www.dergipark.gov.tr
ISSN:2148-3736
El-Cezerî Fen ve Mühendislik Dergisi
Cilt: 9, No: 2, 2022 (669-679)

El-Cezerî Journal of Science and Engineering
Vol: 9, No: 2, 2022 (669-679)
DOI : 10.31202/ecjse.993313

ECJSE

Makale / Research Paper

Position Control of the Suspended Pendulum System with Particle Swarm Optimization Algorithm

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Received/Geliş: 09.09.2022 **Accepted/Kabul:** 12.11.2022

Abstract: In this study, the suspended pendulum system was controlled by PID (Proportional-Integral-Derivative). Obtaining the system parameters was carried out using the PSO (Particle Swarm Optimization) algorithm in the simulation environment. For this purpose, the model of the suspended pendulum system's model was created. The parameters of the real system were found through the model. Viscous damping coefficient, Moment of Inertia and PID coefficients were calculated as parameters. The calculated values were used in the real system and the real system proved the accuracy of simulation the model. Control of the system was carried out in PLC (Programmable Logical Controller). The force required for the pendulum to reach a certain position was obtained by brushless motor control. ESC (Electronic Speed Control) is used as brushless motor driver and PWM (Pulse Width Modulation) is used as control signal of ESC. The encoder is used to measure the position of the system. System data was obtained with MATLAB. Communication between MATLAB and PLC is realized with OPC (OLE for Process Control). End of the study, it was observed that the simulation and optimization algorithm and the control of the real system gave similar results.

Keywords: PSO, Pendulum, PID, Position Control.

Asılı Sarkaç Sisteminin Parçacık Sürüsü Optimizasyon Algoritması ile Konum Kontrolü

Öz: Bu çalışmada asılı sarkaç sisteminin Oransal-İntegral-Türevsel (PID) denetleyici parametreleri ile kontrolü yapılmıştır. Sistem parametrelerinin elde edilmesi simülasyon ortamında PSO (Parçacık Sürü Optimizasyonu) algoritması kullanılarak gerçekleştirilmiştir. Bu amaçla asılı sarkaç sisteminin modeli çıkartılmıştır. Model üzerinden gerçek sistemin parametreleri bulunmuştur. Parametre olarak, Viskoz Sönümleme Katsayısı, Atalet Momenti ve PID katsayıları hesaplanmıştır. Hesaplanan değerler gerçek sistemde kullanılarak ve gerçek sistem ile simülasyon modelinin doğruluğu ispatlamıştır. Sistem kontrolü PLC (Programlanabilir Mantıksal Denetleyici)'de gerçekleştirilmiştir. Sarkacın belli bir konuma gelmesi için gerekli kuvvet, fırçasız motor kontrolü ile elde edilmiştir. Fırçasız motor sürücüsü olarak ESC kullanılmıştır, ESC'nin kontrol işareti olarak PWM kullanılmıştır. Sistemin konumunu ölçmek için ise enkoder kullanılmıştır. Sistem verileri MATLAB ile alınmıştır. MATLAB ile PLC arasında haberleşme OPC ile gerçeklenmiştir. Çalışma sonunda simülasyon ve optimizasyon algoritması ile gerçek sistemin kontrolünün benzer sonuçları verdiği gözlemlenmiştir.

Anahtar kelimeler: PSO, Sarkaç, PID, Konum Kontrolü.

1. Introduction

The hanging pendulum system has made our lives easier in many areas from past to present. The basic working principles of seismic measuring devices, metronome instruments and pendulum clocks are based on this system. Modeling of prosthetic legs and prosthetic leg applications, control of micro-

How to cite this article
Özdemir C., Öztürk S., Şengül Ö., Kuncan F., "Position Control of the Suspended Pendulum System with Particle Swarm Optimization Algorithm" El-Cezerî Journal of Science and Engineering, 2022, 9 (2); 669-679.

Bu makaleye atıf yapmak için
Özdemir C., Öztürk S., Şengül Ö., Kuncan F., "Asılı Sarkaç Sisteminin Parçacık Sürüsü Optimizasyon Algoritması ile Konum Kontrolü" El-Cezerî Fen ve Mühendislik Dergisi 2022, 9 (2); 669-679.
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aircraft, farm tractors, sailboats and even amusement parks can be listed among the areas where the control of the hanging pendulum system is needed [1,2]. Even though the hanging pendulum is a standard topic in advanced physics education, it is often also addressed in many laboratory programs [3]. The location of the center of gravity and the determination of the moment of inertia parameters require complex analyzes [4].

In this study, difficult to calculate parameters such as moment of inertia and viscous are found on simulation using optimization algorithm. It is aimed to operate the system by controlling propeller and the brushless motor with the ESC, to read the angle information of the angle formed by the torque of the motor via the encoder, and to PID control of the system with the help of simulations.

