OPTIMAL SINGULAR ADAPTIVE COMPUTER OBSERVATION AND MODELING OF DISCHARGE PROCESSES IN XENON PULSE TUBES

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Özet: **Bu makale, şimdiye kadar doğrusal ve kalıcı olmayan sürekli süreçler şeklinde düşünülen xenon vurum tüplerindeki deşarj süreçlerinin, ayrık uyarlamalı gözlemi ve modellemesi için yeni bir yaklaşım sunar. Bu amaçla, Varna Teknik Üniversitesi'nde son 25 yılda geliştirilen optimum tekil uyarlamalı bilgisayar gözlemi ve modellemesi teorisinin araçları uygulandı.**

Anahtar Kelimeler: **Xeron vurum tüpleri, deşarj süreçleri, doğrusal olmayan diferansiyel denklemler, ayrık sistemler, optimum tekil uyarlamalı bilgisayar gözlemi, tanımlama, ayrık modelleme, başlangıç durum vektörü değerlendirmesi.**

Abstract: **The paper presents a new approach of discrete adaptive observation and modeling of discharge processes in Xenon pulse tubes considered so far as nonlinear and non-stationary continuous processes. The tools of the Theory of Optimal Singular Adaptive Computer Observation and Discrete Modeling, which have been developed in the recent 25 years in the Technical University Varna are applied.**

Keywords: **Xenon pulse tubes, discharge processes, optimal singular adaptive computer observation, identification, discrete modeling.**

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I. INTRODUCTION

The Xenon pulse tubes have various applications including optically excited lasers operating in pulse mode. Irrespectively of their type (tinctorial or solid state) the requirements to the discharge pulses are:

- Impulse energy;
- Impulse duration;
- Repeatability of impulse parameters.

Considering the pulse tubes as a load in the discharge chain we can isolate two groups of problems:

- The Xenon pulse tubes are nonlinear elements, i.e. it is necessary to model the discharge circuit.

Accepting the concept of a tube constant -λ, defined by the construction features of the tube – distance between electrodes and cross-section of the discharge channel, they can be determined by the following equations [1, 2]:

$$R_\Lambda = \frac{\lambda}{U_\lambda}; \; I_\Lambda = \frac{U_\Lambda^2}{\lambda}; \qquad (1)$$

$$t_i = 4\lambda \frac{W_c}{U_\Lambda^3}; \; P_{av} = W_c.f,$$

where:

R_Λ [Ω] - is the discharge resistance of the pulse tube,
I_Λ [A] - is the tube discharge current,
U_Λ [V] - tube applied voltage,
W_c [J] – energy accumulated in the operating capacitor,
t_i [s] – duration of the discharge impulse $t_i = 2R_\Lambda.C$,
P_{av} [W] - average power dissipated by the tube,
f [Hz] - is the frequency of the discharge impulses,
λ [Ω^2.A] – can be defined by expressions of the type:

$$\lambda = \frac{k^2 l^2}{S}, \qquad (2)$$

S $[m^2]$ is the cross-section of the capillary channel,

$k = 1{,}13\ [\Omega.A^{0,5}]$ is a proportional coefficient.

$\lambda = 200 \div 800\ [\Omega^2.A]$ for the most frequently used Xenon lamps.

The following equation is valid for the simplified circuit – Fig. 1 of the discharge chain:

$$L\frac{dI_\Lambda(t)}{dt} + \sqrt{\lambda I_\Lambda(t)} + R^* I_\Lambda(t) = -U_\Lambda(t),$$
$$I_\Lambda(0) = i_{\Lambda 0}. \qquad (3)$$

where:

R^* - reflects the cumulative resistance of the coil, the capacitor and the connecting wires.

