3D Modeling of Gravity Anomalies Using 2D Synthetic Models

Hasan CAVSAK, Ali ELMAS

Karadeniz Technical University, Engineering Faculty,Geophysical Engineering Department, Trabzon, Turkey
cavsak@ktu.edu.tr, elmas@ktu.edu.tr

Abstract- The advantages of 3D gravity studies have been investigated in this study. For this purpose, first of all, 3D four different synthetic models geometry have been designed in order to determine the importance of 3D gravity study. The studies have been done with the first mass model have been described in this study. Then, 3D gravity anomalies have been calculated by using a special algorithm, for this model geometry by intensity, the algorithm results have been used in the next 2D and 3D inversion algorithm as measurement value. The calculated densities by different inversion algorithm and the other calculated parameters, as shown in the following order, have been compared with each other:

1. The forward solution 2D models have been compared with the forward solution 3D simulation models. This test has been done in order to prove 2D and 3D simulation model geometries are same.

2. The forward solution 3D original model has been compared with the inverse solution 2D and 3D simulation models. Here, as expected, the 2D and 3D simulation model intensities are equal to each other, but 3D original model intensity is different from the others.

3. The forward solution 3D original model has been compared with the inverse solution 3D original model. This final step has been done for the test.

Thus, according to the manner of formation, in item 2 of the above mentioned, why 3D gravity study should be done instead of 2D gravity study have been worked to explain. Giving accurate results of gravity studies of the first condition, model geometries have been defined as 3D. As a result, the following can be said that 3D gravity studies should be done instead of 2D gravity studies.

Keywords- Gravity, Modeling, Inversion, Anomaly.

Gravite Yönteminde 2B Sentetik Modellerin 3B Tasarımlarıyla Testi

Özet- Petrol aramalarında, petrolle ilgili yapıyı, fay ve tuz domlarını ortaya çıkarmak, sismik etütlere yardımcı olmak amacıyla gravite yöntemi uygulanmaktadır. Ayrıca, muhtelif tektonik üniteleri tetkik etmek, büyük fay sistemlerini ortaya çıkarmak, genç tabakalar tarafından örtülmüş havzalarda magmatik kütle sınırını araştırmak, yer kabuğunun kalınlık ve strüktürlerini incelemek amacıyla da gravite yönteminden yararlanılmaktadır. Yer altı yapısı ne kadar iyi bilinirse, bu yapının neden olacagı deprem afetlerinden korunmak, aynı oranda mümkün olur. Ayrıca yer altı zenginliklerimizden de azami derecede faydalanabiliriz.

Bu amaçla 3B gravite çalışmalarının önemini belirlemek için önce üç boyutlu dört değişik sentetik model geometri tasarlanmıştır. Sonra yoğunluk kabulleriyle bu model geometriler için özel bir algoritma (Çavşak H. 1992) kullanılarak üç boyutlu gravite anomalileri hesaplanmış ve bunlar, daha sonraki iki ve üç boyutlu inverziyon hesaplarında ölçü değerleri olarak kullanılmıştır.

Değişik inverziyon hesaplarıyla bulunan yoğunluklar ve diğer hesaplanan parametreler, aşağıdaki sırada görüldüğü gibi birbirleriyle karşılaştırılmıştır :

1. Düz çözüm 2B model, düz çözüm 3B similasyon modeli ile karşılaştırılmıştır. Bu test 2B ve 3B similasyon model geometrilerin aynı olduğunu kanıtlamak acıyla yapılmıştır.

2. Düz çözüm 3B orijinal model, ters çözüm 2B ve 3B similasyon modeller ile karşılaştırılmıştır. Burada, beklendiği üzere, 2B ve 3B similasyon model yoğunluklar birbirlerine eşit ama 3B orijinal model yoğunluğundan farklı çıkmışlardır.