Optimization means an act, process, or methodology of making something (such as a design, system, or decision) as fully perfect, functional, or effective as possible specifically. Optimization algorithms that have become widespread today have also been developed to find the best solution to the problems. PSO- Particle Swarm Optimization is an optimization technique developed by J. Kennedy and R. C. Eberhart [5] in 1995. It is called particle swarm optimization because it resembles the swarm behavior of birds. Different than other evolutionary algorithms, it simulates social behavior. Individuals in the swarm cooperate and compete with each other for generations, and individuals try to reach the best result with the experiences of other individuals. In 1998, Shi, Y. and Eberhart, R. C. [6] added inertial weight to this optimization, providing a balance between local and global search, and as a result, sufficient optimal results are achieved in fewer iterations. This new algorithm is called Enhanced Particle Swarm Optimization. This article will use enhanced particle swarm optimization and will be referred to as PSO for short. While using the PSO algorithm, the thesis [7] was referenced.

2. Literature Studies

Control applications have attracted the attention of researchers for many years. In control applications, many methods are preferred; especially PI, PD and PID control applications. Initially, theoretical studies are carried out for these applications. Afterwards, simulation studies of theoretical studies are carried out with computerized systems. In the last stage, it is tested on the real-time system. After the analysis and examinations obtained, optimal studies and control applications are made for the systems.

It is seen that control applications are widely used on pendulum systems, especially in recent years. These studies are carried out in both academic and industrial applications. The following studies, in which pendulums, optimization algorithms, motor controls using different controllers and PID controls are used, can be given similar study examples.

Erkol conducted a study on Optimization of the Inverted Pendulum System with Artificial Bee Colony Algorithm. Optimization of a Proportional-Integral-Derivative controller for an Inverted Pendulum System was realized by the Artificial Bee Colony algorithm. In the author's study, the control of the inverted pendulum with a trolley was carried out with PID and the PID parameters were adjusted with the Ziegler-Nichols method and the ABC algorithm [8]. Küçüker carried out a thesis study named Position control in compound pendulum system. In this study, model-based adaptive position control of the hanging pendulum system is predicted. First of all, using the mathematical model of a hanging pendulum system realized in the laboratory environment, PID and model-based adaptive control methods were applied in the Matlab/Simulink software environment. Then, the same control methods were carried out in real time using Labview software, and experimental results were obtained for different conditions. It has been observed that the experimental results obtained and the simulation results are quite compatible, and very good results are obtained when the PD controller and model-based adaptive control are used together [4]. Ayyıldız carried out a study named inverted pendulum is controlled with PID using PSO algorithm [9]. Öztürk et al. carried out a Real Time

Controller system based on PSO algorithm for. In that study, permanent magnet direct current motor actuator is implemented by using fuzzy neural network structure. Particle Swarm Optimization (PSO) algorithm is used as training algorithm of fuzzy neural network controller. Learning and control in real time is executed in MATLAB. Dynamic performance of the system is observed for constant and variable reference trajectory of speed [10]. Kuncan et al. carried out a study named Position Determination by Using Image Processing Method in Inverted Pendulum. In this study, the measurement of the inverse pendulum angle and position was performed using image processing method. The algorithm developed in MATLAB Simulink. The position determination process is performed in computer environment by image processing method. The information obtained in this study is targeted to be tested in a real-time system [11]. Çubukçu et al. have successfully implemented DC Motor Speed Control in real time using Image Processing and OPC. In their study taking the speed of the DC motor as reference, Speed control or synchronization of AC motor is intended. Camera is used instead of encoder for speed control. The images taken with the help of the camera are processed with image processing methods in the MATLAB environment and converted into speed information. Speed information is sent from MATLAB to PLC using OPC [12]. Kaplan et al. have successfully implemented the Real Time Position Control of PID and Fuzzy Logic Based DC Motor in real time. In that paper DC Motor's dynamic equations and transfer functions are of Motor. PID and Fuzzy Logic parameters were calculated in software program (MATLAB Simulink) and with STM32F4 microcontroller DC motor position control have been performed. It has been observed that the position control of the DC motor is carried out successfully according to the reference position [13].

