 plane for the simple pure shear. The yield point, Z, in the 3rd principal material direction is equal to Y, due to the same fiber alignment in these directions. In the plastic region, the equations of equilibrium are written as,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0\,, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \quad (32)$$

from the first equation σ_x is found as C(y). As a result of this, at any section σ_x is only the function of y. For a linear strain-hardening material the yield stress is given by the Ludwik equation as ;

$$\sigma_y = \sigma_o + K\varepsilon_p \quad (33)$$

Where σ_o is equal to X which is the yield point in the first principal material directions are ;

$$\sigma_1 = \sigma_x \cos^2\theta$$
$$\sigma_2 = \sigma_x \sin^2\theta \quad (34)$$
$$\tau_{12} = -\sigma_x \cos\theta \sin\theta$$

Substituting them in Eqn. (31) gives the yield condition for the fiber direction oriented as θ ;

$$X_1 = \frac{\sigma_o}{N} = \frac{X}{N} \quad (35)$$

Where ;

$$N = \sqrt{\cos^4\theta - \sin^2\theta\cos^2\theta + \frac{X^2 \sin^4\theta}{Y^2} + \frac{X^2 \sin^2\theta\cos^2\theta}{S^2}} \quad (36)$$

The plastic strain increments in the principal material directions are found by using the potential function f (Owen and Hinton, 1980).

$$\begin{Bmatrix} d\varepsilon_1^p \\ d\varepsilon_2^p \\ d\varepsilon_{12}^p \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_1} d\lambda \\ \dfrac{\partial f}{\partial \sigma_2} d\lambda \\ \dfrac{\partial f}{\partial \tau_{12}} d\lambda \end{Bmatrix} \quad (37)$$

The total strain increments in the principal material directions are written as ;

$$d\varepsilon_1 = \bar{a}_{11} d\sigma_1 + \bar{a}_{12} d\sigma_2 + \frac{2\sigma_1 - \sigma_2}{2\sigma_y} d\lambda$$
$$d\varepsilon_2 = \bar{a}_{12} d\sigma_1 + \bar{a}_{22} d\sigma_2 + \frac{-\sigma_1 + 2\sigma_2 \dfrac{X^2}{Y^2}}{2\sigma_y} d\lambda \quad (38)$$
$$d\varepsilon_{12} = \frac{\bar{a}_{66} d\tau_{12}}{2} + \frac{2\tau_{12}\dfrac{X^2}{S^2}}{2\sigma_y} d\lambda$$

For the orientation angle θ, the stress component σ_x can be written as ;

$$\sigma_x = \frac{\sigma_y}{N}, \quad d\lambda = d\varepsilon_p$$

Putting σ_1, σ_2 and τ_{12} into Eqn (38) and integrating them gives ;

$$\varepsilon_1 = \bar{a}_{11}\sigma_1 + \bar{a}_{12}\sigma_2 + \frac{1}{2N}\left(2\cos^2\theta - \sin^2\theta\right)\varepsilon_p + c_5 \tag{39}$$

$$\varepsilon_2 = \bar{a}_{12}\sigma_1 + \bar{a}_{22}\sigma_2 + \frac{1}{2N}\left(-\cos^2\theta + 2\sin^2\theta\frac{X^2}{Y^2}\right)\varepsilon_p + c_6 \tag{40}$$

$$\varepsilon_{12} = \bar{a}_{66}\frac{\tau_{12}}{2} + \frac{-2\sigma_x\cos\theta\sin\theta\frac{X^2}{S^2}}{2\sigma_y}\varepsilon_p + c_7 \tag{41}$$

Plastic and elastic strain components are equal on the boundary of the elastic and plastic regions. Writing $\varepsilon_p = 0$ and equating the elastic and the plastic strains gives the integration constants;

$$c_5 = X_1\begin{bmatrix}\left(\bar{a}_{11} - a_{11}\right)\cos^2\theta + \left(\bar{a}_{12} - a_{12}\right)\sin^2\theta \\ +\bar{a}_{16}\sin\theta\cos\theta\end{bmatrix} \tag{42}$$

$$c_6 = X_1\begin{bmatrix}\left(\bar{a}_{11} - a_{22}\right)\sin^2\theta + \left(\bar{a}_{12} - a_{12}\right)\cos^2\theta \\ -\bar{a}_{16}\sin\theta\cos\theta\end{bmatrix} \tag{43}$$

$$c_7 = X_1\begin{bmatrix}\left(\bar{a}_{12} - \bar{a}_{11}\right)\sin\theta\cos\theta + \frac{\bar{a}_{16}}{2}\cos^2\theta \\ -\frac{\bar{a}_{66}}{2}\sin\theta\cos\theta\end{bmatrix} \tag{44}$$

