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GENERALIZED FORMULAE FOR THE SHARED EQUITY HOME FINANCING MODEL

ORTAKLIĞA DAYALI EV FİNANSMANI MODELİ İÇİN GENELLEŞTİRİLMİŞ FORMÜLLER

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ABSTRACT

An alternative financing method, called Shared Equity Home Financing (SEHF) model, for today's world of finance to large purchases or business enterprises, such as houses, has received great interest in recent years. Especially, great applications can be seen in the Islamic countries. An explosive growth of applications are also experiencing in the United States and it has potential growth in Europe

In this article, after introducing the SEHF model, general formulae are derived for different cases which are additional payments with zero, an equal, and a geometric-gradient series for the model. In addition, illustrative examples for each case are presented.

ÖZET

Bir alternatif finansman modeli olarak Ortaklığa Dayalı Ev Finansmanı (ODEF) modeli son yıllarda günümüz dünyasının büyük çaplı satın almalarında veya işletme yatırımlarında, ev satın almaları gibi, büyük ilgi odağı olmuştur. Özellikle, büyük çaplı uygulamalar özellikle İslam ülkelerinde görülmektedir. Bununla ilgili uygulamalardaki büyük patlamalar Amerika Birleşik Devletleri'nde yaşanmaktadır ve Avrupa'da potansiyel büyümeye sahiptir.

Bu makalede, ODEF modeli tanıtıldıktan sonra, sıfır ilave ödemeli, eşit, ve geometrik eğimli seriler gibi farklı durumlar için genel formüller türetilmiştir. Ayrıca, her durum için örnekler verilmiştir.

Shared Equity Mortgage, Musharakah Mutanaqisah Partnership, Home Financing, Interest-free Mortgage.
Ortaklığa Dayalı Yatırım, Ev Finansmanı, Faizsiz İpotek.

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1. INTRODUCTION

The normal procedure to finance a purchase of a house is to apply for a mortgage. A buyer and a bank sign a mortgage contract that will require monthly payments over a period of years. The bank, of course, changes interest rate based on the amount of the unpaid principal, whose rate depends on prevailing market conditions and the time period of the loan. In the SEHF model, instead of making a loan and asking to sign a mortgage contract, the bank has an equity share in the house. Here the bank will require some down payment that will be initial equity share. Let's assume that the buyer makes the same down payment of 20 %, or $20,000, the bank puts up the remaining $80,000 for a $100,000 house. Now the buyer and the bank are co-owners. The buyer owns 20 % of the house and the bank owns 80 % of the house. There is no interest to be paid on the coop bank's capital but if the buyer occupies the house, the buyer will be required to pay rent to the owners. Of course, since the buyer is a part of the ownership, a part of that rental income comes back to him/her. At the outset, the bank will get 80 % of the rental payments and the buyer will get 20 %. But the buyer is also allowed to increase his/her ownership share at any time by making additional payments to the bank, in effect, buying out the bank's interest in the house. As the buyer does so, his/her proportionate share increases while the coop bank's share decreases and the distribution of the rent payments will change accordingly.

There is not much research available for the SEHF model in the literature. Abdul-Rahman [1], in Annual Harvard Islamic Finance Forum, states in his study that the SEHF is feasible for the United States. Thomas H. Greco [4], and Matthews and others [6] compared conventional mortgage models with the SEHF and concluded that the SEHF has some advantages over a conventional mortgage. General formulae where additional payments are constant are obtained by Meera and Razzak [7]. Numerical examples are given by Abidin [2], Hijazi and Hanif [5], Meera and Razzak [8], Rammal [9], and Siswantoro and Qoyyimah [10]. Models are developed with constant additional payments in the above works. There would be other models with variable additional payments too. In this study, general formulae for the model with additional payments as geometric gradient series are derived and models are supported with illustrative numerical examples.