3. Düz çözüm 3B orijinal model, ters çözüm 3B orjinal model ile karşılaştırılmıştır. Bu da test için yapılmıştır.

Böylece formasyonların şekline göre, yukarda madde 2 de belirtildiği üzere, iki boyutlu gravite çalışmaları yerine üç boyutlu gravite çalışmaları yapmanın ne kadar gerekli olduğu açıklanmaya çalışılmıştır. Gravite çalışmalarının doğru sonuçlar vermesinin ilk şartı, model geometrilerin 3B olarak tanımlanmasıdır. Ayrıca bu tanımın ayrıntılı olarak yapılması, hesapların yapılacağı profillerin gerektiği kadar ve en uygun yerlerde seçilmesi, sonuçların güvenirliğini daha üst seviyelere çeker. Sonuç olarak bu çalışma, 2B hesapların ne düzeyde güvenilir olmadığını, diğer bir deyişle ne düzeyde kabul edilebilir olduğunu ortaya koymuştur.

Anahtar Kelimeler- Gravite, Modelleme, İnversiyon İşlemi, Anomali.

1. INTRODUCTION

The results were compared with each other, so that the importance of 3D model algorithms were highlighted [1]. In large scale engineering surveys, gravity method has been used to locate large fault zones, buried geological structures, buried channels, bedrock and thickness of the crust [1, 2]. The world-wide geophysical research has been done mostly with 2D modeling [3]. In addition, the gravity method is applied in petroleum search, oil-related construction, to bring out fault and salt structure, to help seismic surveys [4, 5]. Of course, the results are level trusted. Though 2D models require less data and time, we should direct to 3D underground modeling requires more data and effort [6]. While the underground model geometry definition is performed in gravity studies, to be aware very important issues were investigated [7].

Various 2D and 3D gravity inversion algorithms were studied by using synthetic geometries in this study.

2. MATERIALS AND METHODS

In the 3D study, mass surfaces were defined by dividing the triangle surfaces. The more triangle surface is taken, the more precise definition of mass is made. Triangular pyramids were taken into consideration as the 3D master model. This model is formed between each triangle surface and observation point.

2.1. 3D Gravity Algorithm

The 3D gravity algorithm is explained with outlines. The outlines of the 3D gravity algorithm were got from the Ph.D. Thesis (Çavşak 1992). Parameters in equation are obtained after coordinate transformation. The parameters are obtained after coordinate transformation and shown in Fig.1.

The gravity potential is expressed as [1]
Normal unit vector of the vertical (z) component

$$\Delta U = \frac{G\cdot\rho}{h}\int_{\eta_A}^{\eta_C}\int_{\xi^{(1)}}^{\xi^{(2)}}\int_{\zeta=0}^{h}\frac{\zeta\cdot d\zeta\cdot d\xi\cdot d\eta}{(\xi^2+\eta^2+h^2)^{1/2}}\,, \qquad (1)$$

where h is height of tetrahedral; ξ, η and ζ are the coordinate values that define tetrahedral (Fig.2).

With an analytical solution of Eq.(1), we obtain [1]

$$\Delta U = \frac{1}{2}G\cdot\rho\cdot h\cdot F(\eta,\xi)\;. \qquad (2)$$

$F(\eta,\xi)$ is defined with Formula 3 [1]

$$F(\eta,\xi)=\left|\begin{array}{l}\eta\cdot\ln\left[\xi+\sqrt{\xi^2+\eta^2+h^2}\right]+\\ +\xi_2\cdot\cos\beta\cdot\ln\left[\sqrt{\xi^2+\eta^2+h^2}+\dfrac{\eta}{\cos\beta}+\xi_2\cdot\sin\beta\right]\\ +\zeta\cdot\arctan\left[\dfrac{h^2\cdot\tan\beta-\xi_2\cdot\eta}{h\cdot\sqrt{\xi^2+\eta^2+h^2}}\right]\end{array}\right|_{\xi^{(1)^2},\eta_A}^{\xi^{(2)^2},\eta_C}\;. \qquad (3)$$

