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Minimization of Power Loss in Hydrodynamic Bearings Design Using the Genetic Algorithm

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Abstract: The search of bearing geometry and performance which satisfies at best design objective along with design criteria is not so easy task. Design optimization of hydrodynamic bearings is very complex in nature. The complexity and time consuming nature of the design process of hydrodynamic bearings warranted the development of a new methodology. The purpose of this study is to use of the genetic algorithm in the optimal design of a three-lobe preloaded fluid film bearing in essence developing the bearing configurations that optimize power loss along with other design criteria namely fluid film thickness, stability parameter, film temperature, and film pressure. The results obtained and presented in this study are compared to results from numerical optimization methods such as gradient-based method, and show the potential of the genetic algorithm in optimization of three-lobe preloaded hydrodynamic bearings. This robust method has been designed to search for most feasible solutions to problems and has gained recognition in many fields.

Keywords: power loss, genetic algorithm, optimization, hydrodynamic bearing

Genetik Algoritma Kullanılarak Hidrodinamik Yatakların Tasarımında Güç Kaybı Minimizasyonu

Özet: Hidrodinamik yatakların tasarımında önemli olan geometri ve performansın tasarım kriterlerini sağlaması kolay bir iş değildir. Hidrodinamik yatakların tasarım optimizasyonu doğal olarak çok karmaşıktır. Bu karmaşık ve zaman gerektiren hidrodinamik yatakların tasarımı yeni bir tasarım yöntemini gerekli kılmaktadır. Bu çalışmanın amacı genetik algoritma metotunu kullanarak hidrodinamik yatakların tasarımında önemli olan güç kaybını en aza indirmek aynı zamanda yağ filmi kalınlığı, stabilizasyon parametresi, yağ filmi sıcaklığı ve yağ basıncı için en uygun değerleri bulmada önemli rol oynayan yatakların konfigurasyonunu en iyi şekilde tasarlamaktır. Bu çalışmada elde edilen sonuçlar klasik olan numerik metotla elde edilen sonuçlarla karşılaştırılarak genetik algoritmanın potansiyeli ve kabiliyeti gösterilmiştir. Genetik algoritma tabii seleksiyon (seçim) tekniğini kullanarak tanımlanan sınırlar içinde tarama yapan ve genetik fikrine dayalı uygun araştırma tekniğidir. Gün geçtikçe genetik algoritma daha iyi tanınmakta ve bir çok alanda uygulanmaktadır.

Anahtar Kelimeler: güç kaybı, genetik algoritma, optimizasyon, hidrodinamik yataklar

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Introduction

Successful operation with increased efficiency and higher power requirement in modern high-speed rotor-bearing systems is very much dependent upon behavior of the bearings which support the rotor. The bearings provide damping [1], which is adequate for many rotating system designs, and their stiffness properties affect the stability of the rotor-bearing system [2]. The power loss performance objective is an important element in the design and optimization of hydrodynamic bearings. For this study, power loss reduction is a primary goal in the design of three-lobe preloaded bearings.

for more robust and efficient optimization methods. One of these methods is the genetic algorithm, which has

random search technique. It is parameter search procedure based on the idea of natural selection and genetics [3]. It uses objective function information instead of derivatives as in numerical method such as gradient-based method.

Many numerical optimization methods have been developed and used for design optimization of hydrodynamic bearings. Most of these methods are based on gradient techniques. These methods are reasonably effective for well-behaved objective functions. This is because the gradient of the function helps to guide the direction of the search. However, when the continuity and existence of derivatives of the function are not assured, gradient methods lack robustness and may trap in local optima. To overcome these problems, many different approaches exist in the literature.

The development of faster computer has allowed gained recognition as a general problem solving technique in many applications. The genetic algorithm is guided

Numerical search methods are good at "exploitation but not exploration" of the parameter space [4]. They focus on areas around the current design point, using local gradient calculations to move to a better design. Since

there may not be exploration for all regions of the parameter space, they can more easily be trapped in local optima [4]. The genetic algorithm is a class of general purposes algorithm that can provide a remarkable balance between exploration and exploitation of the search space [5]. From this point of view, this study provides use of the genetic algorithm to seek the most feasible solution to this problem. The genetic algorithm is new to the field of bearing system analysis, and in current literature there is limited work in the area of rotor-bearing systems using the genetic algorithm. Interested reader can refer to the studies by Saruhan et al. [6] and Saruhan et al. [7].