Liu et al. carried out a study named Real-Time Controlling of Inverted Pendulum by Fuzzy Logic. The main aim of the study is to balance a real inverted pendulum at the central position. For this purpose, fuzzy logic controller is used. The fuzzy logic controller designed in the Matlab-Simulink environment. In this study, the inverted pendulum mathematical model is built. MATLAB based Hardware in Loop simulation system is designed [14]. El-Hawwary et al. carried out a study named Adaptive Fuzzy Control of the Inverted Pendulum Problem. In their study, the adaptive fuzzy control of an inverted pendulum in a car problem is considered as an under operated mechanical system. An experimental application of AFLC (Adaptive Fuzzy Logic Control) is intended for this well-known problem. Most of the schemes presented in the adaptive fuzzy control literature treat the problem as a quadratic system based on feedback linearization. Such schemes create unstable zero dynamics for car-pole systems, which hinders experimental application. The presented paradigm is also based on a feedback linearization (FBL) scheme, but provides system stabilization. A damping term and an adaptive fuzzy control term are added to ensure asymptotic stability and account for distortions. Experimental results show the success of the proposed controller in stabilizing a reference trajectory and tracking car position [15]. Shen has done a study on the topic of 3D Pendulum. In that study, a new 3D pendulum model is introduced, its key features are analyzed, and several new associated control problems are proposed. The pendulum consists of a rigid body supported on a fixed pivot with three rotational degrees of freedom. This article is the result of ongoing research on a laboratory facility called the Triaxial Attitude Control Test Bed (TACT). TACT was built to provide a testing environment for a variety of physical experiments on attitude dynamics and attitude control. The highlight of the TACT design is that it is supported by a three-dimensional air bearing that acts as an ideal frictionless pivot and provides almost unlimited three-degree rotation [16]. Taşkın has done a study on the Propeller Pendulum. In this paper, a fuzzy PID controller is proposed for angular position control of a nonlinear propeller pendulum system. In order to demonstrate the position control enhancement for the nonlinear system, the proposed controller is compared with classical PID controller using simulation results with and without external disturbance. The simulation results show that the proposed Fuzzy PID controller is more successful in reference tracking than classical PID controller [17].

Ratnayake et al. carried out a study named LQR-Based Stabilization and Position Control of a Mobile Double Inverted Pendulum. This paper described the development and control of a mobile double inverted pendulum. Both simulations and experiments were conducted to evaluate the performance of the system in balancing and position control. The results showed successful balancing of body and pendulum at the upright position and the position control at the desired positions, using LQR [18]. Kafetzis et al. carried out an Inverted Pendulum. Authors mentioned their study; the first step in this work is to determine the equations of motion for the inverted pendulum, using the Euler-Lagrange equations. The next step is to find a linearized model that approximates the original nonlinear system's behavior around the equilibrium located on the upright vertical position. The behavior of the linearized system is simulated using Matlab. The final step is the computation of an optimal control law for the linearized system, using the Linear Quadratic Regulator method [19]. Rabah et al. carried out a study named Comparison of Position Control of a Gyroscopic Inverted Pendulum Using PID, Fuzzy Logic and Fuzzy Logic PID Controllers. In that study, the gyroscopic inverted pendulum is stabilized using PID controller, fuzzy logic controller and fuzzy PID controller. These controllers are compared to determine performance and distinguish which one responds better. Experiments and simulations are performed to examine different controllers and the results are presented under different failure scenarios [20].

Many literature studies made with the pendulum system and position control have been examined above. The side that distinguishes this study from other mentioned studies is the control of the Hanging Propeller Pendulum with the help of the PSO algorithm using encoder, PLC and OPC.

3. Test Setup and Methodology

There is a two-stage application in the proposed study. The first stage is the theoretical calculations of the study. According to these theoretical calculations, the system is tested as a simulation on the computer. It is aimed that the proposed control system for position control in the testing phase is optimal. Simulating the system in a computer environment is the first step. The real-time testing of the simulation studies and the proposed control study is the second phase. At this stage, a prototype test setup designed and manufactured in the laboratory environment was created for the real-time implementation of the system. The success of the system was analyzed by making many applications on this test setup.

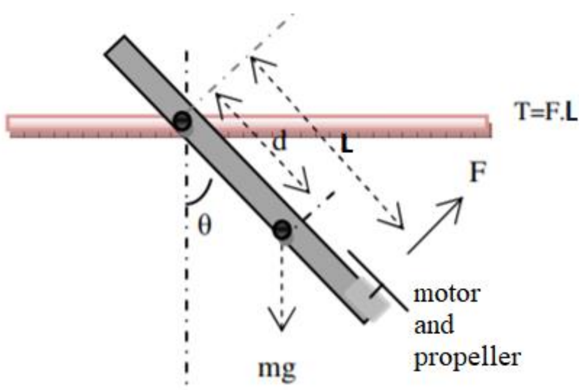

Figure 1. Hanging propeller pendulum system [4]

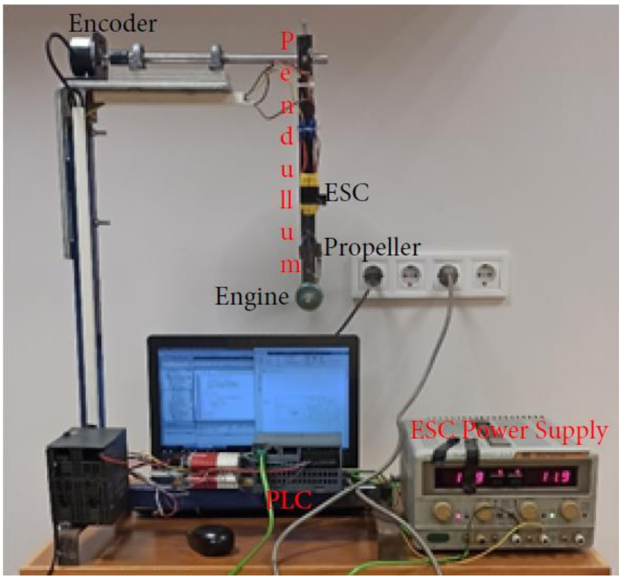

Figure 2. Test Setup

By leaving the pendulum from a certain angle, the oscillation values of the real system are found with the encoder.

