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Welfare Effects of Electricity Transit

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Elektrik Geçişinin Refah Etkileri

Abstract

The liberalization of the European electricity market facilitating the cross-border exchanges of electricity requires the establishment of a system for accessing the European grid. The starting point for the discussion of pricing the access to interconnected networks in this paper is the fact that electricity transit creates external effects to the involved countries which have to be internalized. Using a three countries model we analyze the welfare effects of different compensation scenarios for the electricity transfer between countries under individual and joint welfare maximization. The three countries have different roles: country 1 is an electricity exporting country, country 2 is importing electricity from country 1 and country 3 is the transit country. Individual and joint maximization deliver identical results if and only if export prices and transit charges are equal to marginal production and marginal transit costs. The joint optimum implies that the demand for imported electricity is determined by the sum of all relevant marginal costs: marginal production cost and marginal electricity loss in the export country, marginal transit costs and marginal electricity loss in the import country. The transit country has to be compensated for its electricity loss costs on the basis of marginal costs. Capital costs of transit capacity should only be compensated in the case of capacity shortage in the transit country.

Key Words : Cross-border Trade, Electric Utilities, Industrial Policy, Social Welfare.

JEL Classification Codes : L14, L94, O25, D60.

Özet

Elektriğin ülkeler arasında kolayca mübadele edilmesini sağlamaya yönelik Avrupa elektrik piyasasındaki serbestleştirme Avrupa iletim şebekesine erişimle ilgili bir sistemin kurulmasını gerektirmektedir. Elektrik iletiminin ilgili ülkeler arasında içselleştirilmesi gereken dışsal etkiler yarattığı gerçeği, bu çalışmada, birbirine bağlı şebekelere erişimin fiyatlandırılmasına dair tartışmanın başlangıç noktasını oluşturmaktadır. Çalışmamızda bir üç ülke modeli kullanılmak suretiyle bireysel ve birleşik refah maksimizasyonu altında ülkeler arasındaki elektrik transferinin farklı ödünleşme senaryoları altında yarattığı refah etkileri incelenmektedir. Burada üç ülke farklı rollere sahiptir: ülke 1 elektrik ihraç eden bir ülke, ülke 2 ülke 1'den elektrik ithal eden bir ülke ve ülke 3 ise bir geçiş ülkesidir. Bireysel ve birleşik maksimizasyon, ancak ve ancak ihraç fiyatları ve geçiş bedelleri, marjinal üretim ve marjinal geçiş maliyetlerine eşit olduğunda aynı sonuçları vermektedir. Birleşik optimum, ithal elektrik talebinin ilgili bütün maliyetlerin toplamı (ihraç ülkesindeki marjinal üretim maliyeti ve marjinal elektrik kaybı, ithal ülkesindeki marjinal geçiş maliyetleri ve marjinal elektrik kaybı) tarafından belirlendiğini ifade etmektedir. Geçiş ülkesi elektrik kayıp maliyetlerinin marjinal maliyetler temelinde tazmin edilmesi gerekmektedir. Geçiş kapasitesinin sermaye maliyetleri ise sadece geçiş ülkesinde kapasite açığı olduğu durumda tazmin edilmelidir.

Anahtar Sözcükler : Sınırötesi Ticaret, Elektrik Şirketleri, Endüstri Politikası, Sosyal Refah.

1. Introduction

Economists for some time analysed the question whether a monopoly organization in which the electric power utility operated the generation, transmission, and distribution systems located in a fixed geographic area was efficient. The first country which restructured its nationally owned power system, creating privately owned companies to compete with each other to sell electric energy was the United Kingdom followed by Norway, Australia, New Zealand and then, in 1992, by the United States.

There are different methods of managing the operation of the transmission system in the deregulated power system operating environment. In the paper by Christie-Wollenberg-Wangensteen (Christie, Wollenberg, Wangensteen, 2000: 170-195) three models are analysed: First, the optimal power flow model, used in the United Kingdom, Australia, New Zealand, and some parts of the United States. Second, the price area based model used in Norway, Sweden, and Finland. Third, the transaction based model used in the United States. As shown in this paper, each model maintains power system security but differs in its impact on the economics of the energy market. No clearly superior method has been identified.