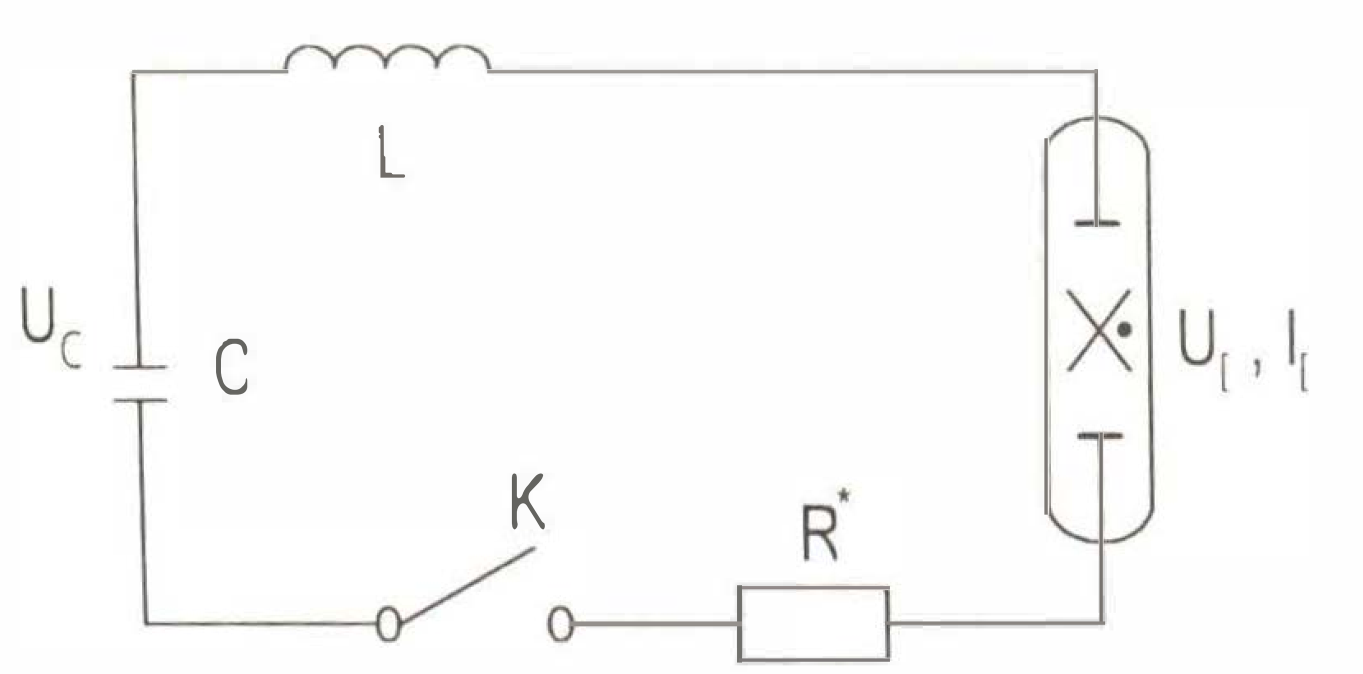

Fig.1

The nonlinear differential equation (3) [1, 2] suggests numerical solution. So, the first problem is that the modeling even of a simplified discharge circuit is a quite complex process. Second problem is to secure repeatability of the discharge pulses. While the initiation of the discharge and the increase of the current carriers are processes depending on various factors [1], the discharge of the main capacitor battery most often flows at certain concentration of the current carriers in the pulse Xenon lamp. This suggests use of either "duty arc" mode I_{da} – Fig.2 or "pre-discharge" ("double pulse") mode – Fig.3. K_1 closes first and after certain delay, K closes as well. The solution of the second problem makes the discharge circuit much more complex. All this makes it clear that the discharge processes interpreted in this paper as objects of identification, adaptive observation, modeling and control, are specified mainly by nonlinear and non-stationary dynamics, by lack of repeatability of the experiments and difficulties for measurement of main variables of the discharge process. These specific peculiarities of the discharge processes explain the limited number of publications committed to the application and development of the contemporary theory of nonlinear assessment for observation of parameters and state of discharge tubes. The existing methods mentioned above are appropriate mainly for continuous models of discharge processes. They do not assess, on the basis of input-output data for the process, its order, parameters and current states. They do not assess also the initial state vector. The influence of the inaccurate initial assessments on the convergence of recurrent algorithms is not yet studied sufficiently in literature. The objective of this publication is to present a new approach for discrete adaptive observation and modeling of discharge processes in Xenon pulse tubes, which uses the Theory for Optimal Singular Adaptive (OSA) Computer Observation and Modeling, developed in the last 25 years in the Technical University Varna.