↓

Simulation and PSO code are prepared on MATLAB.

↓

By comparing the simulation and the actual oscillation values, the parameters we are looking for (Viscosity, Inertia) are found.

↓

The real system is controlled with ESC and angle values are found corresponding to the pwm values.

↓

Using PWM-angle values, driver's mathematical equivalent function is found.

↓

Edited the PSO code and Simulation with the mathematical equivalent function and PID coefficients are found with the PSO algorithm.

↓

By applying the found pid coefficients to the system, the suitability of the simulation and the real system was observed.

Figure 3. Algorithm of the study

Figure 1 shows the model of the hanging propeller pendulum system and Figure 2 shows the real system. If the moment rule (Torque) is applied to the point where the hanging pendulum system in Figure 1 is suspended based on the Archimedean moment rules, the following equation will be obtained [21]. Figure 3 shows the algorithm of the study.

$$j\ddot{\theta} + c\dot{\theta} + mLgd\sin\ \theta = T \tag{1}$$

Here T is the thrust (torque) that will be generated by the rotation of the propeller at the end of the engine. θ represents the angle of rotation that is desired to control. If both sides of formula 1 are divided by j, it becomes the following formula [4].

$$\ddot{\theta} = -\frac{c}{j}\dot{\theta} - \left(\frac{mLgd}{j}\right)\sin\ \theta \tag{2}$$

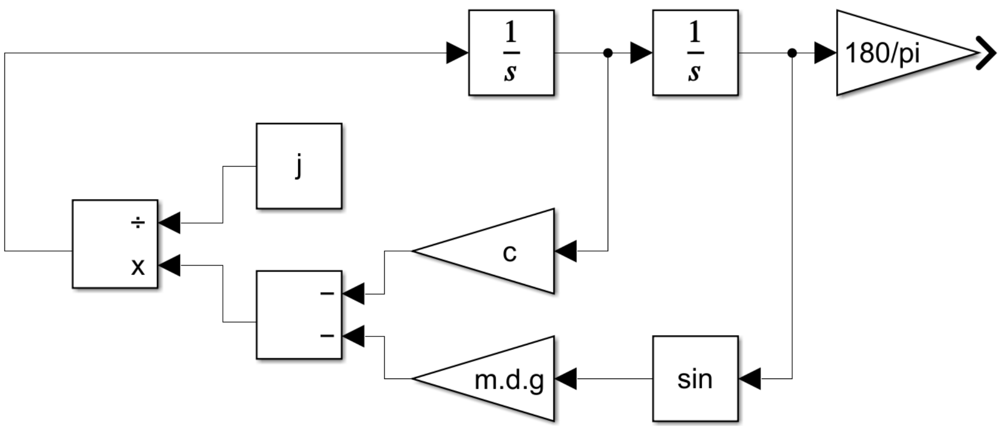

Figure 4. Pendulum Simulink model

Based on this formula 2, the Simulink model has been prepared for simulating pendulum and shown in Figure 4.

An encoder is an electro-mechanical device for converting angular position or motion from analog to digital signal. Oscillation angle information can be measured with the help of the encoder that is connected to the pendulum arm. In order to find the parameters of the system, firstly, an oscillation was made from a certain angle.