The Genetic Algorithm

The genetic algorithm is an efficient search technique which applies the rules of natural genetics to explore a given search space [8]. It is being applied successfully to find solutions to problems in engineering and science [9]. This robust adaptive searching technique has gained recognition as a general problem solving technique in many optimization problems. The genetic algorithm is well behaved for problems with combination of complex, discontinuous, and discrete functions. The genetic algorithm maintains a population of encoded solutions, and guides the population towards the most feasible solution [3]. Thus, it searches the space of possible individuals and seeks to find the best fitness strings. Rather than starting from a single point solution within the search space as in traditional methods, the genetic algorithm begins with an initial set of random solutions of population. The solutions are represented by strings (chromosomes), which are coded as a series of zeros and ones. The genetic algorithm is non-deterministic search optimization method and does not require differential. Viewing the genetic algorithm as an optimization technique, it belongs to the class of zero-order optimization methods [10] and [11], which requires only function evaluations.

The description of the genetic algorithm is outlined in Figure 1. An initial population is chosen randomly in the beginning and the fitness of each individual in initial population members is evaluated. Then an iterative process starts until the termination criteria have been satisfied. There are many different ways to determine when to stop running the genetic algorithm. One method is to stop after a preset number of generations which is used in this study or a time limit. Another is to stop after the genetic algorithm has converged. Convergence is the progression towards uniformity. A string is said to have converged when 95 % of the population share the same value [24]. After the evaluation of each individual fitness in the population, the genetic operators -- selection, crossover, and mutation -- are applied to produce a new generation. Other genetic operators are applied as needed. The newly created individuals replace the existing generation, and re-evaluation is started for fitness of new individuals. In each succeeding generation, the genetic algorithm creates a new set of "chromosomes" using information of the previous generation. The loop is repeated until an acceptable solution is found.

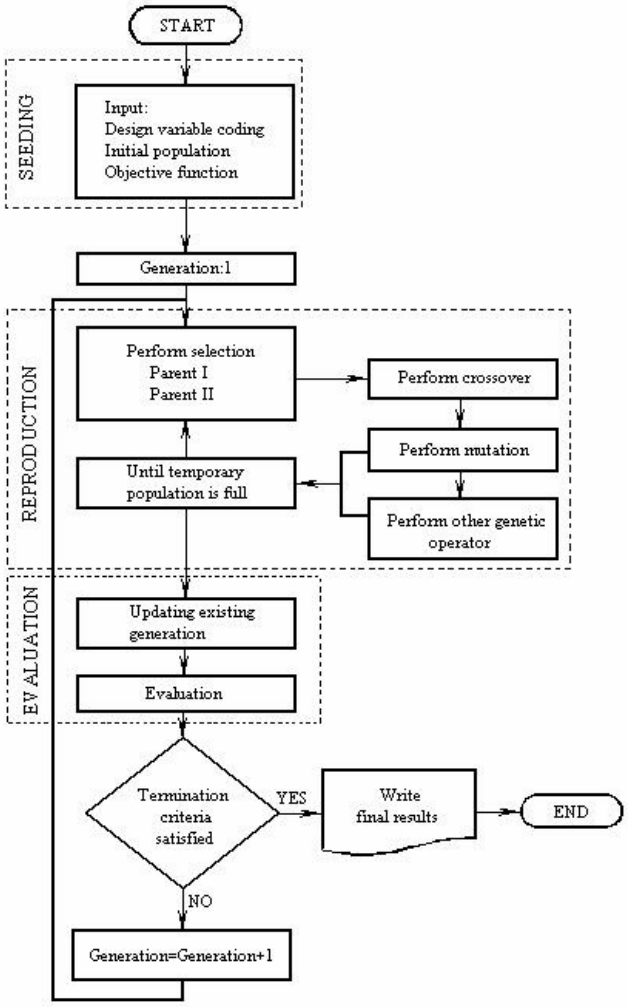

Figure I Flow Chart for the Genetic Algorithm

Problem Statement

The effort here is the use of the genetic algorithm in the optimal design of a three-lobe preloaded fluid film bearing in essence of developing the bearing configurations that optimize minimum power loss objective.