The first step of restructuring the European electricity sector was done by Directive on common rules for the internal market in electricity and appeared in the Official Journal, in January 1997 (E.C., 1996-2003). Additional legislation (in E.C., 1996-2003) appeared in July 2004. An amending Directive strengthened and repeated the 1996 law. The Regulation on conditions for access to the network for cross-border exchanges in electricity (hereafter referred as the Regulation) aims to facilitate the exchanges of electricity among Member States.

Two important issues of the Regulation deal with the establishment of a system for accessing the European grid and improving the management of congestion at the interconnections. In the paper by Daxhelet-Smeers (Daxhelet, Smeers, 2007: 1396-1412) a computational framework encompassing these two features was developed. Pricing the access to interconnected networks does not only involve the coverage of local network costs, but also transit costs induced on other regional networks through loop flows. Several combinations of electricity transmission system operators have been proposed for compensation mechanism by Daxhelet-Smeers ((Daxhelet, Smeers, 2003) and (Daxhelet, Smeers, 2007: 1396-1412)). The problem in (Daxhelet, Smeers, 2007: 1396-1412) is cast in the form of a game between regional Regulators, and is modelled by a two-stage equilibrium problem.

This paper deals with the setting of tariffs to cover operating costs. We analyse the welfare effects of different compensation scenarios for the electricity transfer between countries. Welfare will be defined as aggregate economic profits. Since, in addition we assume perfect competition, profit maximization is equivalent to the maximization of social welfare, defined as the sum of consumer rent and producers rent.

The paper is organized as follows. In the next section the basic three countries model with individual welfare maximization is developed. Section 3 deals with the joint welfare maximization and conclusions in the last section terminate the paper.

2. Modelling Electricity Transfer

We develop a three-country model. The three countries have different roles: country 1 is an electricity exporting country, country 2 is importing electricity from country 1, and country 3 is the transit country. To make calculations as simple as possible, we assume prices for commodities and wages as given and equal in all countries. Since countries may differ in natural resources, size, population and population density, we allow production conditions for commodities, electric energy and electricity transport to vary in the three countries. All three countries produce an aggregate commodity. Production technology is described by a neoclassical production function, with positive, decreasing marginal products and positive mixed second derivatives. Without restriction of generality we assume that the capital stock $\overline{K}_i^C$ is given and constant, so it can be dropped from the production function:

$$X_i = \mathrm{f}_i\left(\mathrm{L}_i, \mathrm{E}_i\right); \quad i = 1,2,3$$

where L_i denotes labor input and E_i is energy input in countries i = 1,2,3 and

$$f_L > 0, f_{LL} < 0, f_E > 0, f_{EE} > 0, f_{LE} = f_{EL} \geq 0, f_i(0, E_i) = 0, f_i(L_i, 0) = 0;$$

Country 1 and country 2 produce electric energy from given natural resources (water, wind,…), labor and a given capital stock for energy production. The extraction of electrical energy from natural resources is assumed to follow a Leontief type production function:

$$E_i = \min\left(\frac{1}{\eta_i} L_i^E, \overline{K}_i^E, N_i\right); \qquad \mathrm{i}=1,3$$

L_i^E …labor inputs for electricity production, $\overline{K}_1^E$ ….production capacity for electricity production and N_i …natural resources.

We assume that $\frac{1}{\eta}$ stands for the (constant) marginal productivity for labor in electricity production, (η is then the marginal labor requirement for electricity production), the production capacity for electricity is given and is expressed in maximum possible units of electricity production and that natural resources have a zero price. Under these assumptions production is limited to $\overline{K}_i^E$ units of electricity.

We assume that country 2 does not have adequate natural resources for electricity production, which would lead to production costs above market prices. Therefore, for the sake of simplicity we assume that country 2 is not producing electric energy at all in our model.

All countries have power grids to transport electricity. We assume costs of electricity transportation having two components: capital costs and energy losses, other variable costs of electricity transport will be subsumed under the latter term. Due to technical reasons electricity losses approximately follow a quadratic function of the transported energy. We assume that these losses have to be compensated by feeding the electricity loss into the network. The loss is assumed to follow the following formula:

$$l_i = \tau_i(E_i); \qquad \tau' > 0, \quad \tau'' > 0; \quad i = 1,2,3.$$

Under these assumptions we can write the following cost function for electricity production and transport:

$$C_i^E = \omega\eta_i \cdot (E_i + l_i) + \rho \cdot \left(\overline{K}_i^E + \overline{E}_i\right) = \omega\eta_i \cdot [E_i + \tau_i(E_i)] + \rho\left(\overline{K}_i^E + \overline{E}_i\right)$$

where ρ denotes the cost of capital.