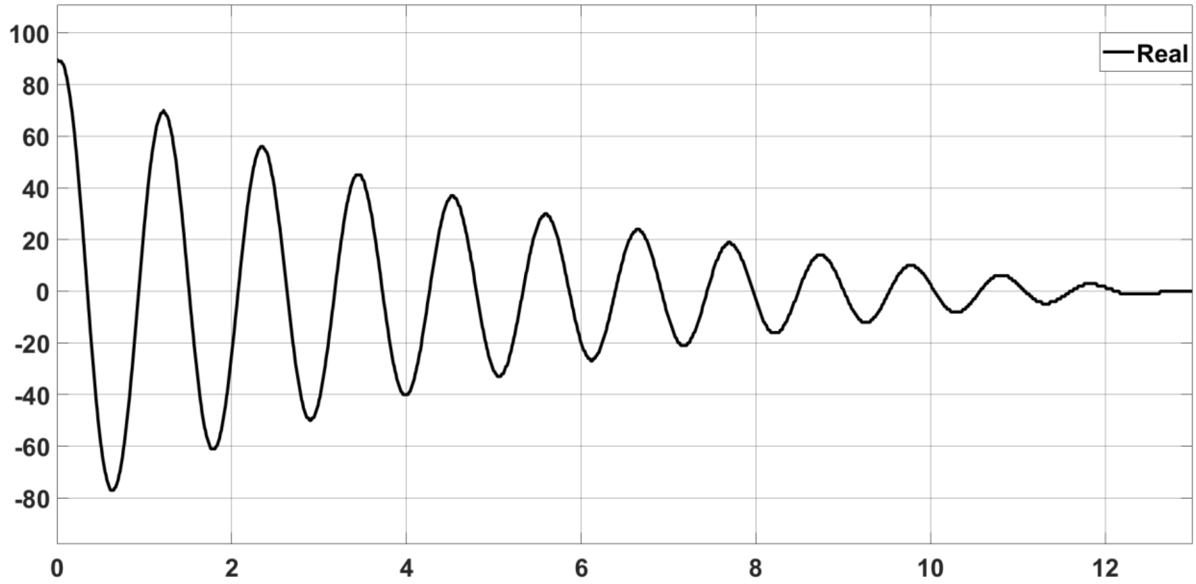

Figure 5. Free oscillation of real-life pendulum from 90° angle

The graph of the information obtained when the real system is left to free oscillation at a 90° angle is seen as Figure 5.

for *each particle* $\boldsymbol{i = 1, \ldots, S}$ ***do***
Initialize the particle's position with a uniformly distributed random vector;
$\boldsymbol{x_i \sim U(b_{lo},\ b_{up})}$
Initialize the particle's best-known position to its initial position: $\boldsymbol{p_i \leftarrow x_i}$
If $f(p_i) < f(g)$ ***then***
update the swarm's best-known position: $\boldsymbol{g \leftarrow p_i}$
Initialize the particle's velocity: $\boldsymbol{v_i \sim U(-|b_{up} - b_{lo}|,\ |b_{up} - b_{lo}|)}$ (3)[22]
while *a termination criterion is not met do:*
for *each particle* $\boldsymbol{i = 1, \ldots, S}$ ***do***
for *each dimension* $\boldsymbol{d = 1, \ldots, n}$ ***do***
Pick random numbers: $\boldsymbol{r_p, r_g \sim U(0, 1)}$
Update the particle's velocity: : $\boldsymbol{v_{i,d} \leftarrow \omega\, v_{i,d} + \varphi_p r_p (p_{i,d} - x_{i,d}) + \varphi_g r_g (g_d - x_{i,d})}$
Update the particle's position: $\boldsymbol{x_i \leftarrow x_i + l_r v_i}$
If $f(x_i) < f(p_i)$ ***then***
Update the particle's best-known position: $\boldsymbol{p_i \leftarrow x_i}$
If $f(p_i) < f(g)$ ***then***
Update the swarm's best-known position: $\boldsymbol{g \leftarrow p_i}$

The PSO algorithm finds "Pbest" (Best of the swarm, personal best) values that lead to the solution closest to the desired result (one with the lowest error) by trying possible solutions for a particular problem (function, etc.) as a swarm. The algorithm compares these Pbest values between different swarms and finds the "Gbest" (Global best) result. Using this information, the PSO algorithm can be used to find the parameters sought in the simulation. The basic PSO algorithm is as given in Equation 3.

When c (viscosity) and j (inertia) values are found using PSO algorithm, the 90° oscillation graph of the simulation looks like Figure 6.

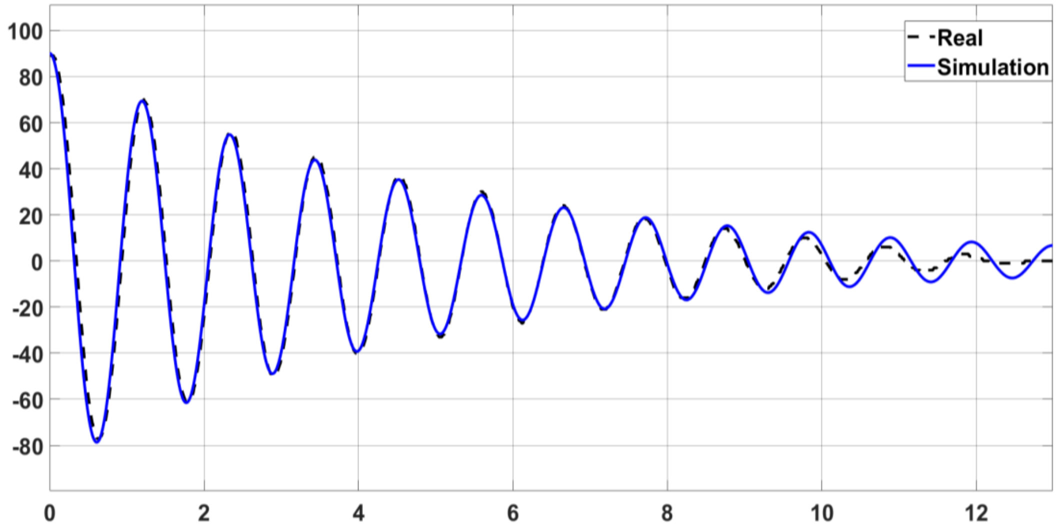

Figure 6. Comparison of the 90° oscillation of pendulum systems

As we can see oscillation of both systems are almost identical as a result, all the necessary parameters for the first simulation have been given in Equation 4.

