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Log-normal and Negative Binomial Distributions of DONUT Data

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Abstract

Since charged particle multiplicity distributions provide important results about the interactions of particles it has been studied in various experiments so far. The analysis of multiplicity distributions, which can be easily obtained in experiments, is therefore very important. There are several parametric models to describe the charged particle multiplicity distributions in literature. Among them, log-normal distribution and negative binomial distribution are the most well-known parametric models. In this study, negative binomial and log-normal distributions are compared and tested in neutrino interactions produced in Direct Observation of Nu-Tau experiment. This analysis was carried out using the real experimental results of the Direct Observation of Nu-Tau experiment collaboration. The results of the analysis show that the neutrino data, reported by Direct Observation of Nu-Tau Collaboration, is well described by negative binomial and log-normal distribution. In terms of fit parameters, the χ/ndf value of the both distribution NBD and LND are close to unity but log-normal distribution data slightly better than negative binomial distribution.

Keywords: Negative binomial, log-normal, multiplicity.

1. Introduction

In particle physics experiments, the multiplicity distribution is an interesting parameter that can be easily obtained in the experiment, but it keeps information about the characteristics of the interaction. To reveal the nature of the interaction, it is necessary to understand the behavior of multiplicity distribution. Several measurements have been collected so far in many experiments at different energies and on different reactions to understand the behavior of charged particle multiplicity distributions. In particular, the shape of the multiplicity distribution is very important as it provides information about the particle production mechanism. Therefore, various theoretical, phenomenological models and rules are proposed to parameterize charged particle multiplicity distributions in the literature. However, results from experiments showed that some parametric models are more successful in describing the data and its energy dependence up to now [1-7].

Among the parametric models, log-normal distribution and negative binomial distribution are the most well-known parametric models, so this study focuses on these two models. The log normal distribution, is a statistical function has a longer tail, more peaked than the negative binomial distribution. It is very common to encounter this distribution in nature. Therefore, it has been found to be applicable in a number of different fields like population ecology, income distribution in some economies and multi-particle production etc. [8,9].

The Log-Normal Distribution (LND) is defined as;

$$P(x) = \frac{1}{x\sqrt{2\pi s^2}} e^{-(lnx-m)^2/2s^2} \quad \mathbf{(1.1)}$$

and here m and s are the free parameters. When multi-particle production is analyzed, the log normal distribution arises in naturally so this leads an important and simple physical interpretation. In 1990, R. Szwed et al. has showed that, LND describes the e^+e^- multiplicity data very well [10,11]. Then, Apparatus for LEP PHysics (ALEPH) Collaboration proved that, LND gives a proper description of the charged particle multiplicity distribution of e^+e^- annihilation in 1991 [12]. In addition, the studies on of νp, $\bar{\nu}p$, and $\bar{p}p$ interactions showed that, charged particle multiplicity data can be parameterized by a log-normal distribution [11,13-15]. Although, the LND has been tested in many experiments, it has been studied very rarely in neutrino interactions.

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To describe the charged particle multiplicity, the other parametric model is Negative Binomial Distribution (NBD) which is a statistical distribution and defined as;

$$P(n;k,\bar{n}) = \left(\frac{\bar{n}+k-1}{\bar{n}}\right) p^{\bar{n}}(1-p)^{k} \quad \textbf{(1.2)}$$

where k and $\bar{n}$ are free parameters of the distribution with p=$(\bar{n}/k)$/($1+(\bar{n}/k)$). It is known that the first application was made to the cosmic-ray muon data by McKeown and Wolfendale in 1966 [16]. In 1985, the UA5 Collaboration obtained a remarkably good negative binomial fit in the $\bar{p}p$ collision and this made an overwhelming impression [17-19]. In the same year, it was shown that NBD was describe the charged multiplicity distribution of e^+e^- annihilation excellently [20]. Similar results were obtained for the charged particle multiplicity distribution of μp and pp collisions [21,22]. Recently, The Oscillation Project with Emulsion-Tracking Apparatus (OPERA) Collaboration also tested the NBD distribution and published the result that NBD describes the charged particle multiplicity in neutrino interactions very well [6].