Y is defined for the whole triangles pyramid as follows (Formula 4) [1]

$$Y = F_1\left(\eta_C,\xi^{(2)}\right)-F_2\left(\eta_A,\xi^{(2)}\right)-F_3\left(\eta_C,\xi^{(1)}\right)+F_4\left(\eta_A,\xi^{(1)}\right)\;. \qquad (4)$$

Taking the first vertical derivative of gravity potential, gravity effect is written as

$$\Delta g = \frac{\partial}{\partial z}(\Delta U)\,, \qquad (5)$$

with derivatives of open expression [1];

$$\Delta g = \frac{1}{2}G\cdot\rho\left\{\frac{\partial}{\partial z}(h)\cdot Y+\frac{\partial}{\partial z}(Y)\cdot h\right\}. \qquad (6)$$

$$\hat{\zeta}_z = \frac{\partial}{\partial z}(h) \text{ ve } Y' = \frac{\partial}{\partial z}(Y) \tag{7}$$

is written;

$$\Delta g = \frac{1}{2} G \cdot \rho \cdot \sum_{i=1}^{n} \left(\hat{\zeta}_{z_i} \cdot Y_i + Y_i' \cdot h_i \right). \tag{8}$$

Gravity can be shown in this way

$$\sum_{i=1}^{n} \left(\hat{\zeta}_{z_i} \cdot Y_i \right) = \sum_{i=1}^{n} \left(Y_i' \cdot h_i \right). \tag{9}$$

The gravity effect after above the required equalities shortly;

$$\Delta g = G \cdot \rho \cdot \sum_{i=1}^{n} \left(\hat{\zeta}_{z_i} \cdot Y_i \right), \tag{10}$$

formulation can be [1].

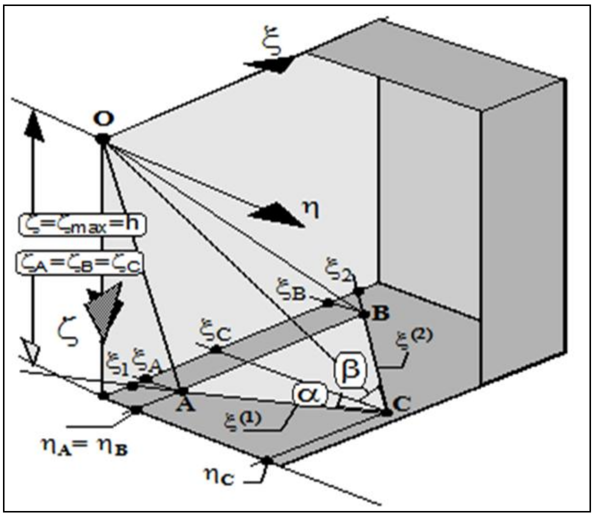

Figure 1. The model geometry is created between triangle mass surface (triangle pyramid) and observation point.

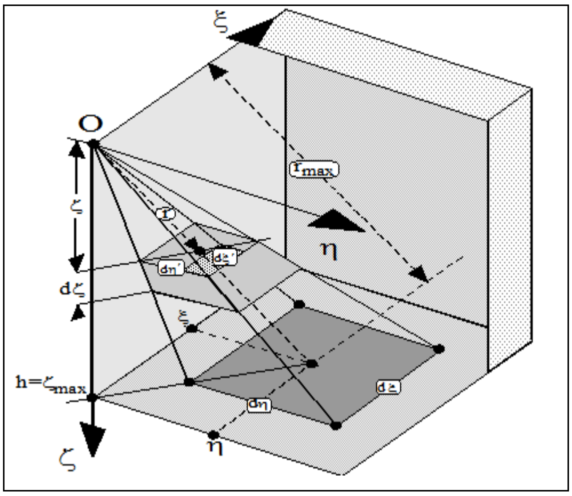

Figure 2. Schematic illustration and parameters of the integration in Eq. (2)

2.2. 3D Gravity Inversion

It performs the smallest sum of the squares of the difference between the observed and calculated values, the least squares method, forms the basis of inversion. Methods are divided into linear and nonlinear solutions. In this study, formation density was taken into consideration as constant. That was a linear solution was implemented. By taking derivatives of the squares of the difference between the observed and calculated values according to parameters, derivative equations to be equal to zero, is intended to perform the smallest mistake.

