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Research Article

A comparison of the performance of entropy measures for interval-valued intuitionistic fuzzy sets

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ABSTRACT

Entropy measure is a significant tool to define unclear information. But, entropy measures for interval-valued intuitionistic fuzzy sets (IVIFSs) cannot be easily understood intuitively. So, it is highly important to compare the existing measures to select a reliable entropy measure in studies. The purpose of this study is to compare the performance of different entropy measures developed for IVIFSs. The numerical examples are presented to show whether entropy measures for IVIFSs are effective in representing the fuzziness degree. In order to understand whether a variation of fuzziness degree of one or more elements of IVIFSs change the ranking results, selected IVIFSs are modified diversely.

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INTRODUCTION

In real-life problems include uncertainty and complexity. However, crisp numbers are not enough to describe uncertainty and imprecision properly. To deal with ambiguity and imprecision, Zadeh [1] proposed fuzzy set (FS) theory in 1965. Although the FSs characterized by a membership function has received attraction from decision experts in the various fields, Atanassov [2] put forward that it was insufficient to represent the state of belonging only with the membership function. So, he offered intuitionistic fuzzy sets (IFSs) theory in 1986. In this theory, elements belonging to a set are described by a membership degree and a non-membership degree in [0,1]. Three years later, Atanassov and Gargov [3] proposed the theory of interval-valued intuitionistic fuzzy sets (IVIFSs) theory in 1989, which is an extension of the IFSs theory. The IVIF sets are characterized by an interval membership degree, an interval non-membership degree, and an interval hesitancy degree. In a famous monograph, Pedrycz [4] stated that expressing membership degree and non-membership with a single value is not realistic enough and technically sufficient. Because the degree of an element belonging to a set is expressed by interval-valued numbers rather than crisp numbers, IVIF sets theory provides a powerful tool to cope with ambiguity and vagueness in real applications [5-9].

The fuzzy entropy presents a global measure of an average amount of intrinsic information which is lost when turning from a classical pattern to the fuzzy pattern. Owing to the similarity of the equation to the Shannon entropy form, this measurement was given the name of 'entropy' [10]. However, Shannon entropy and fuzzy entropy are different in terms of uncertainty. The Shannon entropy

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measures the average uncertainty in bits related to the estimation of outcomes in an experiment, but the fuzzy entropy explains the degree of fuzziness in a fuzzy set [11]. Entropy is an important concept for the fuzzy set theory proposed by Zadeh [1]. Burillo and Bustince [12] extended the fuzzy entropy for intuitionistic fuzzy sets. Then, in order that fuzziness degree of interval-valued intuitionistic fuzzy sets can be measured, Liu et al. [13] developed the interval-valued intuitionistic fuzzy entropy. In order to measure the uncertainty of IVIFSs, many researchers have investigated the definition and formulation of entropy of IVIFSs from various aspects. For instance, Liu et al. [13] first introduced an axiomatic definition of entropy for IVIFSs. Based on this, Wei et al. [14], Gao and Wei [15] and Jin et al. [16] constructed a variety of entropy measures. Zhang and Jiang [17] and Zhang et al. [11] presented the entropy model depending on De Luca and Termini [18] model extending the definition of entropy for FSs. Zhang et al. [19] extended the definition of entropy for IVIFSs by taking inspiration from Burillo and Bustince [12]'s study for IFSs. Zhang et al. [20], Rashid et al. [21] developed distance-based entropy measures for unclear information in the interval-valued intuitionistic fuzzy set environment. Furthermore, more studies on entropy measures for IVIFSs has been introduced in different viewpoints [22-32].

When the literature review, it is seen that there are few studies about entropy measures for IVIFSs. Moreover, these existing entropy measures for IVIFSs cannot be easily understood and compared heuristically. So, this paper is aimed to compare the performance of IVIF entropy measures about representing the fuzziness degree of IVIFSs. This comparative analysis is provided with ten numerical examples constructed by considering the variation of fuzziness degree of one or more elements of an IVIF set. In the studies mentioned in section 5, a comparison has been made with several entropy models to compare the performance of the developed entropy model. But this study is prominent with a comprehensive comparing of the IVIF entropy measures.