Previously, neutrino multiplicity data of Direct Observation of Nu-Tau (DONUT) experiment is compared and tested for KNO-G scaling and the validation of data has been shown [23]. In this study, charged particle multiplicity distribution of neutrino interaction produced in DONUT experiment is analyzed for NBD and LND. Although, these models have been tested in several experiments before, this is the first study for DONUT which is an emulsion based-neutrino experiment.

2. Experimental Data and Analysis

The DONUT experiment was design to observe directly and study about ν_τCC interactions in the SPS Wide Band Neutrino Beam. For this purpose, a number of 800GeV protons from the Tevatron collide into the beam dump so a prompt neutrino beam, composed of ν_μ(%60), ν_e(%35), ν_τ(%5) neutrinos with 53GeV average energy, was created. Then charm particles (D_s) are produced and decays to an anti-tau neutrino and a tau lepton. The tau neutrino, produces in the decays of tau, travels 36 m to reach the emulsion target. The experiment used an emulsion/counter-hybrid-detector and the target was composed of nuclear emulsion as a three-dimensional tracking device. Because of its sub-micron accuracy and high spatial resolution, nuclear emulsion is a very powerful technique for the detection of short-lived tau particle. So, three emulsion target designs were used in the experiment. Each design had different plastic base layer and emulsion layer thickness, called as ECC200, ECC800 and Bulk type. The combination of emulsion designs intent to increase the amount of mass while increasing the precision for tau particle. The first design, (ECC 200) composed of 1 mm thick steel sheets interleaved with emulsions having 100 μm thick emulsion layers on both size 200 μm plastic base. The second design (ECC800) is the same as the first design, only the thickness of the plastic base is 800 μm. The bulk design consists of only 350 μm thick emulsion layers on both size of 90 μm plastic base without steel plates.

DONUT experiment collected data during 1997 and published first and the most important results in 2000. Thanks to the emulsion technology and the detector identified 578 neutrino interactions and the collaboration observed 9 $\nu\tau$CC events with background of 1.5 events. Based on these data, they announced the discovery of the tau particle in 2000 [24-27].

1. Negative Binomial and Log-normal Distributions

The NBD provides a convenient framework for multiplicity distributions with two free parameters. In fact, the two free parameters make NB distribution a highly flexible distribution [28,29]. In order to test negative binomial distribution, the data of located multiplicity distributions of charged particles produced in the DONUT experiment obtained directly from the ref [24] which is given in Figure 1.

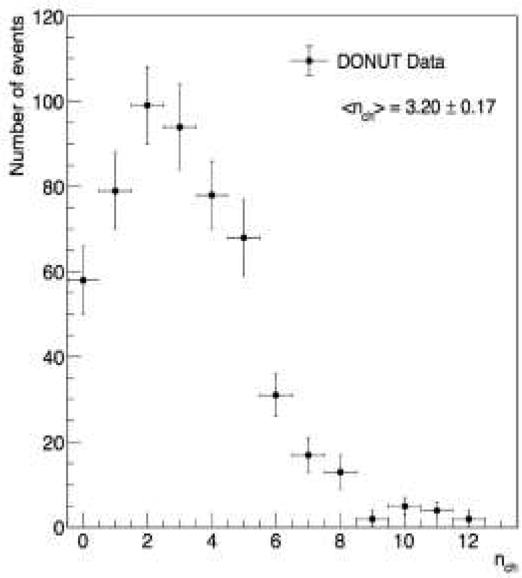

Figure 1. The located multiplicity distributions of charged particles produced in the DONUT experiment.

Then NBD fit applied to the P(n) vs. n distribution of charged particles and the parameters $\bar{n}$ and k are obtained from this fit directly. The fitting procedure was applied, using the χ^2 method, until the best fit was achieved. The data and superimposed fit line are given in Figure 2 and the values of parameters obtained from negative binomial fit is given in Table 1. It can be seen that the fit line shows good agreement with the data set.