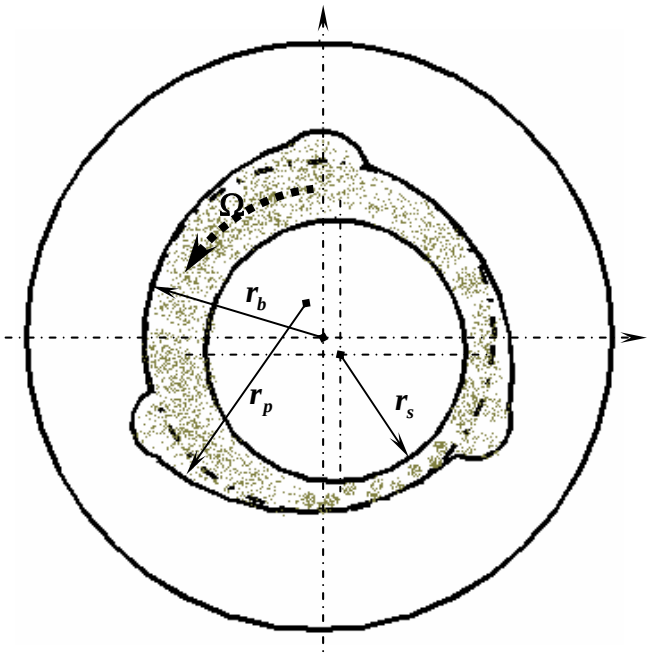

Figure 2 A typical symmetric three-lobe bearing

Design Variables

There is a strong relationship among the design objective and design criteria functions. There are common design variables that influence these, the objective function and the design criteria functions. These variables are the main factor in determining the design problem. The design vector of variables included pad axial length to journal diameter ratio, pad (lobe) arc length, bearing radial clearance, pad offset factor, pad preload factor, and bearing orientation with respect to load expressed as:

$$x(i) = \begin{Bmatrix} \textit{Pad axial length / Journal diameter} \\ \textit{Pad arc length} \\ \textit{Bearing radial clearance} \\ \textit{Pad offset factor} \\ \textit{Bearing orientation wrt. load} \end{Bmatrix} \quad (1)$$

where $i = 1,.....NDV$ (number of design variables).

Pad axial length to journal diameter ratio has effect on fluid induced instability. A key parameter used in describing fixed pad (lobe) bearings is the fraction of converging pad to full pad length. This ratio is called *pad offset factor* and defined as:

$$\alpha_P = L_P / \chi \quad (2)$$

where L_P and χ are length of pad (lobe) with converging film thickness and full pad arc length respectively.
Preload is referred to static loads, which are forces applied to the rotor system. Because the preload can affect the shaft centerline position, stability of rotor system is considered. Preload factor can be expressed as [12]:

$$\textit{Pad preload factor} = \left(c_p - c_b\right)/c_p \quad (3)$$

Bearing pad clearance, c_p, and bearing radial clearance, c_b, can be computed as:

$$c_p = r_p - r_s \quad ; \quad c_b = r_b - r_s \quad (4)$$

where r_p, r_s, and r_b are radius of bearing pad, radius of journal, and radius of bearing at minimum bore, respectively.

State Variables

State variables are the physical quantities, which is describing the bearing configuration, operating conditions, and loading of the rotor-bearing system. These parameters are journal rotational speed, rotor mass, journal external load, journal unbalance, lubricant properties, lubricant pressure, and lubricant temperature.

Constraints

Constraints considered for optimum design of the three-lobe journal bearing in rotor-bearing system include the followings:

$$g_j = \begin{Bmatrix} \textit{Film temperature constraint}, \quad f_t^{lower} \le f_t \le f_t^{upper} \\ \textit{Film pressure constraint}, \quad f_p^{lower} \le f_p \le f_p^{upper} \\ \textit{Lubrication flow constraint}, \quad f_q^{lower} \le f_q \le f_q^{upper} \\ \textit{Orbital displacement constraint}, \quad f_u^{lower} \le f_u \le f_u^{upper} \\ \textit{Geometric inequality}, \\ g_k(\mathrm{x}) \le 0 \qquad k = 1,2,.....,NIC \text{ (number of inequality constraint)} \end{Bmatrix} \quad (5)$$

Bearing temperature is an important criteria that should be met because it can be dangerous enough to give a failure in journal bearings and thus the whole system. The limit of acceptable high temperature assumed in this study is 200 F^0 (93.33 C^0), to avoid oxidation and standing with a good condition. Minimum temperature occurs beyond the inlet groove in the direction of shaft rotation [13], while the maximum temperature occurs in the vicinity of the minimum film thickness [14].
The effect of pressure in fluid film is reflected by the density and viscosity of the lubricant [15]. It is a common

knowledge that in almost all fluid film bearings, as the film thickness decreases the pressure increases. The amount of fluid that needs to be supplied for the bearing is also a factor in bearing performance.
Rouch [16] and Abdul-Wahed [17] suggested that the dynamic response of the system at bearing location should be less than 30 percent of the clearance or less than film thickness otherwise the properties of bearing are not valid. Beside the constraints outlined so far the geometric inequality constraints also are conducted such as bearing pad length, bearing orientation, and attitude angle constraints.