$$\begin{aligned}
&m = 0.36\ \text{kg (Weight of the pendulum arm)}\\
&d = 0.22\ \text{m (Distance from propeller to the center of gravity of the pendulum)}\\
&L = 0.33\ \text{m (Length of pendulum arm)}\\
&g = 9.81\ \text{ms}^2\ \text{(Gravitational force)}\\
&c = 0.0084771\ \text{Nms/Rad (viscosity damping coefficient)}\\
&j = 0.021755\ \text{kg}\cdot\text{m}^2\ \text{(Moment of inertia coefficient)}
\end{aligned} \tag{4}$$

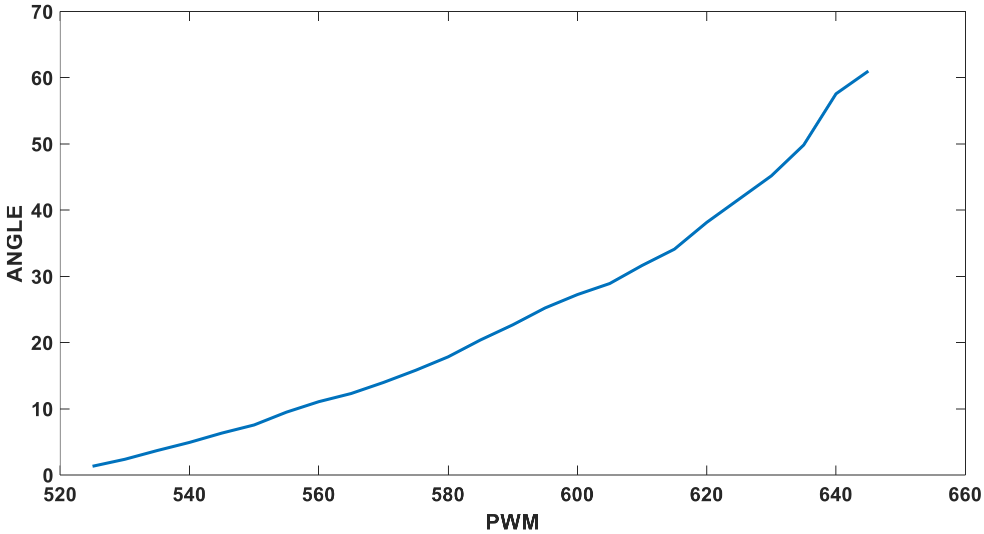

Figure 7. Angle graph corresponding to the PWM value

ESC is an electrical circuit that regulates and controls the speed of the electric motor. Since ESC controls the brushless motor with the PWM signal, the speed of the motor can be controlled by increasing the width of the pulses, thus the angle of the pendulum. In the real system, the PWM values given by increasing in a controlled way and the resulting angles are seen on the graph like Figure 7.

Exponential function (driver's mathematical equivalent function) that can be calculated according to this graph is equation 5 and its fitted graph is shown in Figure 8.

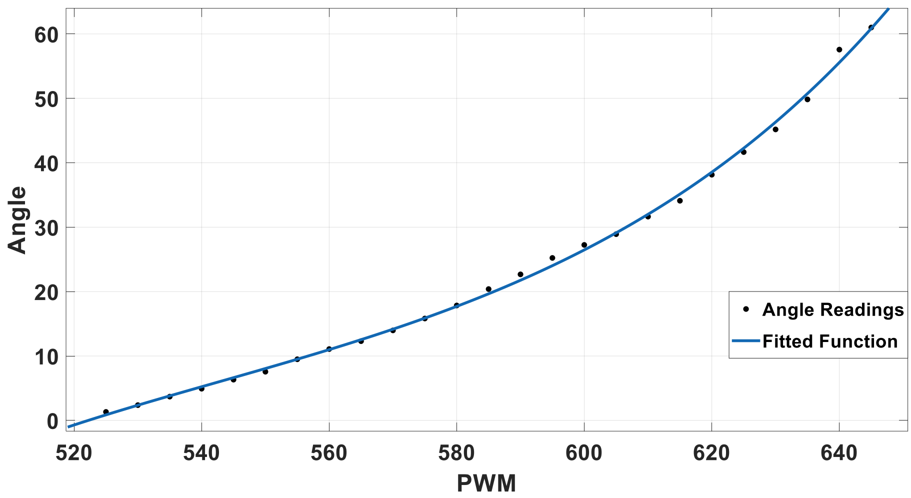

Figure 8. Driver's mathematical equivalent function's graph

$$y = -187^{*}exp\,(-0.02387^{*}x) + 707.6^{*}exp\,(-0.0004631^{*}x) \quad (5)$$

PID is a control loop mechanism widely used in industrial control systems that uses feedback loop. To control the ESC that is connected to the brushless motor and propeller with the PID method driver's mathematical equivalent function (equation 5) is added together with PID blocks to the Simulink model Figure 9 and PSO code is readjusted according to these changes.