2. THE SHARED EQUITY HOME FINANCING MODEL

The model calls for the financial institution to purchase a property jointly with a client. The client owns a title to the property with the financial institution holding a first-position lien. The financial agreement consists of two parts. The first is a loan agreement between the potential buyer and the financial institution, with the buyer providing, for instance, 10 % and financing entity 90 % housing capital. The client agrees to buy back the company's portion over a period of time. The second part is a lease agreement based on an agreed lease rate. Since the house is used as residence,

the buyer will pay fair market rent. A portion of this rent will go toward the rent of financing entity's ownership, and the other portion will go toward to buy out of financing entity's share. Eventually the customer has repaid enough to buy out the bank's share of the house, and the ownership passes to the customer. The key feature here is that the repayments are not based on pre-determined interest rates. At the end of each month both the buyer and the financing entity (or coop bank) will get rental income in respect to their shares. In order to hold 100 % of the property, the buyer either uses his rental payments to the coop bank or adds additional payments in addition to rental income.

There are five components of the SEHF model. These are payment numbers, additional payments, buyer's initial equity, coop bank's (or financing entity's) initial equity, and rental income. Moreover, early studies do not take into account additional payments with a geometric gradient series case. In the following sections, the formulae are developed for different additional payment cases.

The following notations are used in this study:

A : buyer's initial equity

B : financing entity's (or coop bank's) initial equity

C : purchasing price of the property, $C=A+B$

D_k : additional payments to the financing entity at the end of k^{th} month

D : first additional payment

E : monthly rental income of the property

E_k: rental income belong to buyer at the end of k^{th} month

n : number of payments

M_k: buyer's equity at the end of k^{th} month

3. GENERALIZED FORMULAE FOR ADDITIONAL PAYMENTS WITH A GEOMETRIC - GRADIENT SERIES

The case is called as a geometric gradient series payment when periodic payments increase or decrease over a time with a constant percentage.

Let g be the percentage change (or gradient) in the magnitude of the payment from one period to another. Then the following equations are obtained:

$$D_k = D(1+g)^{k-1}\ ,\ k=1,2,\cdots,n\,, \qquad (1)$$

$$M_k = M_{k-1}\left(1+\frac{E}{C}\right)+D_k$$

$$= M_{k-1}P + D_k \ , \ k = 1, 2, \cdots, n \, . \qquad (2)$$

The following expressions can be obtained from equation (2) for k= 0, 1, 2, …, n:

$$M_0 = A$$

$$M_1 = AP + D$$

$$M_2 = AP^2 + DP + D(1+g)$$
$$= AP^2 + D\left[P + (1+g)\right]$$

$$M_3 = AP^3 + D\left[P^2 + (1+g)P\right] + D(1+g)^2$$
$$= AP^3 + D\left[P^2 + (1+g)P + (1+g)^2\right]$$

$$M_4 = AP^4 + D\left[P^3 + (1+g)P^2 + (1+g)^2 P\right] + D(1+g)^3$$
$$= AP^4 + D\left[P^3 + (1+g)P^2 + (1+g)^2 P + (1+g)^3\right]$$

$$\vdots$$

$$M_j = AP^j + D\left(\sum_{k=0}^{j-1}(1+g)^k \, P^{j-k-1}\right)$$
$$= AP^j + \left(\frac{DC}{gC-E}\right)\left[(1+g)^j - P^j\right] \ , \ j = 1, 2, \cdots, n \qquad (3)$$

where $P = 1 + \dfrac{E}{C}$.

Since the buyer's equity will be C at the end of n^{th} month, the following equation is obtained from equation (3):

$M_n = C$ or

$$C = AP^n + \left(\frac{DC}{gC-E}\right)\left[(1+g)^n - P^n\right]. \qquad (4)$$

The following formulae are the result of series derivations from equation (4):

$$A = \frac{C(gC-E) - DC\left[(1+g)^n - P^n\right]}{(gC-E)P^n}, \qquad (5)$$

$$D = \frac{\left(gC - E\right)\left(C - AP^n\right)}{C\left[\left(1+g\right)^n - P^n\right]}. \qquad (6)$$

3.1. Example-1

A family wants to buy a $100,000 house and goes to a financing entity (or coop bank) for financing. In this scenario, let's assume that the family makes a down payment of 20 %, or $20,000, and 80 %, or $80,000, of the house would be financed by the financing entity (or coop bank). Now, the family and the financing entity are co-owners of the house. At the beginning, the family owns 20 % of the house and the financing entity owns 80 % of the house. When family occupies the house, it will be required to pay a rent to the financing entity. Since the family is a part of the ownership, part of that rental income comes back to the family. At the outset, the financing entity will get 80 % of the rental payments and family will get the remaining 20 %. Each month, a portion of this rent goes toward the rent of the financing entity's ownership, and another portion goes toward to buy out of the financing entity's share. Therefore, the buyer's proportion of share increases and the financing entity's share decreases. If family wants to end mortgage period within 10 years (120 months) when monthly additional payments increases 0.4 % what would be the first monthly additional payments? Property's monthly rent is $500.