The costs of electricity production and transport are real wages cost for electricity production including energy losses due to transportation, plus capital costs of energy production and transport.

For our further calculations we will use these building blocks to characterize the three countries and we will compare several scenarios to analyze welfare effects of different ways of transit cost allocation to the countries involved. Basically we look at two cases:

(1) Each country individually maximizes its welfare

(2) All countries jointly maximize welfare

In case (1) we look at two important sub scenarios.

(1a) No binding capacity constraints for electricity transport in the transit country

(1b) Binding capacity constraint in the transit country

Here we will explore the impacts of such a constraint for several versions of allocating transit costs, including the question whether or not the export country and the import country should contribute to the elimination/reduction of the capacity constraint in the transit country.

The basic problem which is tackled here is related to negative external effects inflicted upon the transit country by electricity transit. In addition, an inefficient allocation of transit costs between countries might induce significant welfare losses due to inefficient production in at least the import country and possibly in the export country, too.

Now we derive optimality conditions under the assumption that each country maximizes its welfare individually. We then analyze these conditions with respect to external effects of energy transit.

2.1. Export Country

Country 1 produces commodities and services and electric energy. The welfare function is defined as follows:

$$Y_1 = \mathrm{f}_1(\mathrm{L}_1, \mathrm{E}_1) + \varepsilon_{12} E_{12} - \left[\omega \cdot \left(L_1 + \eta_1 \cdot (E_1 + E_{12} + \tau_1(E_1 + E_{12}) \right) + \rho \cdot \left(\overline{K}_1^C + \overline{K}_1^E + \overline{E}_1 \right) \right]$$

ω.... real wage; ε_{12}....export electricity price; τ_1... energy loss function;
η_1...marginal labor requirement for electricity production;
$\overline{K}_1^C$, $\overline{K}_1^E$...fixed capital for commodities and electricity production; ρ.... cost of capital;
E_1.... domestic electricity inputs; E_{12}.... electricity exports;
$\overline{E}_1$.... maximum transportation capacity; $\left(E_1 + E_{12} \right) \leq \overline{E}_1$.

This specification of the welfare function implies given relative prices and a given export price ε_{12} for electricity exported to country2. Maximization of this welfare function will lead to competitive outcomes because production factors will be used efficiently, i.e. factor inputs will be chosen to equalize marginal products and factor prices. If we assume the given prices to be equilibrium prices we will end up in a situation of a competitive equilibrium. The welfare functions for the other countries are defined analogously.

Maximizing welfare in country 1 requires accommodating the inequality condition for electricity transportation capacity $\overline{E}_1$. For the time being we assume E_{12} as given. The Lagrange function therefore can be written as:

$$\max_{L_1,E_1} \Lambda_1^1 = f_1\left(L_1,E_1\right)+\varepsilon_{12}E_{12}-\left[\omega\cdot\left(L_1+\eta_1\cdot\left(E_1+E_{12}+\tau_1\left(E_1+E_{12}\right)\right)\right)+\rho\cdot\left(K_1^C+K_1^E+\overline{E}_1\right)\right]+$$
$$+\lambda_1^1\cdot\left(\overline{E}_1-E_1-E_{12}\right)$$

It follows from the Kuhn-Tucker conditions:

(1) $\dfrac{\partial\Lambda_1^1}{\partial L_1}=\dfrac{\partial f_1}{\partial L_1}-\omega=0$

(2) $\dfrac{\partial\Lambda_1^1}{\partial E_1}=\dfrac{\partial f_1}{\partial E_1}-\omega\cdot\eta_1\cdot\left(1+\tau_1'\right)-\lambda_1^1=0$

(3) $\lambda_1^1\cdot\left(\overline{E}_1-E_1-E_{12}\right)=0$

Since $\left(E_1+E_{12}\right)\leq\overline{E}_1$ we can conclude that according to the Kuhn-Tucker Theorem $\lambda_1^1\geq 0.$ This reasoning applies to all other models considered in this paper and will not be repeated further. The expression $\omega\cdot\eta_1\cdot\left(1+\tau_1'\right)$ can be interpreted as the marginal cost of electricity provision (production plus transport) in country 1. As mentioned above, the specification of the loss function implies increasing marginal costs of electricity transportation (and electricity provision), given the capacity of the grid.