Objective Function

The objective function for power loss is:

$$F_{objective} = power\ loss\ \ objective \tag{6}$$

$$Fitness\ Function = F_{objective} - P \tag{7}$$

$$P = \sum_{j=1}^{NCON} r_j \left(\max \left[0,\ g_j \right] \right)^2 \tag{8}$$

where $F_{objective}$ is power loss function and $NCON$ is number of constraints.
One of most important aspects of the genetic algorithm is fitness function. Fitness function measures and rates the coded variable vectors in order to select the fittest strings that lead the solution. Constraint optimization problem have been transformed into an unconstrained optimization problem and handled by penalizing the objective function value by quadratic penalty function, P, which is used to ensure that the bearing system meets any imposed constraints. In case of any violation of a constraint boundary, the fitness function of corresponding solution is penalized and kept within feasible regions of design space. The penalty coefficients, r_j, for the j-th constraint have to judiciously selected because the good solutions importantly depends on these values of penalty coefficients.

Construction of Design Variables and the Genetic Algorithm

The first step for applying the genetic algorithm to the assigned design problem is encoding of the design variables as a string. This string typically refers to a solution to the problem. Rather from starting from a single point solution within the search space as in traditional methods, the genetic algorithm is initialized with a population of solutions, which specify the number of strings in each generation. The genetic algorithm uses a selection scheme to select best individuals, strings, from the population to insert into a mating pool by using the fitness function. Individuals from the mating pool are used by selection operators to generate new candidates for forming the basis of the next generation of solution.
Each design variable vector has a specified range so that $x(i)_{lower} \le x(i) \le x(i)_{upper}$. The continuous design variables vector are represented and discretized to a precision of ε ($\varepsilon = 0.01$). The number of digits in the binary string, l, is estimated from the following relationship [18]:

$$2^l \ge \left[\left(x(i)_{upper} - x(i)_{lower} \right) / \varepsilon \right] + 1 \tag{9}$$

where $x(i)_{lower}$ and $x(i)_{upper}$ are the lower and upper bound for design variable vector respectively. Suitable representation, coding, of the design vectors is a success key in the genetic algorithm. The six design vectors of variables are coded into binary digits {0, 1} as shown in Table 1. The binary string representation for the vector of design variables, $x(i)$, can be placed head-to-tail to form one long string, referred to as a chromosome. This chromosome represents a solution to the design problem. Table 2 shows string of 40 binary digits denotes the concatenated design variables vector. A randomly selected set, for this study a 150-string, of potential solutions is initialized to form the starting population as can be seen in Table 2. Population size influences the number of search points in each generation. A guideline for an appropriate population size is suggested by Goldberg [19].

The real value of the design variable vectors can be transformed from binary string by following relationship [20]:

$$x(i) = \left[\left(x(i)_{upper} - x(i)_{lower} \right) / \left(2^l - 1 \right) \right] a(i) + x(i)_{lower} \tag{10}$$

where $a(i)$ represents the decimal value of string for design variable vectors which is obtained by using base-2 form.

Table 1 Coding of design variable vectors into binary digits.

Design Variables Vectors $x(i)$	Binary String	String Lenth (bits) l
Pad axial length / journal diameter	0 1 0 1 0 0	6
Pad arc length	0 1 0 1 0 0	6
Bearing radial clearance	1 0 1 0 0	5
Pad offset factor	0 0 1 0 0 0 1	7
Pad preload factor	0 0 1 0 1 0 1 0	8
Bearing orientation wrt. load	1 0 0 0 1 0 0 1	8

Table 2 A set of starting population.