Assuming that *A=$20,000, B=$80,000, C=$100,000, n=120 months, E=$500* and *g=0.004,* what would be the initial additional payment (*D*)?

D is calculated as *$310.50* from equation (6). The Table 1 summarizes the buyer's equity, rental share, additional payments, and the financing entity's equity and rental income for mortgage period of 120 months.

Table 1: Solution Result for Additional Payments with a Geometric Gradient Series

Payment number (k)	The Buyer				The Financing Entity (Bank)		
	Equity (M_k) [from (3)]	Equity %	Rental Share (E_k)	Additional Payment (D_k) [from (1)]	Equity (C-M_k)	Equity %	Rental Share (E-E_k)
0	$20,000.00	20.000	---	----	$80,000.00	80.00	-----
1	$20,410.50	20.411	$100.00	$310.50	$79,589.50	79.590	$400.00
2	$20,824.30	20.824	$102.05	$311.74	$79,175.70	79.176	$397.95
3	$21,241.41	21.241	$104.12	$312.99	$78,758.59	78.759	$395.88
4	$21,661.86	21.662	$106.21	$314.24	$78,338.14	78.338	$393.79
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25	$31,301.85	31.302	$154.03	$341.72	$68,698.15	68.698	$345.97
26	$31,801.44	31.801	$156.51	$343.09	$68,198.56	68.199	$343.49
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85	$69,054.27	69.054	$341.39	$434.21	$30,945.73	30.946	$158.61
86	$69,835.48	69.835	$345.27	$435.94	$30,164.52	30.165	$154.73
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118	$98,018.24	98.018	$485.19	$495.35	$1,981.76	1.982	$14.81
119	$99,005.66	99.006	$490.09	$497.33	$994.34	0.994	$9.91
120	$100,000.00	100.00	$495.03	$499.32	$0.00	0.00	$4.97

4. GENERALIZED FORMULAE FOR ADDITIONAL PAYMENTS WITH AN EQUAL SERIES

When g is equal to zero in equation (4), the model with geometric addition becomes the model with constant payments as follows:

$$C = \left(A + \frac{DC}{E} \right) P^n - \frac{DC}{E} . \qquad (7)$$

There are five components (A, C, D, E, n) in equation (7). When any four of them are known, it is easy to determine the fifth component. From equation (7), the following formulae are obtained:

$$n = \frac{\log\left[\frac{C(D+E)}{(AE)+(DC)} \right]}{\log(P)}, \qquad (8)$$

$$D = \frac{E(C - AP^n)}{C(P^n - 1)}, \qquad (9)$$

$$A = \frac{C}{E}\left(\frac{D+E}{P^n} - D\right). \qquad (10)$$

Since the components *E* and *C* cannot be obtained analytically, they can be obtained by using trial and error method from equation (7).

4.1. Example-2

What would be the constant additional payment when the additional payment does not increase in Example-1?

In this example: *A=$20,000, B=$80,000, C=$100,000, n=120 months, E=$500, D=?*

The monthly additional payments, *D*, are *$388.164,* from equation (9). The Table 2 summarizes the buyer's equity, rental share, additional payments, and the financing entity's equity and rental income for mortgage period of 120 months.