Under the assumption of non binding capacity constraints in country 1 ($\lambda_1^1=0$), we see – given E_{12} and the export price for electricity ε_{12} – that country 1 would maximize its welfare by expanding labor inputs in commodity production until the marginal productivity of labor equals the real wage rate. Furthermore country 1 would expand electricity input to the point where marginal costs of electricity production plus marginal costs of electricity transportation equal the marginal productivity of electricity inputs. We are not considering a situation with capacity constraints in the electricity exporting country because it is of no importance for the problems to be analyzed.

2.2. Import Country

Country 2 is only producing commodities; its production conditions are described by the same type of production function, as in the other countries. We have assumed that this country is importing electrical energy from country 1 and to do so it must transit country 3. Welfare maximization in country 2 and country 3 involves assumptions about the allocation of transit costs. We model the allocation by use of a Kronecker symbol δ_{12}. If $\delta_{12} = 1$, we assume that country 3 will be completely compensated for energy losses by the importing country. If $\delta_{12} = 0$ no compensation for transit costs is given to the transit country. The second question with regard to the compensation of transit costs touches upon the question of compensation of cost of capital of the network. We will discuss this issue later.

Country 2 has to import more electricity than it actually uses for production, since it has to make up for energy losses inside its own grid. So the imported quantity is defined by:

$$E_{12} = E_2 + \tau_2(E_2);$$

E_2...electricity used for production in country 2, E_{12}...electricity imports

Using this adjustment to the same building blocks discussed earlier, the welfare function of country 2 can be written as follows:

$$\max_{L_2, E_2} \Lambda_2^1 = f_2(L_2, E_2) - \left[\omega L_2 + (\varepsilon_{12} + \delta_{12}\varepsilon_3) \cdot (E_2 + \tau_2(E_2)) + \rho \cdot \left(K_2^C + \overline{E}_2 \right) \right] + \\ + \lambda_2^1 \cdot \left(\overline{E}_2 - (E_2 + \tau_2(E_2)) \right)$$

For the time being, we assume ε_{12} and ε_3 as given. In the later analysis, we will specify these prices differently. From the Kuhn-Tucker conditions we obtain:

$$(4) \quad \frac{\partial \Lambda_2^1}{\partial L_2} = \frac{\partial f_2}{\partial L_2} - \omega = 0$$

$$(5) \quad \frac{\partial \Lambda_2^1}{\partial E_2} = \frac{\partial f_2}{\partial E_2} - (\varepsilon_{12} + \delta_{12}\varepsilon_3) \cdot (1 + (\tau_2'(E_2))) - \lambda_2^1 = 0$$

$$(6) \quad \lambda_2^1 \cdot \left(\overline{E}_2 - (E_2 + \tau_2(E_2)) \right) = 0$$

Again, the interpretation of these conditions appears to be straightforward. Condition (4) describes optimal labor input. If we assume sufficient grid capacity in country 2, the Lagrange multiplier λ_2^1 will be zero. This allows the following interpretation of condition (5). Electricity input will be extended, until the marginal cost of electricity (consisting of country one's export price plus the transit price for one unit corrected for marginal network losses in country 2) equals the marginal product of electricity inputs.

The first order conditions can be solved for labor and electricity inputs and electricity import demand of country 2, given the specification of the production functions:

$$L_2^* = L_2^*\left(\omega, \varepsilon_{12}, \delta_{12}\varepsilon_3\right)$$
$$E_2^* = E_2^*\left(\omega, \varepsilon_{12}, \delta_{12}\varepsilon_3\right)$$
$$E_{12}^* = E_2^* + \tau_2(E_2^*)$$

These functions show that the optimal levels of labor and electricity inputs depend on real wages, electricity prices and transit charges (if at all levied), optimal electricity imports depend on the energetic losses in country 2's grid.