I_1, I_2, I_3 are taken into consideration to be measurement values,

$$\begin{aligned} l_1 &= a_1x+b_1y+c_1z \\ l_2 &= a_2x+b_2y+c_2z \\ &\vdots \\ l_n &= a_nx+b_ny+c_nz \end{aligned} \quad , \qquad (11)$$

a small error is made absolutely in measurements. These errors must be added to the equation.
Errors are placed to measurement equations,

$$\begin{aligned} l_1 &= a_1x+b_1y+c_1z+\vartheta_1 \\ l_2 &= a_2x+b_2y+c_2z+\vartheta_1 \\ &\vdots \\ l_n &= a_nx+b_ny+c_nz+\vartheta_n \end{aligned} \quad , \qquad (12)$$

ϑ_n error amounts is put to the left side of the equations,

$$\begin{aligned} \vartheta_1 &= l_1 - a_1x - b_1y - c_1z \\ \vartheta_2 &= l_2 - a_2x - b_2y - c_2z \\ &\vdots \\ \vartheta_n &= l_n - a_nx - b_ny - c_nz \end{aligned} \quad , \qquad (13)$$

the aim is to zero the sum of the error.

$$\sum_{i=1}^{n} \vartheta_n = 0 \ , \qquad (14)$$

for this process in mathematics,

$$\sum_{i=1}^{n} (\vartheta_i^2) = \min \ , \qquad (15)$$

If this equality will be written in the general case,

$$\sum_{i=1}^{n} (\vartheta_i^2) = \sum \left(l_i - a_i x - b_i z - c_i y \right)^2 = \min \qquad (16)$$

then

$$\left[\vartheta.\vartheta\right] = \left[aa\right]x^2 + 2\left[ab\right]xy + 2\left[ac\right]xz - 2\left[al\right]x + \left[bb\right]y^2 + 2\left[bc\right]yz - 2\left[bl\right]y \ , \qquad (17)$$
$$+\left[cc\right]z^2 - 2\left[cl\right]z + \left[ll\right] = \min$$

partial derivatives are got as to the unknown and are equaled to zero,

$$\begin{aligned} \frac{\partial(\vartheta\vartheta)}{\partial x} &= 2\left[aa\right]x + 2\left[ab\right]y + 2\left[ac\right]z - 2\left[al\right] = 0 \\ \frac{\partial(\vartheta\vartheta)}{\partial y} &= 2\left[ab\right]x + 2\left[bb\right]y + 2\left[bc\right]z - 2\left[bl\right] = 0 \\ \frac{\partial(\vartheta\vartheta)}{\partial z} &= 2\left[ac\right]x + 2\left[bc\right]y + 2\left[cc\right]z - 2\left[cl\right] = 0 \end{aligned} \quad , \qquad (18)$$

equation is rearranged,

$$\begin{aligned} [aa]\ x + [ab]\ y + [ac]\ z &= [al] \\ [ba]\ x + [bb]\ y + [bc]\ z &= [bl] \ . \\ [ca]\ x + [cb]\ y + [cc]\ z &= [cl] \end{aligned} \qquad (19)$$

Where (al), (bl) and (cl) are known gravity measurements, (aa), (ab), (ac), (ba), (bb), (bc), (ca), (cb) and (cc) are matrix factors, x is function constant, and y and z are mass densities.

Matrix is obtained. This matrix can be solved with various solution methods. For example, the equation teams can be solved by the method of gauss elimination (See Formula 19) [1].