	Initial Population Concatenated variables vectors head-to-tail					
	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$
	010100	**010100**	10100	**0010001**	00101010	**10001001**
1	0 1 0 1 0 0 **0 1 0 1 0 0** 1 0 1 0 0 **0 0 1 0 0 0 1** 0 0 1 0 1 0 1 0 **1 0 0 0 1 0 0 1**					
	100111	**100010**	01001	**1100011**	10101010	**01110110**
2	1 0 0 1 1 1 **1 0 0 0 1 0** 0 1 0 0 1 **1 1 0 0 0 1 1** 1 0 1 0 1 0 1 0 **0 1 1 1 0 1 1 0**					
.	.	.	.	.	.	.
.	.	.	.	.	.	.
	001101	**0111000**	10011	**0011100**	00011001	**10001010**
150	0 0 1 1 0 1 **0 1 1 1 0 0 0** 1 0 0 1 1 **0 0 1 1 1 0 0** 0 0 0 1 1 0 0 1 **1 0 0 0 1 0 1 0**					

he selection scheme used in the algorithm code is a tournament selection with a shuffling technique for choosing random pairs for mating. Shuffling technique rearrange the population in random order for selection. Tournament selection approach works as follows: a pair of individuals from mating pool is randomly picked and the best-fit two individuals from this pair will be chosen as a parent. Each pair of parent creates two Child as described in the method of uniform crossover shown in Figure 3. A specialized mechanism, elitism, is added to the genetic algorithm. Elitism forces the genetic algorithm to retain the best individual in a given generation to proceed unchanged into the following generation [21]. This ensures the genetic algorithm that converges to appropriate solution. In other words, elitism is a safeguard against operation of crossover and mutation operators that may jeopardize the current best solution.

A uniform crossover operator is used in this study. Crossover is very important in the success of the genetic algorithm. This operator is primary source of new candidate solutions and provides the search mechanism that efficiently guides the evolution through the solution space towards the optimum. In uniform crossover, every bit of each parent string has chance of being exchanged with corresponding bit of the other parent string. Procedure is to obtain any combination of two parent strings (chromosomes) from the mating pool at random and generate new Child strings from these parent strings by performing bit-by-bit crossover chosen according to a randomly generated crossover mask [22]. Where there is a 1 in the crossover mask, the child bit is copied from the first parent string, and where there is a 0 in the mask, the Child bit is copied from the second parent string. The second Child string uses the opposite rule to the previous one as shown in Figure 3. For each pair of parent strings a new crossover mask is randomly generated.

Crossover mask	10010111001001011100100101111001001011100
Parent 1	**10100011101010001110101000111010100011110**
Parent 2	01010100110101010011010101001101010100011
Child 1	**11000011111100001111110000111111000011**11
Child 2	**00110**100**100011**0100**100011**0100**100011**010**10**

Figure 3 Uniform crossover

Crossover operator with different probability, (0.5, 0.7, and 0.9), were tested for genetic algorithm performance. The results showed that the crossover probability, 0.7, performs better than the 0.5 and 0.9.

Preventing the genetic algorithm from premature convergence to a non-optimal solution, which may diversity lost by repeated application of selection and crossover operators, mutation operator is used. Mutation is basically a process of random altering a part of individual to produce a new individual by switching the bit position from a 0 to a 1 or vice versa. Mutation probabilities of 0.001, 0.01, and 0.1 were tested for the genetic algorithm performance. For this study, the results showed that the mutation probability of 0.001 gives preferable results compared to 0.1 and 0.01. It should be noted that if a mutation rate 0.1 is selected, many good strings are never evaluated. In other words many random perturbations are happened with mutation rate 0.1. This causes the losing of parent resemblance and is disastrous for obtaining the optimum point.

In summary, the setting parameters of genetic algorithm for this study are chosen as follows: Chromosome length = 40, population size = 150, number of generation = 150, crossover probability = 0.7, and mutation probability = 0.001.

Results

The computation was performed on a personel computer equipped with an Intel (R) Pentium (R) 4 CPU 3.00 GHz 512 MB RAM and registered an execution approximately for 92 minutes for total of 22500 functions evaluation.

The distribution of normalized fitness function values for generation number one, fifty, and one hundred and fifty is given in Figure 4. Figure 5 provides average and best fitness function values in each generation as optimization proceeds. From the plot, it can be seen that the fitness function has converged to a uniform solutions with similar values throughout generations. The genetic algorithm found the optimal power loss at generation number 25. Comparison of the best overall solution found with numerical optimization by Roso [23] and this genetic algorithm technique is given in Table 3. The results of objective function for both methods are presented. As can be seen from these results, the genetic algorithm was able to obtain in some respect better results than those obtained by numerical optimization. The result from optimization by the genetic algorithm showed that power loss ended with 2.52 hp while numerical method with 2.65 hp. It should be noted that the selected rotor assembly operating at a rotational speed of 28155 rpm is absorbing 450 hp.