2.3. Transit Country

The transit country produces commodities, electricity and electricity transit services. In contrast to the export and import countries, binding capacity constraints play an important role for transit related welfare effects. Whenever the capacity constraint for electricity transport is binding, we have to consider two situations. In the first situation the transit country cannot control electricity transit. A binding capacity constraint means that domestic electricity usage is limited and cannot be fully adjusted to equal the marginal product. In the second case we can assume that the transit country has the technical and/or contractual means to control transit flows which allows the transit country to optimally adjust its domestic electricity usage.

The welfare maximization can be written as follows:

$$\begin{aligned}\max_{L_3, E_3} \Lambda_3^1 = \mathrm{f}_3\left(\mathrm{L}_3, \mathrm{E}_3\right) + \delta_{12}\varepsilon_3 \cdot E_{12} - \omega \cdot \left[L_3 + \eta_3 \cdot \left(E_3 + \tau_3\left(E_3 + E_{12}\right)\right)\right] \\ - \rho\left(\overline{K}_3^C + \overline{K}_3^E + \overline{E}_3\right) + \lambda_3^1\left(\overline{E}_3 - E_3 - E_{12}\right)\end{aligned}$$

$$where \qquad E_{12} \equiv \left(E_2 + \tau_2\left(E_2\right)\right)$$

The necessary conditions for a maximum of welfare in the transit country are as shown below:

(7) $\dfrac{\partial \Lambda_3^1}{\partial L_3} = \dfrac{\partial f_3}{\partial L_3} - \omega = 0$

(8) $\dfrac{\partial \Lambda_3^1}{\partial E_3} = \dfrac{\partial f_3}{\partial E_3} - \omega \eta_3 \left[1 + \tau_3'(E_3 + E_{12})\right] - \lambda_3^1 = 0$

(9) $\lambda_3^1 \cdot \left(\overline{E}_3 - E_3 - E_{12}\right) = 0$

As in country 1 the expression $\omega \cdot \eta_3 \cdot \tau_3(E_3 + E_{12})$ can be interpreted as the marginal cost of electricity transport in the transit country, whereby $E_{12} = (E_2 + \tau_2(E_2))$ is taken as given.

First we consider the case of a non-binding capacity constraint. In this case $\left(E_3 + E_{12}\right) < \overline{E}_3$, which implies by condition (3) that $\lambda_3^1 = 0$. This leads to the standard optimality conditions where marginal products of factor inputs equal factor costs. If we look at the case of full compensation of energy losses, which means $\delta_{12} = 1$, we can see that the revenue of transiting electricity cancels out, if the price ε_3 is equal to the average costs of transiting electricity.

$$\varepsilon_3 = \frac{1}{E_3 + E_{12}} \omega \eta_3 \cdot \tau_3\left(E_3 + E_{12}\right)$$

It should be noted here, that the marginal energy losses τ_3' are higher than in a case without transit, even if energy losses, induced by transit are fully compensated by the export/import country. We will deal with the non-compensation case later, but it appears to be clear, that even in the case of full compensation, country 3 will incur a significant welfare loss, caused by an uncompensated increase of marginal and average costs of electricity transport, which leads to lower production.

If the Lagrange multiplier λ_3^1 is positive we have the case of a binding capacity constraint $\left(\overline{E}_3 - E_3 - E_{12}\right) = 0$. Now we have again to distinguish between the two cases mentioned above. In the first case, the transit country cannot control transit flows. This implies that the domestic use of electricity is defined by $E_3^\circ = \left(\overline{E}_3 - E_{12}\right)$. Condition (8) then implies that the use of electric energy is lower than in the unconstrained optimum. This means a welfare loss for the transit country, because its electricity inputs and its commodity production are lower than in the welfare optimum of the unconstrained case. The (marginal) welfare loss is equal to the Lagrange multiplier, which is the shadow price of capacity shortage.

$$(8') \quad \lambda_3^1 = \frac{\partial f_3}{\partial E_3^{\circ}} - \omega \eta_3 \left[1 + \tau'(\overline{E}_3)\right] > 0$$

In the case of full control over the transit quantities, the transit country can stay in the optimal position, but both the export and the import countries will suffer from welfare losses, which will be shown later.