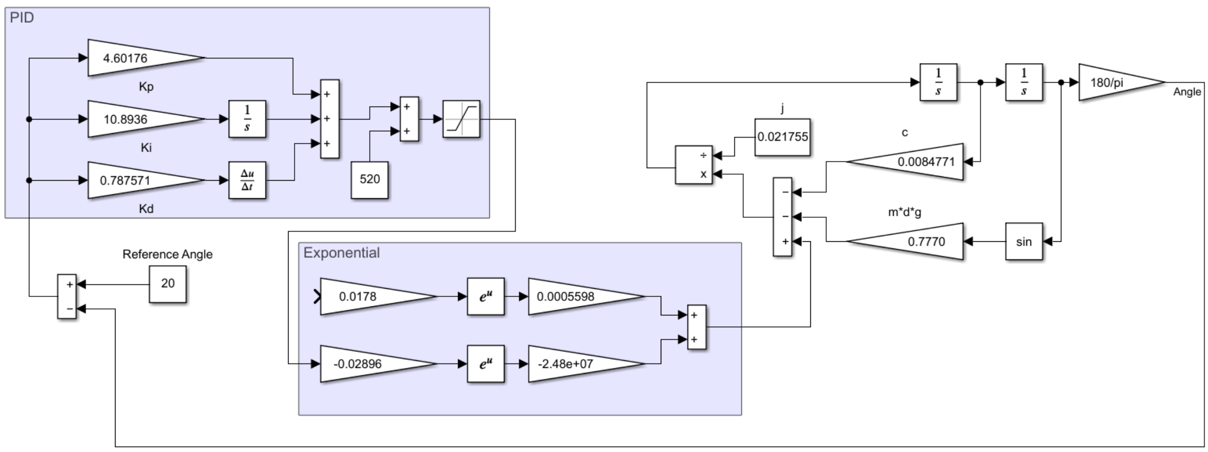

Figure 9. PID and exponential Simulink model

When the PID parameters are found in MATLAB (P=4.6 I=10.8 D=0.7) it can be seen that the pendulum simulation is positioned at 20° angle in 3.3 seconds Figure 10.