Impression about the status and the development of the OSA theory of computer observation and modeling can be formed from [3÷16].

II. DISCRETE MODELING OF THE DYNAMICS OF THE DISCHARGE PROCESSES IN XENON PULSE TUBES

The approach requires presentation of the discrete linearized observable sub-models of the studied discharge process in the state space as follows:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}U_\Lambda(k),\ \mathbf{x}(0) = \mathbf{x}_0, \qquad (4)$$

$$I_\Lambda(k) = \mathbf{c}^T\mathbf{x}(k),\ k=0,1,2,..., \qquad (5)$$

where $\mathbf{x}(k) \in R^n$ is a unknown current state vector, $\mathbf{x}(0) \in R^n$ is a unknown initial state vector, $U_\Lambda(k)[V] \in R^1$ is a control scalar input, $I_\Lambda(k)[A] \in R^1$ is the measured scalar output reaction of the discrete model. **A** is a unknown matrix of the type:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{n-1} \\ \hline \multicolumn{2}{c}{\mathbf{a}^T} \end{bmatrix},\ \mathbf{a}^T = [a_1, a_2, ..., a_n], \qquad (6)$$

where I_{n-1} is a single (n-1) x (n-1) matrix.

$$\mathbf{b}^T = [0, ..., 0, 1], \qquad (7)$$

$$\mathbf{c}^T = [1, 0, ..., 0]. \qquad (8)$$

The set of algorithms of the discrete computer-mathematical model (4÷8) uses the instruments of the OSA theory for observation and modeling.