In addition we can see the effect of different compensation schemes for transit costs. If there is full compensation ($\delta_{12} = 1$), we have shown, that the optimal factor allocation in the transit country is only indirectly affected by electricity transit by the increase in marginal energy losses τ_3' induced by the transit quantity E_{12}. If there is no compensation, there is a second welfare effect. This is a direct reduction of welfare by the transit costs. Again, there will be an increase in marginal energy losses leading to a change in the factor allocation, which in general will reduce welfare.

2.4. Model Interaction and Welfare Considerations

A look on the individual welfare maximization models of the three countries shows the interdependency of the system. The export country depends on the decision of the import country, the import country's optimal decision depends on the energy price ε_{12} and on the transit price ε_3, which may or may not be controlled by the export or transit country respectively. Welfare in the transit country 3 inter alia depends on electricity transit and on compensation mechanisms. The functioning of the system can be described as follows: given ε_{12} and ε_3 the import country decides on the optimal electricity inputs $E_2\left(\varepsilon_{12}, \varepsilon_3, \ldots\right)$, which corrected for energy losses in its own country gives the optimal electricity import $E^*_{12} = E_2 + \tau_2(E_2)$. Depending on this decision, countries 1 and 3 maximize their own welfare.

The welfare implications of electricity transit in the independent maximization scenario can now in detail be traced. For a start, we examine a situation with no binding capacity constraints in the transit country. This means that country 2 can buy and country 1 can sell the amounts of electricity they optimally wish to buy/sell and are in the position to transit this optimal amount of electricity through country 3. Depending on the compensation scheme for transit costs different welfare effects can be observed. In the following analysis the effects of a compensation for the cost of capital attributable to transit is excluded and deferred to a later section.

(i) $\delta_{12} = 0$: This means country 3 is compensated neither for energy losses and other variable transit costs, nor for costs of capital attributable to transit. In this case welfare optimization in the import country is based on real wages and the electricity price: ε_{12}, which is lower than the actual cost of electricity in country 2, because transit costs are carried by country 3. This leads to a distorted resource allocation in the three countries: country 2 will produce too many commodities compared to an economically efficient allocation; country 1 consequently will export to much electric energy to country 2. In addition in this scenario country 3 will incur welfare reductions by the amount of transit energy losses and other variable transit costs.

(ii) $\delta_{12} = 1$: In this situation country 3 is fully compensated for transit energy losses and other variable transit costs. If ε_3 equals average transit costs and ε_{12} equals production costs in country 1, the resource allocation in all three countries is similar to a situation of a joint maximum of all three countries.

(iii) We can now study a situation where all three countries start their interaction in the electricity sector with national grids which have been dimensioned to accommodate their countries' energy needs. This means that: $\overline{E}_1$ will cover total electricity production capacity for domestic demand and exports, $\overline{E}_2$ will cover import demand (country 2 does not produce electric energy) and $\overline{E}_3$ will accommodate only domestic electricity demand but not electricity transit E_{12}. This case is designed to analyze the welfare effects of (missing) transit capacity. In the most extreme case there is no idle transport capacity in country 3, which could be used for transit. This means country 1 cannot export electricity to country 2 and consequently there is no production in country 2 at all. Country 3 is not affected at all. We can now analyze the induced welfare effects compared to an optimal allocation, including transit.

$$\Delta Y = -\left[Y_2^* + \left(\varepsilon_{12} - \omega\eta_1\right) E_{12}^* - \frac{E_{12}^*}{E_1^* + E_{12}^*} \tau_1 \left(E_1^* + E_{12}^*\right) \right]$$

Starred variables indicate values of an optimal allocation.

The welfare effects consist of a complete stop of production in country 2 plus lost profits for electricity exports in country one, which are partly offset by not incurred transit costs. Country 3 has no incentive to increase its transportation capacity since it optimally fits its domestic requirements. An increase in capacity would lead to a welfare loss of $\rho \cdot d\overline{E}_3$. We could derive an estimate for the total welfare effect of an increase of transit capacity in country 3 by taking the partial derivatives of the welfare functions of all three countries with respect to $\overline{E}_3$. For this purpose we simply have to recognize that in this case, E_{12} is a function of $E_{12} = E_{12}\left(\overline{E}_3\right),\ E'_{12} = 1$. This will be shown in detail in a later section for joint welfare maximization.