Table 2: Solution Results for Additional Payments with an Equal Series

Payment number (*k*)	**The Buyer**				**The Financing Entity (or coop bank)**		
	Equity (M_k) [from (3)]	**Equity %**	**Rental Share (E_k)**	**Additional Payment (D_k)**	**Equity (C-M_k)**	**Equity %**	**Rental Share (E-E_k)**
0	$20,000.00	20.000	---	----	$80,000.00	80.000	-----
1	$20,488.16	20.488	$100.00	$388.164	$79,511.84	79.512	$400.00
2	$20,978.77	20.979	$102.44	$388.164	$79,021.23	79.021	$397.56
3	$21,471.83	21.472	$104.89	$388.164	$78,528.17	78.528	$395.11
4	$21,967.35	21.967	$107.36	$388.164	$78,032.65	78.033	$392.64
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25	$32,965.20	32.965	$162.07	$388.164	$67,034.80	67.035	$337.93
26	$33,518.19	33.518	$164.83	$388.164	$66,481.81	66.482	$335.17
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85	$71,547.34	71.547	$354.03	$388.164	$28,452.66	28.453	$145.97
86	$72,293.24	72.293	$357.74	$388.164	$27,706.76	27.707	$142.26
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118	$98,236.91	98.237	$486.81	$388.164	$1,763.09	1.763	$13.19
119	$99,116.25	99.116	$491.18	$388.164	$883.75	0.884	$8.82
120	$100,000.00	100.00	$495.58	$388.164	$0.00	0.000	$4.42

5. GENERALIZED FORMULAE FOR ZERO ADDITIONAL PAYMENTS

In some circumstances, having 100 % of the property can be succeeded only with the rental income shares of the buyer. In this case, D will be equal to zero in equation (7). Then, the following equation will be obtained:

$$C = AP^n, \tag{11}$$

The following formulae can be obtained from equation (11):

$$n = \frac{\log\left(\frac{C}{A}\right)}{\log(P)}, \tag{12}$$

$$E = C\left[\left(\frac{C}{A}\right)^{1/n} - 1\right], \tag{13}$$

$$A = \frac{C}{P^n}. \tag{14}$$

5.1. Example-3

Assuming that the parameters *C, E* and *n* have not been changed in Example-2. Then, what would be the buyer's initial equity payment?

Since *C=$100,000, n=120 months, and E=$500, A* is calculated as *$54,963.27* by using equation (14). The Table 3 summarizes the buyer's equity, rental share, additional payments, and the financing entity's equity and rental income for mortgage period of 120 months.

Table 3: Solution Results for Zero Additional Payments

Payment number (k)	**The Buyer**			**The Financing Entity (or coop bank)**		
	Equity (M_k) [from (3)]	**Equity %**	**Rental Share (E_k)**	**Equity (C-M_k)**	**Equity %**	**Rental Share (E-E_k)**
0	$54,963.27	54.963	---	$45,036.73	45.037	-----
1	$55,238.09	55.238	$274.82	$44,761.91	44.762	$225.18
2	$55,514.28	55.514	$276.19	$44,485.72	44.486	$223.81
3	$55,791.85	55.792	$277.57	$44,208.15	44.208	$222.43
4	$56,070.81	56.071	$278.96	$43,929.19	43.929	$221.04
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25	$62,262.15	62.262	$309.76	$37,737.85	37.738	$190.24
26	$62,573.46	62.573	$311.31	$37,426.54	37.427	$188.69
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85	$83,982.31	83.982	$417.82	$16,017.69	16.018	$82.18
86	$84,402.23	84.402	$419.91	$15,597.77	15.598	$80.09
---	-------	------	-------	-------	------	-----
118	$99,007.45	99.007	$492.57	$992.55	0.993	$7.43
119	$99,502.49	99.502	$495.04	$497.51	0.498	$4.96
120	$100,000.00	100.00	$497.51	$0.00	0.000	$2.49

6. CONCLUSION

The religious law forbids observant Muslims to pay interest. As a result, financing homes for Muslim families presents a challenge to the mortgage industry. Recognizing the need to expand home ownership opportunities for this market, two of the mortgage loan companies, Freddie Mac and Fannie Mae, are delivered around $110 million in 2002 to finance Islamic home mortgages in the United States. Also, HSBC bank entered this huge business with $20 million in New York State in 2002 [3]. It is expected that Islamic mortgages will experience explosive growth in the west in the near future.

In this paper, we formulized the current system and extended it for different cases of SEHF model. Each case has been supported by illustrative examples. It is clear that the SEHF has been an alternative home financing model not only for Muslims but also for individuals who seek alternatives to conventional mortgages.

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