The article is organized as follows: Section 2 briefly reviews some basic concepts related to IVIF sets. Section 3 presents the properties of the IVIF entropy measure and some IVIF entropy measures to be compared. To understand whether a variation of fuzziness degree of one or more elements of IVIFSs changes the ranking results, ten different tests are conducted in Section 4. Test results are discussed and conclusions are presented, in the last section.

PRELIMINARIES

In this section, some basic concepts related to interval-valued intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy numbers are reviewed in order to facilitate further discussions.

Definition 1. [3] Let $D[0,1]$ be the set of all closed sub-intervals of the interval [0,1] and X be a universe of discourse. An IVIFS $\tilde{A}$ in X is an object having the form $\tilde{A}=\{\langle x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)\rangle | x\in X\}$ where $\mu_{\tilde{A}}: X\to D[0,1]$ and $\nu_{\tilde{A}}: X\to D[0,1]$ under the condition $0\le \sup(\ _{\tilde{A}}(x))+\sup(\nu_{\tilde{A}}(x))\le 1$ for all $x\in X$. The closed intervals ( ) and $\nu_{\tilde{A}}(x)$ denote the degrees of membership and nonmembership of the x to $\tilde{A}$, respectively. Lower and upper end points of these intervals are denoted by $\mu_{\tilde{A}}(x)=[\mu_{\tilde{A}}^L(x),\mu_{\tilde{A}}^U(x)]$ and $\nu_{\tilde{A}}(x)=[\nu_{\tilde{A}}^L(x),\nu_{\tilde{A}}^U(x)]$. Thus an interval-valued intuitionistic fuzzy set can be denoted as below:

$$\tilde{A}=\left\{\left\langle x,\left[\mu_{\tilde{A}}^L(x),\mu_{\tilde{A}}^U(x)\right],\left[\nu_{\tilde{A}}^L(x),\nu_{\tilde{A}}^U(x)\right]\right\rangle \middle| x\in X\right\} \quad (1)$$

For convenience, Xu [33] called $\tilde{A}=[\mu_{\tilde{A}}^L(x),\mu_{\tilde{A}}^U(x)],[\nu_{\tilde{A}}^L(x),\nu_{\tilde{A}}^U(x)]$ an interval-valued intuitionistic fuzzy number (IVIFN), where $0\le \mu_{\tilde{A}}^U(x)+\nu_{\tilde{A}}^U(x)\le 1$, $\mu_{\tilde{A}}^L(x),\nu_{\tilde{A}}^L(x)\ge 0$ for all . It is clear that if $\mu_{\tilde{A}}(x)=\mu_{\tilde{A}}^L(x)=\mu_{\tilde{A}}^U(x)$ and $\nu_{\tilde{A}}(x)=\nu_{\tilde{A}}^L(x)=\nu_{\tilde{A}}^U(x)$, then the given interval-valued intuitionistic fuzzy set $\tilde{A}$ is reduced to an ordinary intuitionistic fuzzy set.

Definition 2.[3] Hesitation degree of each element x in interval-valued intuitionistic fuzzy set $\tilde{A}$ is given as:

$$\begin{aligned}\pi_{\tilde{A}}(x)&=\left[\pi_{\tilde{A}}^L(x),\pi_{\tilde{A}}^U(x)\right]\\ &=\left[\left(1-\mu_{\tilde{A}}^U(x)-\nu_{\tilde{A}}^U(x)\right),\left(1-\mu_{\tilde{A}}^L(x)-\nu_{\tilde{A}}^L(x)\right)\right]\end{aligned} \quad (2)$$

Theorem 1. [34,35] Let $\Gamma: IFS(X) \to IVIFS(X)$ is given by $\Gamma(A)=\{x,\mu_{\Gamma(A)},\nu_{\Gamma(A)} | x\in X\}$ for all $A\in IFS(X)$ where