In general we can expect that the welfare gain of an additional unit of transit capacity will outweigh the cost of capacity increases. Since the welfare effects occur in the export and import country they can offer a full compensation of capital costs of transit capacity increases to country 3 (or create the transit capacity themselves).

3. Joint Welfare Maximization

We now consider the situation of an economic area, where not individual countries maximize their welfare but total welfare of the economic area will be maximized. This implies that electricity exports and transit services will drop out by consolidation. The joint maximization problem can be written as:

$$
\begin{aligned}
\max_{L_1, L_2, L_3, E_1, E_2, E_3} \Lambda = & \\
= & f_1\left(L_1, E_1\right) - \left[\omega \cdot \left(L_1 + \eta_1 \cdot \left(E_1 + E_{12} + \tau_1\left(E_1 + E_{12}\right)\right)\right) + \rho \cdot \left(K_1^C + K_1^E + \overline{E}_1\right)\right] + \\
& + f_2\left(L_2, E_2\right) - \left[\omega L_2 + \rho \cdot \left(K_2^C + \overline{E}_2\right)\right] + \\
& + \mathrm{f}_3(\mathrm{L}_3, \mathrm{E}_3) - \omega \cdot \left[L_3 + \eta_3 \cdot \left(E_3 + \tau_3(E_3 + E_{12})\right)\right] - \rho\left(\overline{K}_3^C + \overline{K}_3^E + \overline{E}_3\right) + \\
& + \lambda_1 \cdot \left(\overline{E}_1 - E_1 - E_{12}\right) + \lambda_2 \cdot \left(\overline{E}_2 - E_{12}\right) + \lambda_3\left(\overline{E}_3 - E_3 - E_{12}\right)
\end{aligned}
$$

Maximization is now over six variables: $L_1, L_2, L_3; E_1, E_2, E_3;$ and we have to take into account that $E_{12} = E_2 + \tau_2(E_2)$. We will present and discuss the first order conditions separately according to labor input and electricity inputs.

If we look on the first order conditions for labor inputs, we clearly see that they are identical to the conditions under individual welfare maximization. Under our assumptions about the characteristics of the production functions for commodities, the efficient labor allocation under joint maximization will deviate from the allocation under individual welfare maximization if optimal energy inputs are different.

$$\left.\begin{aligned}\frac{\partial\Lambda}{\partial L_1} &= \frac{\partial f_1}{\partial L_1} - \omega = 0\\ \frac{\partial\Lambda}{\partial L_2} &= \frac{\partial f_2}{\partial L_2} - \omega = 0\\ \frac{\partial\Lambda}{\partial L_3} &= \frac{\partial f_3}{\partial L_3} - \omega = 0\end{aligned}\right\} \quad \frac{\partial f_1}{\partial L_1} = \frac{\partial f_2}{\partial L_2} = \frac{\partial f_3}{\partial L_3} = \omega$$

If we now calculate the first order conditions for optimal energy inputs and compare them with the individual maximization case, we see that only the condition for E_2 is different.

$$\frac{\partial\Lambda}{\partial E_1} = \frac{\partial f_1}{\partial E_1} - \omega\cdot\eta_1\cdot\left(1+\tau_1'\right) - \lambda_1 = 0$$

$$\frac{\partial\Lambda}{\partial E_2} = \frac{\partial f_2}{\partial E_2} - \omega\,\eta_1\cdot(1+\tau_1')\cdot E_{12}' - \omega\,\eta_3\cdot\tau_3'\cdot E_{12}' - E_{12}'\sum_{i=1}^{3}\lambda_i = 0$$

$$\frac{\partial\Lambda}{\partial E_3} = \frac{\partial f_3}{\partial E_3} - \omega\,\eta_3\cdot\left[1+\tau_3'\left(E_3+E_{12}\right)\right] - \lambda_3 = 0$$

Neglecting for the moment the Lagrange multipliers we see that the condition for E_2 can be written as:

$$\frac{\partial f_2}{\partial E_2} = E_{12}'\left(\omega\eta_1\cdot(1+\tau_1')\cdot+\omega\eta_3\cdot\tau_3'\right) = 0$$

We see that marginal product of electricity inputs must be equal to the marginal costs of electricity in the system of all three countries, which consists of marginal production costs in country 1: $\omega\eta_1$, marginal energy losses in country 1: $\omega\eta_1\tau_1'$ and marginal transit costs through country 3: $\omega\eta_3\tau_3'$. The sum of these components is then to be corrected for marginal energy losses in country 2.