2.3. Vascular Mass Model

First, a vascular shaped model mass was designed as a model mass. In order to avoid edge effect, the boundaries of source body were extended to ±3000 km as parallel to earth's surface. When measuring network is created, the observation profile has been considered on falls into center profile of model mass. The gravity anomaly generated by vascular shaped body, with a density contrast of 1 gr/cm^3 was computed at an interval of 1 km on a grid of 80 km × 140 km extent. The accuracy of algorithms which perform 2D and 3D calculation were tested. For this purpose, vertical cross-section under y = 0 profile was extended to ±∞ in y direction as parallel to earth's surface, gravity was calculated by 2D algorithm for density contrast to give 1.0 gr/cm^3. Then, vertical cross-section under y = 0 profile was extended to ±3000 km in y direction as parallel to earth's surface, gravity was calculated by 3D algorithm for density contrast to give 1.0 gr/cm^3. In both cases, the maximum gravity values are 16.88491 mGal. Also the minimum gravity values were found as 0.01479. The found 2D and 3D gravity values were compared. Obtained 3D simulation model geometry and 3D gravity anomaly of this model can be shown in Fig.3 and Fig.4. Calculated gravity values for test for both cases are seen in Fig.5. In the original 3D model geometry, mass limits were extended to ±3000 km in y direction as parallel to earth's surface; gravity was calculated by 3D algorithm for density contrast to give 1.0 gr/cm^3.

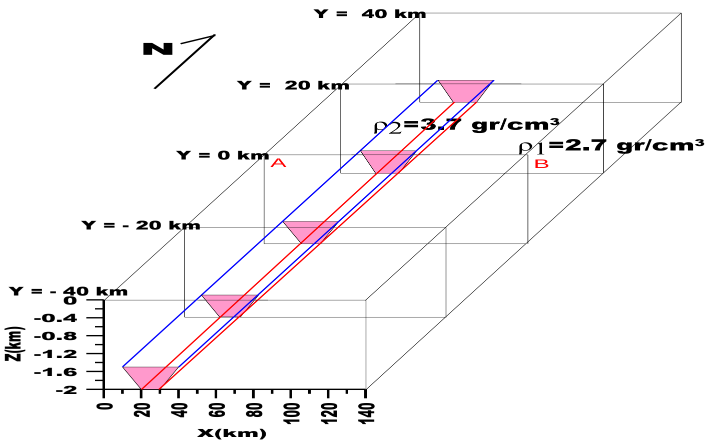

Figure 3. The design of simulation 3D model of vascular mass model.

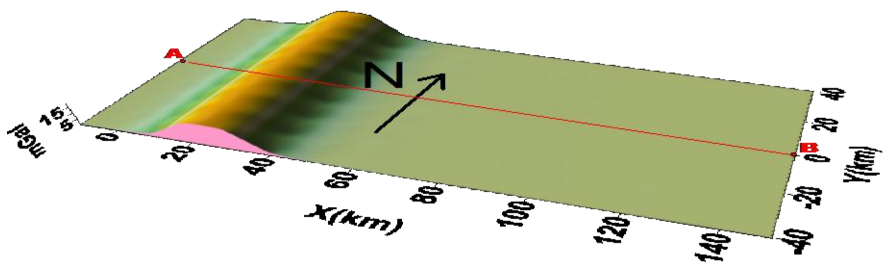

Figure 4. The gravity of simulation 3D model of vascular mass model.