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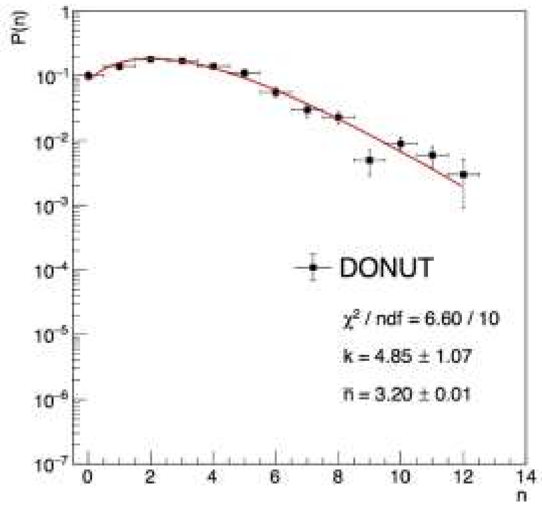

Figure 2. The data distribution with the negative binomial fit.

Another parametric model that is well known and has many applications in the literature is LND [30].

Since shape of charged multiplicity distributions is showed well described by LND, in this study LND fit applied to the data of located multiplicity distributions of charged particles produced in the DONUT experiment.

Table 1. The negative binomial fit results.

Applied Fit	$\bar{n}$	k	χ^2/ndf
Negative Binomial	3.20±0.01	4.85±1.07	6.60/10

The fitting procedure was applied, using the χ^2method, until the best fit was achieved. The data and the resulting fit are shown in Figure 3 and fit parameters are given in Table 2. It is observed that, LND fit seen to be very good which reflects the shape of charged particle multiplicity distribution.

Table 2. The log-normal fit results.

Applied Fit	m	s	χ^2/ndf
Log-normal	1.80±0.02	0.38±0.01	8.52/10

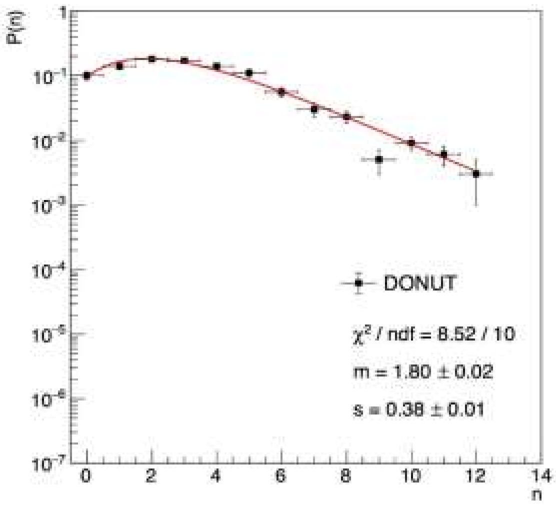

Figure 3. The data distribution with the log-normal fit.

3. Conclusion

In this study, the charged particle multiplicity data of DONUT experiment tested for two well-known parametric models which are log-normal distribution and negative binomial distribution. For the analysis, the real data directly obtained from the DONUT reference paper [24], NBD **(1.1)** and LND **(1.2)** fits were applied one by one until the best fit was achieved and the results are presented in the form of tables. It has been shown that, charged particle multiplicity distribution of DONUT experiment can be described in terms of the k and $\bar{n}$ parameters of NBD and can be described in terms of m and s parameters of LND. Although both fit lines show good agreement with data, χ^2/ndf results shows that log-normal distribution gives a better description of data than the negative binomial distribution. The χ^2/ndf values of NBD and LND are close to unity, but the fit value of the lognormal distribution are relatively better for DONUT neutrino interactions. At the same time, the dependence of the free parameters on energy are reported in the tables for the neutrino interactions.