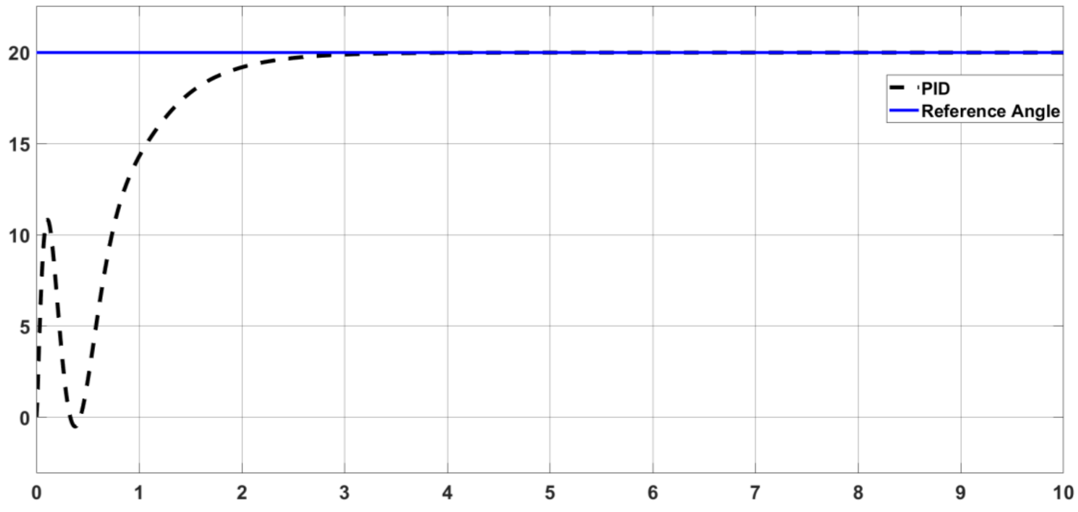

Figure 10. PID controlled simulation results

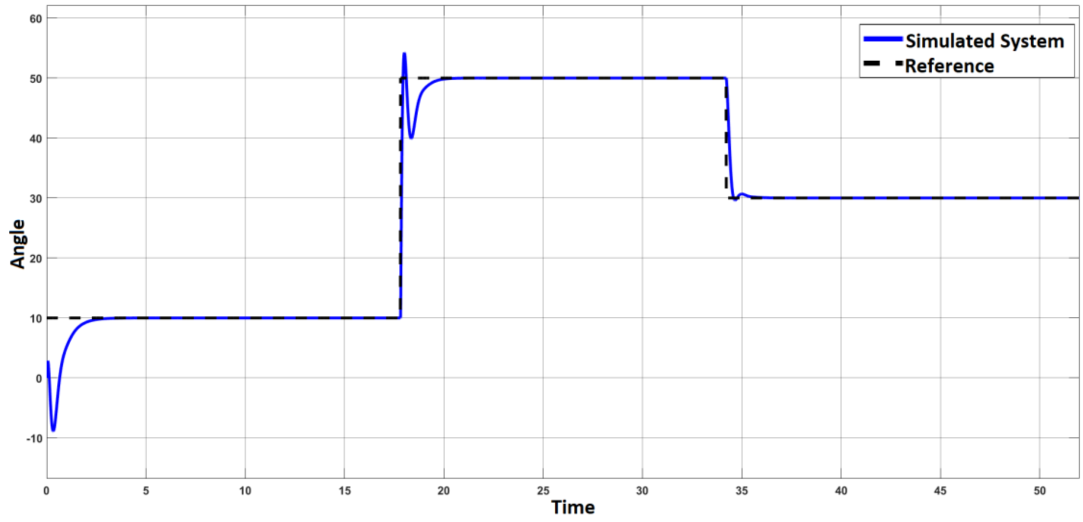

Figure 11. Simulation results with PID

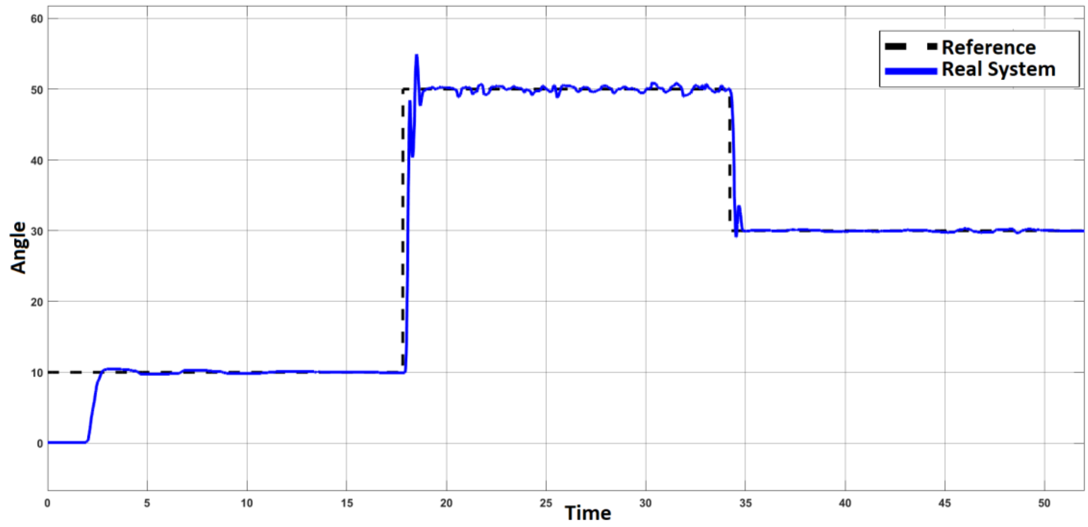

Figure 12. Real system results with PID

4. Results

Given the same reference angles, the simulation results are shown in Figure 11 and the results of the real system are shown in Figure 12. Real system shows slight fluctuations. The reason for this is the inhomogeneity of the air in the environment and the wind reflected from the surroundings.

5. Conclusion

Control applications have attracted the attention of researchers for many years. In control applications, many methods are preferred; especially PI, PD and PID control applications. Initially, theoretical studies are carried out for these applications. Afterwards, simulation studies of theoretical studies are carried out with computerized systems. In the last stage, it is tested on the real-time system. After the analysis and examinations obtained, optimal studies and control applications are made for the systems. It is seen that control applications are widely used on pendulum systems, especially in recent years. These studies are carried out in both academic and industrial applications. The following studies, in which pendulums, optimization algorithms, motor controls using different controllers and PID controls are used, can be given similar study examples.

In this study, the suspended pendulum system was controlled by PID. Obtaining the system parameters was carried out using the PSO algorithm in the simulation environment. Calculated values were used in the real system and the real system proved the accuracy of the simulation results. At the end of the study, it was seen that the simulation results tested in the MATLAB model of the system and the results tested in the real system were very close to each other and there were acceptable differences in certain places. It was seen that PSO algorithm provides convenience to the user in PID optimization and saves time while finding certain parameters.

It is predicted that the method proposed in the study can be used in different application areas. In addition, the study is thought to be a study area with a high potential to be used in industrial applications.

Authors' Contributions

CÖ, SÖ, ÖŞ and FK wrote up the article. Both authors read and approved the final manuscript.

Competing Interests

The authors declare that they have no competing interests.

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