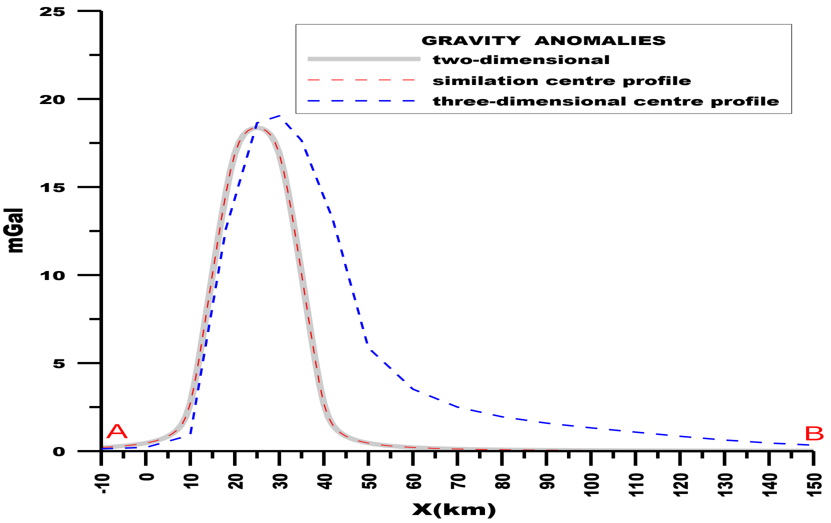

Figure 5. The gravity of vascular mass model on the (A B) centre profiles of the gravity values over the middle profiles which were derived from 2B, simulation 3D and original 3D models.

Gravity anomaly calculated from vascular mass model was adopted as observed values. Then an inverse solution of this anomaly was applied to estimate the density distribution in the subsurface for each model. Calculated with 3D algorithm Δg data are accepted as measure value for the same model geometry. They are used in inversion accounts. Here the purpose, density used in forward solutions, calculation is to test whether the same density cannot be calculated. So the inversion program is to determine working correctly. Calculated maximum gravity value is 79.74457 mGal and minimum gravity value is 0.01479 mGal. Also here, these values can be seen on the y = 0 profile from calculated gravity values in Fig.5. All these values are shown in Table 1.

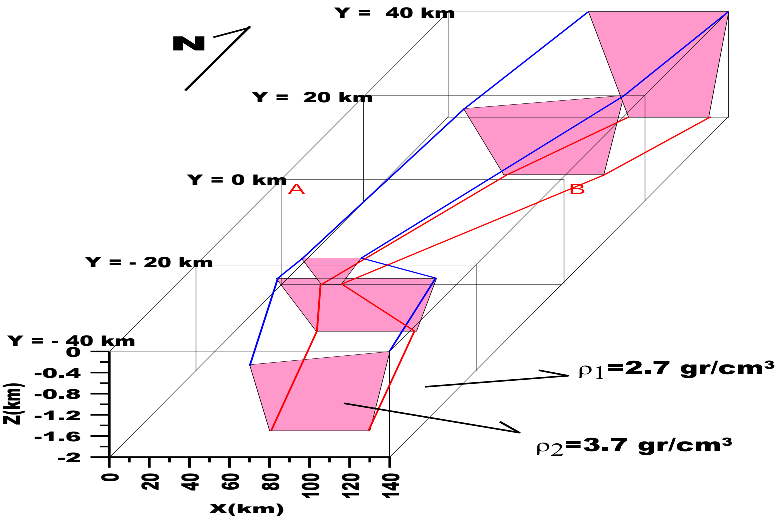

Figure 6. The design of original 3D model of vascular shaped model mass.

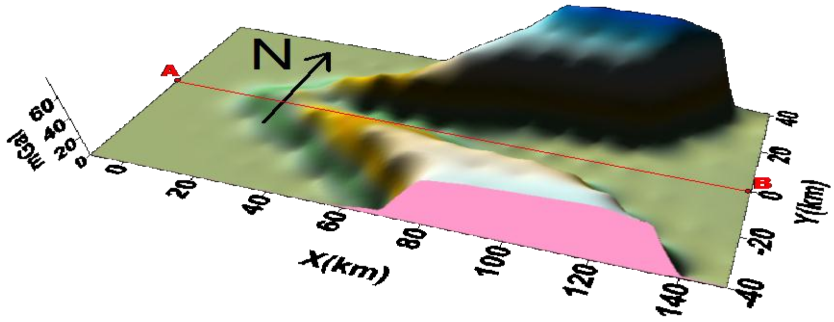

Figure 7. The gravity anomaly of original 3D model of vascular shaped model mass.