(i) $\mu_{\Gamma(A)}^L(x)=|a+b\mu_A(x)-a_x\pi_A(x)|$, with fixed $a,b\in\mathbb{R}$ for all $A\in IFS(X)$

(ii) $\mu_{\Gamma(A)}^U(x)=|a+b\mu_A(x)+a_x\pi_A(x)|$, with fixed $a,b\in\mathbb{R}$ for all $A\in IFS(X)$

(iii) $\nu_{\Gamma(A)}^L(x)=|a'+b'\mu_A(x)-\beta_x\pi_A(x)|$, with fixed $a',b'\in\mathbb{R}$ for all $A\in IFS(X)$

(iv) $\nu_{\Gamma(A)}^U(x)=|a'+b'\mu_A(x)+\beta_x\pi_A(x)|$, with fixed $a',b'\in\mathbb{R}$ for all $A\in IFS(X)$

(v) If $A\in IFS(X)$ then $\Gamma(A)=A$

and by adding the modulus into the formula, the value of IVIFN will appear such that;

Let $a=a'=0, b=b'=1$ and with the condition $a_x+\beta_x\in[0,1]$, if $a_x=0.5$ and $\beta_x=0.5$ as the fuzzification, then:

(i) $\mu_{\Gamma(A)}^L(x)=|\mu_A(x)-a_x\pi_A(x)|$

(ii) $\mu_{\Gamma(A)}^U(x)=|\mu_A(x)+a_x\pi_A(x)|$

(iii) $\nu_{\Gamma(A)}^L(x)=|\mu_A(x)-\beta_x\pi_A(x)|$

(iv) $\nu_{\Gamma(A)}^U(x)=|\mu_A(x)+\beta_x\pi_A(x)|$

Definition 3. The operational relations are defined in [36-38] for $\tilde{A}_1,\tilde{A}_2\in IVIFS(X)$ and $\delta>0$.

$$\tilde{A}_1+\tilde{A}_2=\left(\begin{array}{l}\left[\begin{array}{l}\mu_{\tilde{A}_1}^L(x)+\mu_{\tilde{A}_2}^L(x)-\mu_{\tilde{A}_1}^L(x)\mu_{\tilde{A}_2}^L(x),\mu_{\tilde{A}_1}^U(x)\\ +\mu_{\tilde{A}_2}^U(x)-\mu_{\tilde{A}_1}^U(x)\mu_{\tilde{A}_2}^U(x)\end{array}\right],\\ \left[\nu_{\tilde{A}_1}^L(x)\nu_{\tilde{A}_2}^L(x),\nu_{\tilde{A}_1}^U(x)\nu_{\tilde{A}_2}^U(x)\right]\end{array}\right) \quad (3)$$

$$\tilde{A}_1\tilde{A}_2=\left(\begin{array}{l}\left[\mu_{\tilde{A}_1}^L(x)\mu_{\tilde{A}_2}^L(x),\mu_{\tilde{A}_1}^U(x)\mu_{\tilde{A}_2}^U(x)\right],\\ \left[\begin{array}{l}\nu_{\tilde{A}_1}^L(x)+\nu_{\tilde{A}_2}^L(x)-\nu_{\tilde{A}_1}^L(x)\nu_{\tilde{A}_2}^L(x),\nu_{\tilde{A}_1}^U(x)\\ +\nu_{\tilde{A}_2}^U(x)-\nu_{\tilde{A}_1}^U(x)\nu_{\tilde{A}_2}^U(x)\end{array}\right]\end{array}\right) \quad (4)$$

$$\tilde{A}_1\cap\tilde{A}_2=\left(\begin{array}{l}\left[\min(\mu_{\tilde{A}_1}^L(x),\mu_{\tilde{A}_2}^L(x)),\min(\mu_{\tilde{A}_1}^U(x),\mu_{\tilde{A}_2}^U(x))\right],\\ \left[\max(\nu_{\tilde{A}_1}^L(x),\nu_{\tilde{A}_2}^L(x)),\max(\nu_{\tilde{A}_1}^U(x),\nu_{\tilde{A}_2}^U(x))\right]\end{array}\right) \quad (5)$$