The transformation matrix from the principal material axes to x-y axes is written as ;

$$\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy}\end{Bmatrix} = \begin{bmatrix}\cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta\sin^2\theta\end{bmatrix}\begin{Bmatrix}\varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12}\end{Bmatrix} \tag{45}$$

By using this matrix, the strain components in the plastic region are found as ;

$$\varepsilon_x = \bar{a}_{11}\sigma_x + B_1\varepsilon_p \text{ , } \varepsilon_y = \bar{a}_{12}\sigma_x + B_2\varepsilon_p \text{ ,} \quad \varepsilon_{xy} = \frac{\bar{a}_{16}}{2}\sigma_x + B_3\varepsilon_p \tag{46}$$

Where ;

$$B_1 = \frac{\begin{pmatrix}2\cos^4\theta - 2\cos^2\theta\sin^2\theta + 2\sin^4\theta\frac{X^2}{Y^2} \\ +4\sin^2\theta\cos^2\theta\frac{X^2}{S^2}\end{pmatrix}}{2N} \tag{47}$$

$$B_2 = \frac{\begin{pmatrix}2\cos^2\theta\sin^2\theta - \sin^4\theta - \cos^4\theta \\ +2\sin^2\theta\cos^2\theta\frac{X^2}{Y^2} - 4\sin^2\theta\cos^2\theta\frac{X^2}{S^2}\end{pmatrix}}{2N} \tag{48}$$

$$B_3 = \frac{\begin{pmatrix}2\cos^3\theta\sin\theta - \sin^3\theta\cos\theta - 2\sin^3\theta\cos\theta\frac{X^2}{Y^2} \\ +\left(-2\sin\theta\cos^3\theta + 2\sin^3\theta\cos\theta\right)\frac{X^2}{S^2}\end{pmatrix}}{2N} \tag{49}$$

The stress component σ_x varies linearly in the elastic region, it can be written as ;

$$\sigma_x = \varepsilon_x E = \frac{\varepsilon_x}{\bar{a}_{11}} = \frac{y}{\rho\bar{a}_{11}} \tag{50}$$

If the plastic region expands from the lower and upper surfaces up to h_1 and h_2, at the yield point ;

$$\sigma_x = X_1 = \frac{h}{\rho\bar{a}_{11}} \tag{51}$$

Hence from the above relation, it is found that $h_1 = h_2 = h$. The curvature of the radius is determined as ;

$$\rho = \frac{h}{X_1\bar{a}_{11}} \tag{52}$$

The total strain in the plastic region is equal to the summation of the elastic and plastic strain as ;

$$\frac{y}{\rho} = \bar{a}_{11}\frac{\sigma_y}{N} + B_1\varepsilon_p \tag{53}$$

Writing σ_y in the relation, ε_p is determined as a linear function of y ;

$$\varepsilon_p = a + by \Rightarrow a = \frac{-\sigma_o\bar{a}_{11}}{K\bar{a}_{11} + B_1N}, b = \frac{N}{\rho\left(K\bar{a}_{11} + B_1N\right)} \tag{54}$$

For small deformations, the relation between the strain and displacement components can be written as ;

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{y}{\rho} \tag{55}$$

the integration gives;

$$u = \frac{y}{\rho}x + c_8(y) \tag{56}$$

ε_y is related to v as ;

$$\varepsilon_y = \bar{a}_{12}\frac{\sigma_y}{N} + B_2\varepsilon_p \qquad (57)$$

Putting ε_p in the equation, and integrating v gives ;

$$v = \frac{\bar{a}_{12}\,\sigma_o}{N}y + \left(\frac{K\,\bar{a}_{12}}{N} + B_2\right)\left(ay + \frac{by^2}{2}\right) + c_9(x) \qquad (58)$$

$$c_9(x) = K_1 x - \frac{x^2}{2\rho} + c_{10} \qquad (59)$$

$$c_8(y) = G_1 y + \left(\frac{\bar{a}_{16}K}{N} + 2B_3\right)\frac{by^2}{2} + c_{11} \qquad (60)$$

Where ;

$$K_1 + G_1 = \frac{\bar{a}_{16}}{N}\sigma_o\left(K\frac{\bar{a}_{16}}{N} + 2B_3\right)a \qquad (61)$$

the integration of these equations gives u and v as ;

$$\begin{aligned} v = {} & \frac{\bar{a}_{12}\sigma_o}{N}y + \left[K\frac{\bar{a}_{12}}{N} + B_2\right]ay \\ & + \left[K\frac{\bar{a}_{12}}{N} + B_2\right]\frac{by^2}{2} - \frac{x^2}{2\rho} + K_1 x + c_{10} \end{aligned} \qquad (62)$$

$$\begin{aligned} u = {} & \frac{y}{\rho}x + \left[K\frac{\bar{a}_{16}}{N} + 2B_3\right]\frac{by^2}{2} + \\ & \left[\frac{\bar{a}_{16}}{N}\sigma_o + K\frac{\bar{a}_{16}}{N}a + 2B_3 a - K_1\right]y + c_{11} \end{aligned} \qquad (63)$$