Table 1. Estimated values for parameters of vascular shaped model mass using forward and inverse solution

THE VASCULAR MASS MODEL VALUES ON THE (A B) CENTER PROFILES						
	FORWARD			INVERSION		
UNIT	2D M1	SIMILA. 3D M2	ORJINL 3D M3	2D M1	SIMILA. 3D M2	ORJINL 3D M3
Mea.P. Number	17*1=17	17*9=153	17*9=153	17*1=17	17*1=17	17*9=153
Density. (gr/cm^3)	1.000	1.000	1.000	0.8581	0.8581	1.000
Borders (km)	∞	±3000	±3000	∞	±3000	±3000
Volume (km^3)	∞	60000.0	4747916.	∞	60000.0	4747916.
Max value (mGal)	16.88491	16.88491	79.74457	19.04175	19.04175	79.74457
Min value (mGal)	0.01479	0.01479	0.09669	0.12976	0.12976	0.09669
Aver. Error (mGal)	-	-	-	3.0997	3.0997	0.0

3D simulation model geometry was created. The calculated density is 0.8581 gr/cm3, maximum gravity value is 19.04175 mGal and minimum gravity value is 0.12976 mGal. The average error is 3.0997 mGal in both. In addition to calculated BA values from the original 3D model, as measure values, in order to perform inversion, was given again to the original 3D model itself. All these values are shown in Table 1. 3D original model geometry and 3D gravity anomaly of this model can be shown in Fig.6 and Fig.7.

3. DISCUSSION AND CONCLUSIONS

2D and 3D gravity algorithms of the 3D model geometry that was drawn schematically were compared. As is known, on the profile, a single 2D gravity algorithm can be made for the mass of the vertical cross-section under the profile. Therefore, vertical cross-section was taken at the thinnest place of the model geometry. 2D gravity anomaly was calculated with intensity difference assumption on this vertical cross-section. Then, 3D model was created by this vertical cross-section in 2D design, in the ± y direction, in much larger distances (coincide to infinity) according the size of mass in the x and y directions were extended as parallel to earth's surface. The differ of this model from the 2D model, it is designed in a length that can be counted to infinity instead of infinity in y direction. Of course, while the gravity of 3D model geometry is calculated in cartesian coordinates, it must very close the calculated gravity of 2D model geometry. This proximity is a proof to be proper of the 3D algorithm.

The aim of this study is to examine advantage of the 3D gravity algorithm from 2D gravity algorithm. Therefore, 3D gravity of model geometry that was drawn schematically was calculated on a network. AB profile shown on these networks is over full considering vertical cross-section in 2D model geometry. Therefore, 2D and 3D gravity calculations were compared with each other on the same profile. This similarity is seen that the 2D and 3D calculated gravity anomalies are on top of conflict. Also, deviation is shown between the gravity of original model and the first two gravity calculated. As a result, it can be said here that, if the calculated 2D or 3D gravity anomalies are used in their inversion algorithm, different density differences will be calculated as proportional with deviation between them. Right thing is one calculated by the use the 3D gravity anomaly. Because, this anomaly define the original model geometry in the best way.

Underground formations often have very complex geometry. Therefore, also vertical cross-sections under profiles shows great differences. This means is not enough of the 2D algorithm. As a result, the algorithm that best represents the original measurement values is the 3D one.

Today, most of the geophysical survey is resulted as 2D underground modeling. However, in 2D gravity study, density change is ignored in the third dimension. This situation produces a very unhealthy consequences. Even 2D gravity survey gives different results than desired results in cases that the mass is very complex. 3D gravity survey should be done for the illusion is to minimize. That the better one is also the use of spherical coordinates that is consider the tilt of earth. Only cartesian coordinates were satisfied in the 3D gravity algorithm. Also the use of spherical coordinates will been considered in the future studies are.

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