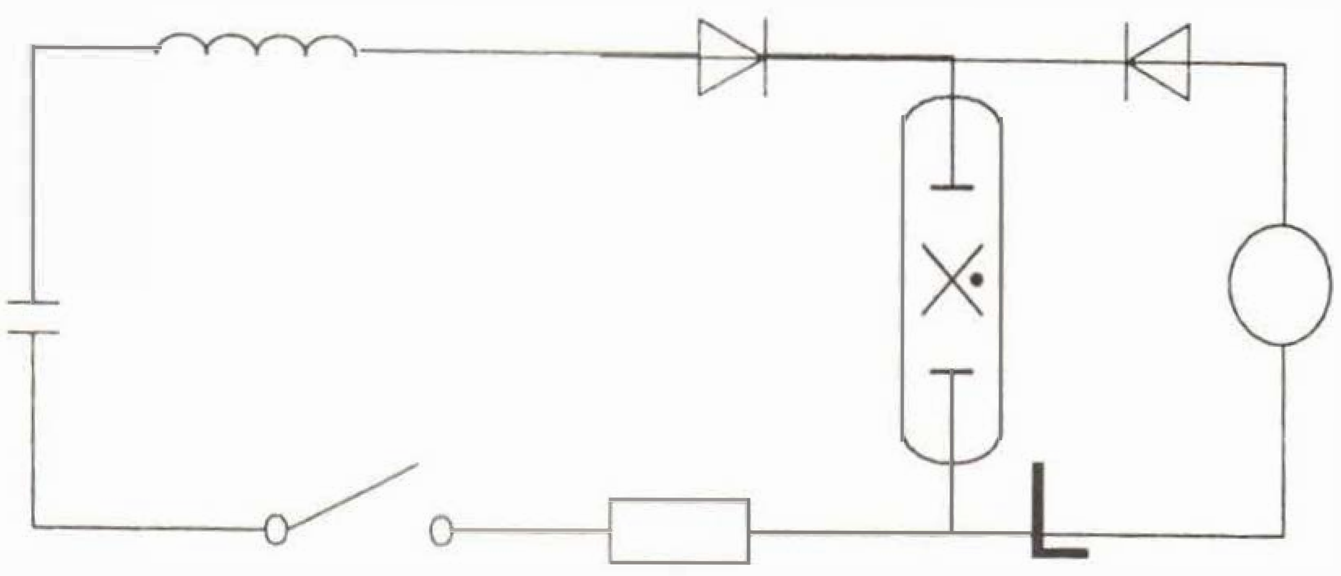

Fig.2

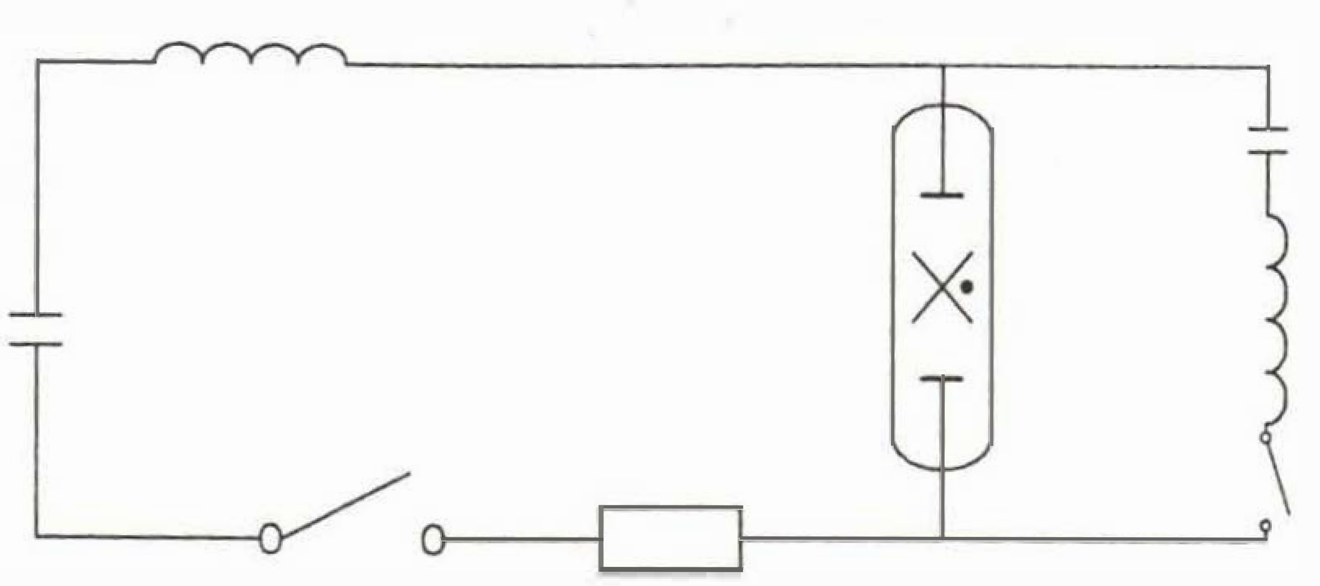

Fig.3

III. ALGORITHMIC SYNTHESIS OF THE OPTIMAL SINGULAR ADAPTIVE M3A1 COMPUTER OBSERVER OF LYUBOMIR SOTIROV FOR DISCHARGE PROCESSES IN XENON PULSE TUBES

Step 1. Formation of input-output data arrays:

$$\mathbf{Y}_1^T = [I_\Lambda(0), I_\Lambda(1), ..., I_\Lambda(n-1)],$$
$$\mathbf{Y}_2^T = [I_\Lambda(n), I_\Lambda(n+1), ..., I_\Lambda(2n-1)],$$
$$\mathbf{V}^T = [U_\Lambda(0), U_\Lambda(1), ..., U_\Lambda(n-1)]$$

$$\mathbf{Y}_{12} = \begin{bmatrix} I_\Lambda(0) & I_\Lambda(1) & . & . & I_\Lambda(n-1) \\ I_\Lambda(1) & I_\Lambda(2) & . & . & I_\Lambda(n) \\ I_\Lambda(2) & I_\Lambda(3) & . & . & I_\Lambda(n+1) \\ . & . & & & . \\ I_\Lambda(n-1) & I_\Lambda(n) & . & . & I_\Lambda(2n-2) \end{bmatrix},$$

where $\mathbf{Y}_{12}$ is a n x n Hankel matrix.