$$\tilde{A}_1\cup\tilde{A}_2=\left(\begin{array}{l}\left[\max(\mu_{\tilde{A}_1}^L(x),\mu_{\tilde{A}_2}^L(x)),\max(\mu_{\tilde{A}_1}^U(x),\mu_{\tilde{A}_2}^U(x))\right],\\ \left[\min(\nu_{\tilde{A}_1}^L(x),\nu_{\tilde{A}_2}^L(x)),\min(\nu_{\tilde{A}_1}^U(x),\nu_{\tilde{A}_2}^U(x))\right]\end{array}\right) \quad (6)$$

$$\delta\tilde{A}=\left(\begin{array}{l}\left[\left(1-(1-\mu_{\tilde{A}}^L(x))^\delta\right),\left(1-(1-\mu_{\tilde{A}}^U(x))^\delta\right)\right],\\ \left[(\nu_{\tilde{A}_j}^L(x))^\delta,(\nu_{\tilde{A}_j}^U(x))^\delta\right]\end{array}\right) \quad (7)$$

$$\tilde{A}^\delta=\left(\begin{array}{l}\left[(\mu_{\tilde{A}_j}^L(x))^\delta,(\mu_{\tilde{A}_j}^U(x))^\delta\right],\\ \left[\left(1-(1-\nu_{\tilde{A}}^L(x))^\delta\right),\left(1-(1-\nu_{\tilde{A}}^U(x))^\delta\right)\right]\end{array}\right) \quad (8)$$

ENTROPY FOR IVIFSs

In this section, the definition and properties of the IVIF entropy measure introduced by Liu et al. [13] are presented. In addition, the IVIF entropy measures to be compared are also listed chronologically.

Definition 1. [13] Let $A\in IvIFSs(X)$. A real-valued function $E:IvIFS(X)\to[0,1]$ is called an entropy for $IvIFSs(X)$, if E is satisfies the following requirements:

(1) $E(A)=0$, if $A=([1,1],[0,0])$ or $A=([0,0],[1,1])$ for each $x\in X$,

(2) $E(A)=1$, if and only if $\left[\mu_A^L(x),\mu_A^U(x)\right]=\left[\nu_A^L(x),\nu_A^U(x)\right]$ and $\nu_A(x)=[0,0]$ for all $x\in X$,

(3) $E(A)=E(A^c)$ for all $A\in IvIFSs(X)$,

(4) For two $IvIFSs$ A and B on X, if $A\le B$, then $E(A)\ge E(B)$

Definition 2. Suppose $A\in IvIFSs(X)$ and some of the existing entropy measures for IVIFSs are presented chronologically.

- Entropy measure developed by Liu et al. [13]:

$$E_{LZX}(A)=\frac{\sum_{i=1}^{n}\left[\begin{array}{l}2-\max(\mu_A^L(x_i),\nu_A^L(x_i))\\ -\max(\mu_A^U(x_i),\nu_A^U(x_i))\end{array}\right]}{\sum_{i=1}^{n}\left[\begin{array}{l}2-\min(\mu_A^L(x_i),\nu_A^L(x_i))\\ -\min(\mu_A^U(x_i),\nu_A^U(x_i))\end{array}\right]} \quad (9)$$

- Entropy measure developed by Ye [39]:

$$E_Y(A)=\frac{1}{n}\sum_{i=1}^{n}\left\{\left[\sqrt{2}\cos\frac{\begin{array}{c}\mu_A^L(x_i)+\mu_A^U(x_i)\\ -\nu_A^L(x_i)-\nu_A^U(x_i)\end{array}}{8}\pi-1\right]\times\frac{1}{\sqrt{2}-1}\right\} \quad (10)$$