Author's Contributions

Çağın Kamışçıoğlu: Drafted and wrote the manuscript, performed, and interpreted the whole experiments and analysis.

Ethics

There are no ethical issues after the publication of this manuscript.

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References

[1]. Ansorge RE., Åsman B., Burow L., Carlson P., DeWolf RS. et al, 1989. Charged Particle Multiplicity Distributions at 200-GeV and 900-GeV Center-Of-Mass Energy *Z. Journal of Physics C.*; 43, 357. doi: 10.1007/BF01506531.

[2]. Berger Ch., Lackas W., Raupach F., Wagner W., Alexander G. et al., 1978. A study of jets in electron positron annihilation into hadrons in the energy range 3.1 to 9.5 GeV (PLUTOColl.) *Physics Letters B*; 78,176. doi:10.1016/0370-2693(78)90377-5.

[3]. Brandelik RW, Braunschweig K, Gather V, Kadansky K, Lübelsmeyer et al, 1980. (TASSOColl.) Rapid growth of charged particle multiplicity in high energy e+e− annihilations. *Physics Letters B*; 89 (3-4): 418-422. doi:10.1016/0370- 2693(80)90156-2.

[4]. Allen P, Blietschau J, Grassler H, Lanske D, Schulte R, et al, 1981. Multiplicity distributions in neutrino-hydrogen interactions. *Nuclear Physics B*; 181 (3): 385-402. doi:10.1016/0550-3213(81)90532-0.

[5]. Kayis-Topaksu A, Önengüt G, Dantzig van R, Jong de M, Oldeman RGC et al., 2007. (CHORUS Coll.) Charged Particle Multiplicities in Charged-Current Neutrino and Anti-Neutrino Nucleus Interactions. *The European Physical Journal C*; 51: 775-785. doi:10.1140/epjc/s10052-007-0366-8.

[6]. Agafonova N, Aleksandrov A, Anokhina A, AokiS, Ariga A et al., 2018. (OPERA Coll.). Study of Charged Hadron Multiplicities in Charged-Current Neutrino-Lead Interactions in the OPERA Dedector. *The European Physical Journal C*; 78: 62. doi:10.1140/epjc/s10052-017-5509-y

[7]. Kamışcıoğlu Ç., 2020. Study of KNO scaling in the emulsion based neutrino experiments. *Turkish Journal of Physics*; 44:222-228. doi: 10.3906/fiz-1912-12.

[8]. Carius S. and Ingelman G., 1990. The log-normal distribution for cascade multiplicities in hadron collisions.*Physics Letters B;* 252:657. doi:10.1016/0370-2693(90)90500-6.

[9]. Huberman BA. 1988. The Ecology of Computation.North Holland (March 15, 1988).342 pages.

[10]. Szwed R.,Wrochna G., Wroblewski AK., 1991. New AMY and DELPHI multiplicity data and the lognormal distribution. *Modern Physics Letters A*;6: 245-258. doi:10.1142/S021773239100021X.

[11]. Szwed R.,Wrochna G., Wroblewski AK., 1990. Genesis Of The Lognormal Multiplicity Distribution In The e+ e- Collisions And Other Stochastic Processes. *Modern Physics Letters A*;5:1851-1870. doi:10.1142/S0217732390002110.

[12]. Decamp D., Deschizeaux B, Goy C, Lees JP, Minard N et al., 1991. Measurement of the charged particle multiplicity distribution in hadronic Z decays. *Physics Letters B* 273(1-2):181-192. doi:https://doi.org/10.1016/0370-2693(91)90575-B.

[13]. Jones GT, Jones RWL, Kennedy BW, Morrison DRO, Mobayyen MM. et al., 1992. Multiplicity distributions of charged hadrons in νp and charged current interactions. *Zeitschrift für Physik C Particles and Fields.* 54:45–54.doi:10.1007/BF01881707.

[14]. Buskulic D, Casper D, Bonis DI, Decamp D, Ghez P et al., 1995. Measurements of the charged particle multiplicity distribution in restricted rapidity intervals.*Zeitschrift für Physik C Particles and Fields* 69:15–25.doi:/10.1007/BF02907382.