Step 2. Calculation of the assessments of the n-dimensional vector **a**, as a solution of the system of linear algebraic equations:

$$\mathbf{Y}_{12}\hat{\mathbf{a}} = \mathbf{Y}_2 - \mathbf{V}.$$

Step 3. Calculation of the elements of the n-dimensional vector **x**(0), with the help of the optimal computer evaluator of the type:

$$\hat{\mathbf{x}}(0) = \mathbf{Y}_1.$$

Step 4. Assessment of the n–dimensional current state vector **x**(k), k=1,2,.., with the help of the full optimal singular adaptive computer observer of the type:

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{F}}\hat{\mathbf{x}}(k) + \mathbf{b}U_\Lambda(k) + \mathbf{g}I_\Lambda(k), \quad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0,$$

k=0,1,2,....,

where:

$$\hat{\mathbf{F}} = \hat{\mathbf{A}} - \mathbf{g}\mathbf{c}^T,$$
$$\mathbf{g}^T = [g_1, g_2 ... g_n].$$

The choice of vector **g** elements is arbitrary and can be effected by one of the following ways:

1) **g**=**b**.
2) **g**=0.
3) The vector **g** elements could be selected so that the $\hat{\mathbf{F}}$ matrix possesses eigenvalues located inside the unit circle.

Step 5. The degenerative OSA computer observer (at **g**=0) of the type:

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{A}}\hat{\mathbf{x}}(k) + \mathbf{b}U_\Lambda(k), \quad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0,$$

k=0,1,2,....,

could be considered as a degenerative case of the full OSA computer observers (at **g**=0) and can be constructed easier.It follows from the analysis of the derived algorithm:

***M3A1 Theorem*.** An only solution of the formulated task for optimal singular adaptive computer observation and modeling with the help of M3A1 algorithm, exists if and only if the matrix $\mathbf{Y}_{12}$ is not singular or:

$$\det(\mathbf{Y}_{12}) \neq 0. \qquad (9)$$

IV. MATLAB-INTERPRETATION OF THE OPTIMAL SINGULAR ADAPTIVE M3A1 COMPUTER OBSERVER

Step 1. The one-dimensional arrays and the bi-dimensional array of the input-output data are formed by the following Matlab-module:

```
y1 = Ilambda(1 : n);
y2 = Ilambda(n + 1; 2 * n);
v = Ulambda(1 : n);
for i = 1 : n
        for j = 1 : n

Y12(i, j) = Ilambda(i + j - 1);
        end
end
```

Step 2. Set up and solution of the respective linear system of algebraic equations:

```
ae = inv(Y12) * (y2 - v);
```

Step 3. Assessment of the initial state vector:

```
xe0 = y1;
```

Step 4. The full OSA computer observer is interpreted by the following Matlab-module:

```
b = [zeros(n - 1, 1);1];
c = [1; zeros(n - 1, 1)];
g = zeros(n, 1);
I = eye(n - 1);
Ae = [zeros(n - 1, 1), I; ae];
Fe = Ae - g * c';
xe(:, 1) = xe0;
for k = 1 : 2 * n - 1
xe(:, k + 1) = Fe * xe(:, k) + b * Ulambda(k) +
+ g * Ilambda(k);
end

e(k) = norm(Ilambda(k) - Ilambdae(k))/
/norm(Ilambda(k));
end
```

Step 5. The relative error of the OSA observation is calculated by the following Matlab-модул:

```
for k = 1 : 2 * n
      Ilambdae = c'*xe(:, k);
end
for k = 1 : 2 * n
```

V. RESULTS OF THE OPTIMAL SINGULAR ADAPTIVE COMPUTER OBSERVATION AND MODELING OF A DISCHARGE PROCESS IN A XENON PULSE TUBE IFP800.