- Entropy measure developed by Zhang et al. [11]:

$$E_{ZJJL}(A)=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\begin{array}{l}\min(\mu_A^L(x_i),\nu_A^L(x_i))\\ +\min(\mu_A^U(x_i),\nu_A^U(x_i))\end{array}}{\begin{array}{l}\max(\mu_A^L(x_i),\nu_A^L(x_i))\\ +\max(\mu_A^U(x_i),\nu_A^U(x_i))\end{array}}\right\} \quad (11)$$

- Entropy measure developed by Zhang & Jiang [17]:

$$E_{ZJ}^1(A)=1-\frac{1}{n}\sum_{i=1}^{n}\max\left(\left|\mu_A^L(x_i)-\nu_A^L(x_i)\right|,\left|\mu_A^U(x_i)-\nu_A^U(x_i)\right|\right) \quad (12)$$

$$E_{ZJ}^2(A)=1-\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A^L(x_i)-\nu_A^L(x_i)\right|+\left|\mu_A^U(x_i)-\nu_A^U(x_i)\right|\right) \quad (13)$$

- Entropy measure developed by Zhang et al. [19]:

$$E_{ZMSZ}(A)=\frac{1}{n}\sum_{i=1}^{n}\left(1-M_A(x_i)-N_A(x_i)\right)e^{1-M_A(x_i)-N_A(x_i)}$$

$$where: M_A(x_i)=\mu_A^L(x_i)+\delta\left(\mu_A^U(x_i)-\mu_A^L(x_i)\right),N_A(x_i)=\nu_A^L(x_i)+\delta\left(\nu_A^U(x_i)-\nu_A^L(x_i)\right),\delta\in[0,1] \quad (14)$$

- Entropy measure developed by Wei et al. [14]:

$$E^1_{WWZ}(A)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\min\left\{\mu^L_A(x_i),\nu^L_A(x_i)\right\}+\min\left\{\mu^U_A(x_i),\nu^U_A(x_i)\right\}+\pi^L_A(x_i)+\pi^U_A(x_i)}{\max\left\{\mu^L_A(x_i),\nu^L_A(x_i)\right\}+\max\left\{\mu^U_A(x_i),\nu^U_A(x_i)\right\}+\pi^L_A(x_i)+\pi^U_A(x_i)}\right) \quad (15)$$

$$E^2_{WWZ}(A)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{2-\left|\mu^L_A(x_i)-\nu^L_A(x_i)\right|-\left|\mu^U_A(x_i)-\nu^U_A(x_i)\right|+\pi^L_A(x_i)+\neq^U_A(x_i)}{2+\left|\mu^L_A(x_i)-\nu^L_A(x_i)\right|+\left|\mu^U_A(x_i)-\nu^U_A(x_i)\right|+\pi^L_A(x_i)+\pi^U_A(x_i)}\right) \quad (16)$$

- Entropy measure developed by Sun & Liu [40]:

$$E_{SL}(A)=1-\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left|\mu^L_A(x_i)-\nu^L_A(x_i)\right|+\left|\mu^U_A(x_i)-\nu^U_A(x_i)\right|}{2+\pi^L_A(x_i)+\pi^U_A(x_i)+\min\left\{\mu^L_A(x_i),\mu^U_A(x_i),\nu^L_A(x_i),\nu^U_A(x_i)\right\}}\right) \quad (17)$$

- Entropy measure developed by Jing [41]:

$$E_J(A)=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{2+\pi^L_A(x_i)+\pi^U_A(x_i)-\left|\mu^L_A(x_i)-\nu^L_A(x_i)\right|-\left|\mu^U_A(x_i)-\nu^U_A(x_i)\right|}{2+\pi^L_A(x_i)+\pi^U_A(x_i)}\right\} \quad (18)$$

- Entropy measure developed by Chen et al. [26]:

$$E^1_{CYWY}(A)=\frac{1}{n}\sum_{i=1}^{n}\cot\left(\frac{\pi}{4}+\frac{\left|\mu^L_A(x_i)-\nu^L_A(x_i)+\mu^U_A(x_i)-\nu^U_A(x_i)\right|}{4(4-\mu^L_A(x_i)-\nu^L_A(x_i)-\mu^U_A(x_i)-\nu^U_A(x_i))}\pi\right) \quad (19)$$