If we compare this formula with the equivalent first order condition under individual maximization, again disregarding the possibility of capacity shortages in country 2:

$$\frac{\partial\Lambda_2^1}{\partial E_2} = \frac{\partial f_2}{\partial E_2} - (\varepsilon_{12} + \delta_{12}\varepsilon_3)\cdot E_{12}' = 0\ ,$$

we see, that the results of individual and joint welfare maximization are identical if

$$\varepsilon_{12} = \omega\eta_1 \cdot (1+\tau_1'),$$

$$\delta_{12} = 1,$$

$$\varepsilon_3 = \omega\eta_3 \cdot \tau_3';$$

which means that country 1 sells its exported electricity at marginal cost and country 3 is compensated for its transit services and charges its marginal cost. This in fact means that prices for electricity would be those prices, which will be ruling in a competitive economy.

Countries 1 and 3 face the same first order conditions for optimal electricity input, as long as we have no binding capacity constraint.

The Kuhn – Tucker conditions finally are identical to the individual maximization case.

4. Capital Cost and Welfare Effects

We can now analyze the issue of the compensation of capital costs of transit capacity. Both maximization approaches show, that capital costs do not play any role for the allocation of resources. If there is no capacity restraint in the transit country, it would have to pay the total capital costs of the grid, if there were no compensation for the transited volumes of electricity. In effect country 3 would subsidize the export and import countries.

The compensation of capital costs of transit capacities is obviously not an allocation problem. Nevertheless, capital costs of transit capacities are important for the distribution of welfare between countries. This type of problem is normally analyzed with the instruments of game theory, which allows modeling strategies, power relations etc. This type of analysis is beyond the scope of this paper.

What we can do in our model is to analyze a situation, where country 3 has not sufficient grid capacity to convey transiting electricity. In other words there is a binding constraint in country 3.

If $\lambda_3 > 0$, we have a binding capacity constraint in country 3. From our discussion above, we already know that this would lead to welfare reductions in the export and import countries, so we will not duplicate the analysis here.

According to the Kuhn-Tucker Theorem, the effect on total welfare $Y = Y_1 + Y_2 + Y_3$ of a marginal increase $d\overline{E}_3$ of grid capacity in the transit country is equal to λ_3, which is the shadow price of a small addition to capacity in the transit country.

If $\frac{dY}{d\bar{E}_3} > 0$, it would be profitable to invest into new transit capacity. This question cannot be answered analytically; it largely depends on the size of the shortage of capacity and the marginal product of electricity in the import country. As a general rule it can be said, the in the presence of significant capacity shortages there will be an economic incentive to take this investment.

5. Conclusions

The optimal solutions/allocations under individual and joint welfare maximization are generally different; welfare in the case of the joint maximization is never inferior to individual maximization. This is due to fact that electricity transit creates external effects to the involved countries, which have to be internalized in an appropriate way.

The decisive factors for an efficient allocation are electricity prices and costs. Individual and joint maximization deliver identical result if and only if export prices and transit charges are equal to marginal production and marginal transit costs. This appears to be the first best solution for the internalization of the external effects mentioned above. It is no surprise that the joint maximum is equal to the solution under the assumption of competitive markets for electricity transport.

The joint optimum implies that that the demand for imported electricity is determined by the sum of all relevant marginal costs: marginal production cost and marginal electricity loss in the export country, marginal transit costs, and marginal electricity loss in import country.

Any deviation of this rule would cause inefficient allocations. The inefficiencies originate from “wrong” prices, i.e. profit maximizing transit charges, profit maximizing electricity prices, average transit costs as price base, etc. One can easily conceive situation, where e.g. the export country would be better off by profit maximizing prices than in the joint maximum. This shows that also distribution and power aspects have to be taken into account.

In the joint optimum the transit country has to be compensated for its electricity loss costs on the basis of marginal costs. There should be no compensation of capital cost if there is abundant transit capacity in the transit country. Capital costs of transit capacity should only be compensated in the case of capacity shortage in the transit country. Creation of transit capacity, which is exclusively used for transit, may under some circumstances be financed out of the welfare gains of the import country.

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