The following input-output data about the discharge process were obtained from experiments: (Table1)

They were processed further in the mode of three randomly selected samples. The evaluated order of the three discrete sub-models is $\hat{n} = 2$. The degenerative OSA computer observer (at g=0) was selected for assessment of the current state vector and of the three discrete sub-models. It is known that all OSA observers calculate the results of the adaptive observation with almost equal accuracy thus the OSA computer observer with the simplest structure for realization was selected.

We shall give the following representative results for the first sample, at k=0,1,2,3:

$$\hat{\mathbf{a}} = \begin{bmatrix} -6.3105 \\ 2.0000 \end{bmatrix}, \quad \hat{\mathbf{x}}(0) = \begin{bmatrix} 0 \\ 20.0000 \end{bmatrix}, \quad \hat{\mathbf{x}}(1) = \begin{bmatrix} 20.0000 \\ 40.0000 \end{bmatrix},$$

$$\hat{\mathbf{x}}(2) = \begin{bmatrix} 40.0000 \\ 60.0000 \end{bmatrix}, \quad \hat{\mathbf{x}}(3) = \begin{bmatrix} 60.0000 \\ 17.7800 \end{bmatrix}.$$

They are obtained with the following relative errors:

$$e(k) = \frac{\left| I_\Lambda(k) - \hat{I}_\Lambda(k) \right|}{\left| I_\Lambda(k) \right|} \cong 10^{-15}, \quad k = 0,1,2,3.$$

at that,

$$\mathrm{cond}(\mathbf{Y}_{12}) = 5.8284,$$

i.e. the value of the number of condition of the respective Hankel matrix tells that the considered here task for OSA observation and modeling of discharge processes of Xenon pulse tubes, modeled by a continuous nonlinear mathematical model [1,2], is well determined in the case of discrete presentation, which leads to a better computer interpretation, e.g. to a minimal computational complexity and to reduction of the operating computer memory to an order of magnitude, compared with the existing method for matrix inversion, solution of linear systems of algebraic equations with a special, dense structure.

The eigenvalues of the state matrix of this sub-model are

$$\chi_{11} = 1.0000 + 2.3045i,$$

$$\chi_{12} = 1.0000 - 2.3045i.$$

Table I

$U_A(0) = 0$ [V]	$U_A(17) = 437{,}90$ [V]	$U_A(34) = 619{,}29$ [V]
$U_A(1) = 106{,}21$ [V]	$U_A(18) = 450{,}60$ [V]	$U_A(35) = 628{,}33$ [V]
$U_A(2) = 150{,}20$ [V]	$U_A(19) = 462{,}95$ [V]	$U_A(36) = 637{,}24$ [V]
$U_A(3) = 183{,}96$ [V]	$U_A(20) = 474{,}97$ [V]	$U_A(37) = 646{,}03$ [V]
$U_A(4) = 212{,}41$ [V]	$U_A(21) = 486{,}70$ [V]	$U_A(38) = 654{,}71$ [V]
$U_A(5) = 237{,}49$ [V]	$U_A(22) = 498{,}16$ [V]	$U_A(39) = 663{,}26$ [V]
$U_A(6) = 260{,}15$ [V]	$U_A(23) = 509{,}35$ [V]	$U_A(40) = 671{,}71$ [V]
$U_A(7) = 281{,}00$ [V]	$U_A(24) = 520{,}31$ [V]	$U_A(41) = 680{,}06$ [V]
$U_A(8) = 300{,}40$ [V]	$U_A(25) = 531{,}04$ [V]	$U_A(42) = 688{,}30$ [V]
$U_A(9) = 318{,}62$ [V]	$U_A(26) = 541{,}55$ [V]	$U_A(43) = 696{,}45$ [V]
$U_A(10) = 335{,}86$ [V]	$U_A(27) = 551{,}87$ [V]	$U_A(44) = 712{,}46$ [V]
$U_A(11) = 352{,}25$ [V]	$U_A(28) = 562{,}00$ [V]	$U_A(45) = 720{,}33$ [V]
$U_A(12) = 367{,}91$ [V]	$U_A(29) = 571{,}94$ [V]	$U_A(46) = 728{,}12$ [V]
$U_A(13) = 382{,}94$ [V]	$U_A(30) = 581{,}72$ [V]	$U_A(47) = 735{,}83$ [V]
$U_A(14) = 397{,}39$ [V]	$U_A(31) = 591{,}34$ [V]	$U_A(48) = 743{,}45$ [V]
$U_A(15) = 411{,}34$ [V]	$U_A(32) = 600{,}80$ [V]	$U_A(49) = 751{,}00$ [V]
$U_A(16) = 424{,}83$ [V]	$U_A(33) = 610{,}11$ [V]	