[15]. Acton PD, Alexander G, Allison J, Allport PP, Anderson KJ et al., 1992. A Study of charged particle multiplicities in hadronic decays of the Z0. *Zeitschrift für Physik C.* 53:539-554. doi:10.1007/BF01559731.

[16]. MacKeown PK. and Wolfendale AW., 1966. The interpretation of the charge ratio of cosmic-ray muons. *Proceedings of the Physical Society,* 553.doi:10.1088/0370-1328/89/3/312.

[17]. Alner, GJ, Alpgard K, Anderer P, Ansorge RE, Asman B. et al., 1985.Multiplicity distributions in different pseudorapidity intervals at a CMS energy of 540 GeV. *Physics Letters B,*160(1-3): 193-198.doi: 10.1016/0370-2693(85)91491-1.

[18]. Alner, GJ, Alpgard K, Anderer P, Ansorge RE, Asman B. et al., 1985.A new empirical regularity for multiplicity distributions in place of KNO scaling. *Physics Letters B;*160(1-3): 199-206.doi: 10.1016/0370-2693(85)91492-3.

[19]. Alner GJ, Ansorge RE, Asman B, Bockmann K, Booth CN et al., 1986. Scaling Violations in Multiplicity Distributions at 200-GeV and 900-GeV. *.Physics .Letters .B,* 167:476-480.doi:10.1016/0370-2693(86)91304-3.

[20]. Chew CK and Lim YK., 1985.Charged-particle multiplicity distributions in \$e^+\$\$e^-\$ annihilation and negative binomial distributions. *.Physics Letters B,*163(1-4):257-260. doi:10.1016/0370-2693(85)90233-3.

[21]. Grosse-Oetringhaus JF, Reygers K., 2010.Charged-Particle Multiplicity in Proton-Proton Collisions. *Journal of Physics G* 37:083001.doi:10.1088/0954-3899/37/8/083001.

[22]. Arneodo M., 1987. Comparison of multiplicity distributions to the negative binomial distribution in muon-proton scattering. *Zeitschrift für Physik C Particles and Fields*; 35:335–345.doi:10.1007/BF01570769.

[23]. Kamışcıoğlu Ç., 2021. Study of KNO-G scaling in neutrino interactions. *Modern Physics Letters A.* 36(9):2150062.doi:10.1142/S0217732321500620.

[24]. Kodama K, Ushida N, Andreopoulos C, Saoulidou N, Tzanakos G et al., 2008. Final tau-neutrino results from the DONuT experiment. *PHYSICAL REVIEW D*;78:052002. doi: 10.1103/PhysRevD.78.052002.

[25]. Kodama K, Saoulidou N, Tzanakos G, Baller B, Lundber B et al., 2002. Detection and analysis of tau–neutrino interactions in DONUT emulsion target. *Nuclear Instruments and Methods in Physics Research A.* 493:45–66.doi:10.1016/S0168-9002(02)01555-3.

[26]. Kodama K, Ushida N, Andreopoulos C, Saoulidou N, Tzanakos G et al., 2001. Observation of tau neutrino interactions. *Physics Letters B*; 504: 218–224.doi:10.1016/S0370-2693(01)00307-0.

[27]. Kodama K, Andreopoulos C, Giokaris N, Saoulidou N, Tzanakos G et al., 2004. Identification of neutrino interactions using the DONUT spectrometer. *Nuclear Instruments and Methods in Physics Research A.;*516 :21–33.doi:10.1016/j.nima.2003.07.035.

[28]. Kittel W., De Wolf EA, 2005. Soft multihadron dynamics.World Scientific Press.

[29]. Szwed R.1986. Proceedings of the XVII international symposium on multiparticle dynamics.pp. 663.

[30]. Wroblewski AK, 1990..High-energy physics. Proceedings, 25th International Conference.pp. 125.Singapore.