The elastic and plastic displacement components have to be equal on the boundary of the elastic and plastic regions. By using this condition, the integration constants are determined and u and v are written as ;

$$\begin{aligned} u = {} & \frac{y}{\rho}x + \left(K\frac{\bar{a}_{16}}{N} + 2B_3\right)\frac{by^2}{2} - \left(K\frac{\bar{a}_{16}}{N} + 2B_3\right)\frac{bh^2}{2} \\ & + \left(\frac{\bar{a}_{16}}{N}\sigma_o + K\frac{\bar{a}_{16}}{N}a + 2B_3 a - K_1\right)y \\ & \frac{hL}{\rho} + -\left(\frac{\bar{a}_{16}}{N}\sigma_o + K\frac{\bar{a}_{16}}{N}a + 2B_3 a - K_1\right)h \end{aligned} \qquad (64)$$

$$\begin{aligned} v = {} & \frac{\bar{a}_{12}\sigma_o}{N}y + \left[K\frac{\bar{a}_{12}}{N} + B_2\right]ay + \left[K\frac{\bar{a}_{12}}{N} + B_2\right]\frac{by^2}{2} \\ & -\frac{x^2}{2\rho} + K_1 x + \frac{L^2}{2\rho} - K_1 L - \frac{\bar{a}_{12}\sigma_o}{N}h \\ & -\left[K\frac{\bar{a}_{12}}{N} + B_2\right]ah - \left[K\frac{\bar{a}_{12}}{N} + B_2\right]\frac{bh^2}{2} \end{aligned} \qquad (65)$$

3. 1. Determination of h

The moment of σ_x at any section to be equal to the bending moment (-Px). The moment of σ_x is obtained as ;

$$-Px = 2\left[\frac{X_1\,h^2\,t}{3} + \int_h^c \frac{\sigma_o + K(a+by)}{N}ty\,dy\right] \qquad (66)$$

The integration gives a third order algebraic equation as;

$$h^3 + Bh + D = 0 \qquad (67)$$

Where ;

$$B = \frac{\dfrac{\sigma_o c^2}{2N} + \dfrac{Kac^2}{2N} - \dfrac{Px}{2t}}{\dfrac{X_1}{3} - \dfrac{\sigma_o}{2N} - \dfrac{Ka}{2N} - \dfrac{Kd_1}{3N}},$$

$$D = \frac{\dfrac{Kd_1c^3}{3N}}{\dfrac{X_1}{3} - \dfrac{\sigma_o}{2N} - \dfrac{Ka}{2N} - \dfrac{Kd_1}{3N}}, \quad d_1 = \frac{X_1\,N\,\bar{a}_{11}}{K\,\bar{a}_{11} + B_1 N}$$

The root of the equation is found as ;

$$h = \sqrt[3]{\frac{-D + \sqrt{D^2 + \dfrac{4B^3}{27}}}{2}} + \sqrt[3]{\frac{-D - \sqrt{D^2 + \dfrac{4B^3}{27}}}{2}} \qquad (68)$$

4. PRODUCTION OF LAMINATED PLATES

The composite layer consists of stainless steel fiber and aluminum matrix. The production has been realized by using moulds which consist of upper and lower parts. Electrical resistance has been used to heat the moulds and material which are insulated, as illustrated in Figure 2.

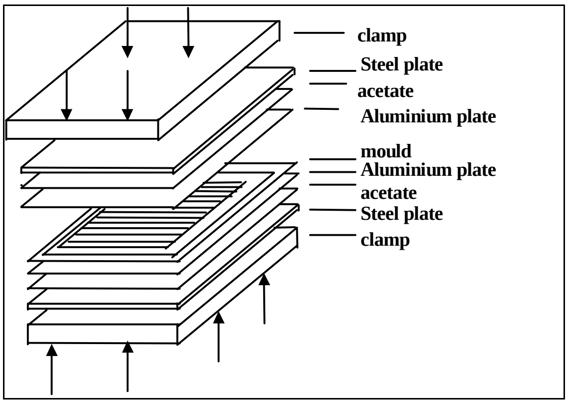

Figure 2. Press operation

The hydraulic press has been used to obtain a pressure of 30 MPa to the upper mould. Manufacturing set has been heated to 600 °C. In this conditions, the yield strength of aluminum is exceeded and good bonding between matrix and fiber has been realized. The mechanical properties, yield points and plastic parameters are given in Table 1. It is assumed that the yield point Z (in the z direction) is equal to the yield point Y (in the y direction), the yield points of τ_{xz}, τ_{yz} are equal to S.