$I_A(0) = 0$ [A]	$I_A(17) = 340$ [A]	$I_A(34) = 680$ [A]
$I_A(1) = 20$ [A]	$I_A(18) = 360$ [A]	$I_A(35) = 700$ [A]
$I_A(2) = 40$ [A]	$I_A(19) = 380$ [A]	$I_A(36) = 720$ [A]
$I_A(3) = 60$ [A]	$I_A(20) = 400$ [A]	$I_A(37) = 740$ [A]
$I_A(4) = 80$ [A]	$I_A(21) = 420$ [A]	$I_A(38) = 760$ [A]
$I_A(5) = 100$ [A]	$I_A(22) = 440$ [A]	$I_A(39) = 780$ [A]
$I_A(6) = 120$ [A]	$I_A(23) = 460$ [A]	$I_A(40) = 800$ [A]
$I_A(7) = 140$ [A]	$I_A(24) = 480$ [A]	$I_A(41) = 820$ [A]
$I_A(8) = 160$ [A]	$I_A(25) = 500$ [A]	$I_A(42) = 840$ [A]
$I_A(9) = 180$ [A]	$I_A(26) = 520$ [A]	$I_A(43) = 860$ [A]
$I_A(10) = 200$ [A]	$I_A(27) = 540$ [A]	$I_A(44) = 900$ [A]
$I_A(11) = 220$ [A]	$I_A(28) = 560$ [A]	$I_A(45) = 920$ [A]
$I_A(12) = 240$ [A]	$I_A(29) = 580$ [A]	$I_A(46) = 940$ [A]
$I_A(13) = 260$ [A]	$I_A(30) = 600$ [A]	$I_A(47) = 960$ [A]
$I_A(14) = 280$ [A]	$I_A(31) = 620$ [A]	$I_A(48) = 980$ [A]
$I_A(15) = 300$ [A]	$I_A(32) = 640$ [A]	$I_A(49) = 1000$ [A]
$I_A(16) = 320$ [A]	$I_A(33) = 660$ [A]	

The discharge process in this time interval is oscillating with increasing amplitude, i.e. unstable. The eigenvalues of the degenerative OSA computer observer (at g=0), are obviously the same, but the stated OSA observer instability does not impact negatively the exceptionally high accuracy of the described observation and modeling

We shall give the following representative results for the second sample, at k=29,30,31,32:

$$\hat{\mathbf{a}} = \begin{bmatrix} 12.9270 \\ -12.4160 \end{bmatrix}, \qquad \hat{\mathbf{x}}(29) = \begin{bmatrix} 580.0000 \\ 600.0000 \end{bmatrix},$$

$$\hat{\mathbf{x}}(30) = \begin{bmatrix} 600.0000 \\ 620.0000 \end{bmatrix},$$

$$\hat{\mathbf{x}}(31) = \begin{bmatrix} 620.0000 \\ 640.0000 \end{bmatrix}, \quad \hat{\mathbf{x}}(32) = \begin{bmatrix} 640.0000 \\ 659.0000 \end{bmatrix}.$$