The result from optimization showed that logarithmic decrement ended with 1.20 while numerical method with 0.7393. This significant outcome allows the rotor to maintain stability. Also it can be seen that the genetic algorithm method produced a higher bearing radial clearance. Increasing bearing radial clearance provides a relatively higher film thickness and lower film pressure. The power loss natuarlly increased by increasing the axial length of the insert and the length of pad arc. The design variables represented by length and diameter, along with the effect produced by them on the power loss, bounded by the temperature and pressure allowed limits. Design variables are all systematically vary to identify the effects of each combination such as: the length of diameter ratio and radial clearance increase as the minimum film thickness increases. The instability treshold increases with larger preloads while tends to decreases as the offset factor increases.

Table 3 Comparison of the best overall solution found for optimized geometry of bearing, design criteria, and objective function by numerical and the genetic algorithm optimization methods

Bearing Optimized Geometry and Performance	Optimization Method: Numerical Optimization	Optimization Method: **Genetic Algorithm**
Radius at minimum bore, in. (20.68mm)	0.8141	0.8142
Pad axial length, in. (20.63mm)	0.8125	0.8125
Pad (lobe) arc length, deg.	90.00	89.95
Radial clearance at minimum bore, in. (0.0449mm)	0.00165	0.00177
Pad (lobe) clearance, in. (0.0866mm)	0.00294	0.00341
Pad (lobe) offset factor	0.7551	1.0000
Pad (lobe) preload factor	0.4392	0.4824
Bearing orientation, deg.	112.1	89.3
Logarithmic decrement	0.7393	1.20
Film thickness, in. (0.0223mm)	0.00077	0.00088
Power Loss, hp	**2.65**	**2.52**
Film temperature, degF. ($84.41C^{o}$)	199.9	183.940
Film pressure, psi. ($67.98kg/cm^{2}$)	1028.0	967.0

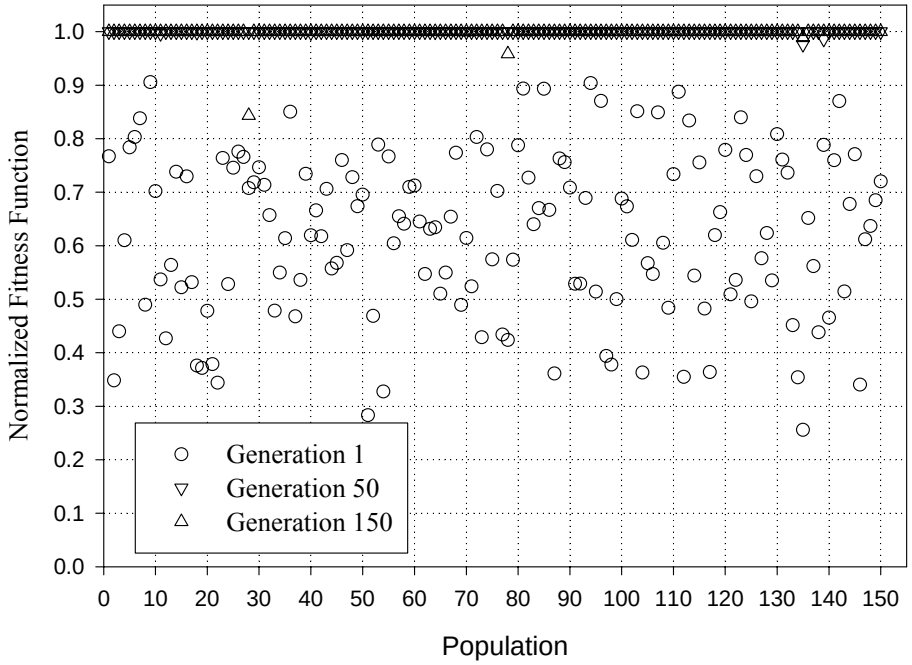

Figure 4 Distribution of normalized fitness function values for generation number one, fifty, and one hundred and fifty

Conclusions

This study shows the implementation of the genetic algorithm and the feasibility of this technique considering a three-lobe preloaded fluid film bearing in essence of developing the bearing configurations that optimize minimum power loss objective.
The overall results obtained in this study are superior to those from a gradient-based optimization method. Instead of using a starting point from which progress is made toward the identification of the values of the design variables that optimize the objective as in the numerical optimization method, the genetic algorithm uses an entire population of points, moves the population in the direction of the optimum, and

tries continuously to refine a population of solutions.

The genetic algorithm has been shown to be capable of solving complex problems where numerical methods have experienced difficulties. Thus, the genetic algorithm provides the designer an alternative design optimization approach to bearings design and, it could be used for an initial search followed by traditional methods to locate the optimum.

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