Table 1. Mechanical Properties and Yield Points of The Composite Beam

E_1	86 GPa	X	230 MPa
E_2	74 GPa	Y	24 MPa
G_{12}	32 GPa	S	48.9 MPa
υ_{12}	0.3	K	1254 MPa

5. RESULTS AND DISCUSSION

Elastic-plastic stress analysis is carried out analytically in the cantilever beam. The yield points from the free end are given at Table 2.

The intensity of the residual stress component of σ_x are given in Table 3, for 0°, 30°, 45°, 60° and 90°. as seen from this Table the intensity of the residual stress component of σ_x maximum at the upper and lower surface of the beam.

It is greatest for the orientation angle of 0° as –78, 16 MPa at the upper surface for h = 4 mm, the equivalent plastic strain is the greatest for the orientation angle of 90° as 2, 356.10^{-5} for h = 4 mm.

Table 2. The Distance Between The Free End and Yield Points

Yield point at the upper and lower	0°	30°	45°	60°	90°
surfaces (mm)	225.40	70.00	42.00	30.00	23.52

Table 3. Elastic, Elastic-Plastic and Residual Stress Components and The Plastic Strain At Upper Surface (X = 0, Y = -C)

Orientation Angles θ	x (mm)	h (mm)	ε_p	σ_{x_e}	σ_{x_p}	σ_{x_r}
0°	255.21 281.22 302.00	6.00 5.00 4.00	0.00040 0.00100 0.00200	260.41 286.95 308.16	230.00 230.00 230.00	-30.41 -56.95 -78.16
30°	79.77 87.71 94.22	6.00 5.00 4.00	0.00003 0.00007 0.00014	81.73 89.83 96.47	71.80 71.80 71.80	-9.93 -18.03 -24.67
45°	47.83 52.58 56.46	6.00 5.00 4.00	0.00001 0.00003 0.00006	48.46 53.31 57.27	43.09 43.09 43.09	-5.37 -10.22 -14.18
60°	34.21 37.60 41.36	6.00 5.00 4.00	0.00000 0.00001 0.00003	35.09 38.55 42.38	30.82 30.82 30.82	-4.26 -7.72 -11.56
90°	26.64 29.28 31.48	6.00 5.00 4.00	0.00000 0.00001 0.00002	27.18 29.88 32.12	24.00 24.00 24.00	-3.18 -5.88 -8.12

The displacement components u and v for orientation angles of 0°, 30°, 45°, 60° and 90° in the elastic and the plastic regions at the free end are given in Table 4. It is seen from this Table that the vertical displacement v is greater than the horizontal displacement v. v is the greatest for the orientation angle of 0° as –34.627 mm for h = 3 mm. The vertical displacement components v in the plastic region is greater than v in the elastic region at the free end.

Table 4. Displacement at The Free End at Points Middle and Upper

			Elastic Displacements at Middle Point (x = 0, y = 0)		Plastic Displacements at Upper Point (x = 0, y = -c)	
Orientation Angles θ	x (mm)	h (mm)	u (mm)	v (mm)	u (mm)	v (mm)
0°	255.21	6.00	0.00	-18.72	1.010	-23.10
	281.22	5.00	0.00	-18.72	1.010	-27.72
	302.00	4.00	0.00	-18.72	1.010	-34.62
30°	79.77	6.00	0.00	-19.71	0.308	-7.57
	87.71	5.00	0.00	-19.71	0.374	-9.09
	94.22	4.00	0.00	-19.71	0.473	-11.36
45°	47.83	6.00	0.00	-23.50	0.198	-4.73
	52.58	5.00	0.00	-23.50	0.239	-5.66
	56.46	4.00	0.00	-23.50	0.301	-7.09
60°	34.21	6.00	0.00	-21.11	0.143	-3.48
	37.60	5.00	0.00	-21.11	0.178	-4.18
	41.36	4.00	0.00	-21.11	0.224	-5.22
90°	26.64	6.00	0.00	-21.49	0.12	-2.75
	29.28	5.00	0.00	-21.49	0.12	-3.31
	31.48	4.00	0.00	-21.49	0.12	-4.14

6. CONCLUSION

In the present investigation the following conclusions are obtained from the analytical solution of the composite cantilever beam.

1. The intensity of the residual stress residual stress is maximum at the upper and lower surfaces of the beam.
2. The intensity of the residual stress component of σ_x is greatest for the orientation angle of 0°.
3. The horizontal displacement component of u is lower than the vertical displacement component of v.
4. The vertical displacement component in the plastic region is greater than vertical displacement in the elastic region at the same section.
5. The vertical displacement component at the upper point (x = 0, y = -c) is the lower for the orientation angle of 90°.

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