They are obtained with the following relative errors:

$$e(k) = \frac{\left|I_\Lambda(k) - \hat{I}_\Lambda(k)\right|}{\left|I_\Lambda(k)\right|} \cong 10^{-14}, \quad k = 29,30,31,32.$$

At that,

$$\operatorname{cond}(\mathbf{Y}_{12}) = 3.6020.10^3,$$

i.e. in the considered time interval the task for OSA computer observation and modeling is not well determined but as it is evident from the enclosed results this has no negative impact on the exceptionally high computational accuracy which is a result of the robust features of OSA observation, studied for the general case in [3÷16].

The eigenvalues of the state matrix of this sub-model are:

$$\chi_{21} = 0.9660,$$
$$\chi_{22} = -13.3820,$$

i.e. the discharge process is unstable again, but is not oscillating.

We shall give the following representative results for the third sample at k=44,45,46,47:

$$\hat{\mathbf{a}} = \begin{bmatrix} 16.5220 \\ -15.9155 \end{bmatrix}, \qquad \hat{\mathbf{x}}(44) = \begin{bmatrix} 900.0000 \\ 920.0000 \end{bmatrix},$$

$$\hat{\mathbf{x}}(45) = \begin{bmatrix} 920.0000 \\ 940.0000 \end{bmatrix},$$

$$\hat{\mathbf{x}}(46) = \begin{bmatrix} 940.0000 \\ 960.0000 \end{bmatrix}, \quad \hat{\mathbf{x}}(47) = \begin{bmatrix} 960.0000 \\ 979.0000 \end{bmatrix}.$$

They are obtained with the following relative errors:

$$e(k) = \frac{\left|I_\Lambda(k) - \hat{I}_\Lambda(k)\right|}{\left|I_\Lambda(k)\right|} \cong 10^{-12}, \quad k = 44,45,46,47.$$

At that,

$$\operatorname{cond}(\mathbf{Y}_{12}) = 8.4660.10^3.$$

The eigenvalues of the state matrix of this third sub-model are:

$$\chi_{31} = 0.9780,$$
$$\chi_{32} = -16.8935.$$

Obviously the comments regarding the behavior of the discharge process in this time interval are analogical to the previous two discrete structures approximating the dynamic behavior of the discharge process of the studied Xenon lamp in the respective time sub-intervals.

VI. CONCLUSION

The results of the computer processed input-output data on the basis of the proposed approach, M3A1 algorithm and Matlab software illustrates the possibility to solve the task for optimal singular adaptive computer observation and discrete modeling of discharge processes in Xenon pulse tubes with guaranteed accuracy. These computational results illustrate the mathematical and program consistency of the algorithmic synthesis of OSA computer observers.

The computer processing of arbitrary, sequential or sequential with overlapping samples of input-output data for the discharge process allows to interpret the considered class of nonlinear and non-stationary continuous systems with a limited set of discrete sub-models with variable structure, variable parameters and variable initial and current states..

The respective processing of experimental information of this type can be used for design, modeling and realization of adaptive systems for regulation, stabilization of discharge processes in Xenon pulse tubes and other objects on the basis of identifiers, optimal computer evaluators of initial state, optimal singular adaptive computer observers of current state, regulators and stabilizers..

It can be shown [3÷16], that the assessments received above are optimal in terms of the minimum of the quadratic functional of the type:

$$\left\|\mathbf{e}(k)\right\|^2 = \left\|\mathbf{x}(k) - \hat{\mathbf{x}}(k)\right\|^2, \quad k = 0,1,2,..., \qquad (10)$$

where

$$e(k) = x(k) - \hat{x}(k), \quad k = 0,1,2,..., \qquad (11)$$

and the norm is defined as:

$$\|e(k)\|^2 = \sqrt{< e(k), e(k) >}, \quad k = 0,1,2,..., \qquad (12)$$

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