- Entropy measure developed by Chen et al. [42]:

$$E^2_{CYWY}(A)=1-\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left(\left(\mu^L_A(x_i)-\nu^L_A(x_i)\right)\times\left(1-\pi^L_A(x_i)\right)\right)^2+\left(\left(\mu^U_A(x_i)-\nu^U_A(x_i)\right)\times\left(1-\pi^U_A(x_i)\right)\right)^2} \quad (20)$$

- Entropy measure developed by Guo & Song [43]:

$$E_{GS}(A)=\frac{1}{n}\sum_{i=1}^{n}\left[1-\frac{1}{2}\left(\left|\mu^L_A(x_i)-\nu^L_A(x_i)\right|+\left|\mu^U_A(x_i)-\nu^U_A(x_i)\right|\right)\right]\frac{1+0.5\left(\pi^L_A(x_i)+\pi^U_A(x_i)\right)}{2} \quad (21)$$

- Entropy measure developed by Zhang et al. [20]:

$$E^1_{ZXLYT}(A)=1-2\left\{\frac{1}{4n}\sum_{i=1}^{n}\left[\left(\mu^L_A(x_i)-0.5\right)^2+\left(\mu^U_A(x_i)-0.5\right)^2+\left(\nu^L_A(x_i)-0.5\right)^2+\left(\nu^U_A(x_i)-0.5\right)^2\right]\right\}^{1/2} \quad (22)$$

$$E^2_{ZXLYT}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}\left[\left|\mu^L_A(x_i)-0.5\right|+\left|\mu^U_A(x_i)-0.5\right|+\left|\nu^L_A(x_i)-0.5\right|+\left|\nu^U_A(x_i)-0.5\right|\right] \quad (23)$$

$$E^3_{ZXLYT}(A)=1-\frac{1}{n}\sum_{i=1}^{n}\left[\max\left(\left|\mu^L_A(x_i)-0.5\right|,\left|\mu^U_A(x_i)-0.5\right|\right)+\max\left(\left|\nu^L_A(x_i)-0.5\right|,\left|\nu^U_A(x_i)-0.5\right|\right)\right] \quad (24)$$

- Entropy measure developed by Jin et al. [16]:

$$E^1_{JPCZ}(A)=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\min\left\{F_Q(\mu_A(x_i)),F_Q(\nu_A(x_i))\right\}+\pi_{F_Q}(A)}{\max\left\{F_Q(\mu_A(x_i)),F_Q(\nu_A(x_i))\right\}+\pi_{F_Q}(A)}\right\} \quad (25)$$

$$E^2_{JPCZ}(A)=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{1-\left|F_Q(\mu_A(x_i))-F_Q(\nu_A(x_i))\right|+\pi_{F_Q}(A)}{1+\left|F_Q(\mu_A(x_i))-F_Q(\nu_A(x_i))\right|+\pi_{F_Q}(A)}\right\} \quad (26)$$

where $\pi_{F_Q}(A)=1-F_Q(\mu_A(x_i))-F_Q(\nu_A(x_i))$;

$$F_Q(\mu_A(x_i))=F_Q\left[\mu^L_A(x_i),\mu^U_A(x_i)\right]=\lambda\mu^L_A(x_i)+(1-\lambda)\mu^U_A(x_i),$$

λ is the attitudinal parameter of basic unit interval monotonic (BUM) function Q.

$$E^1_{JPCZ}(A)_{\lambda=0.05}=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\min\left\{\mu_A^L(x_i),\nu_A^L(x_i)\right\}+\min\left\{\mu_A^U(x_i),\nu_A^U(x_i)\right\}+\pi_A^L(x_i)+\pi_A^U(x_i)}{\max\left\{\mu_A^L(x_i),\nu_A^L(x_i)\right\}+\max\left\{\mu_A^U(x_i),\nu_A^U(x_i)\right\}+\pi_A^L(x_i)+\pi_A^U(x_i)}\right\} \quad (27)$$

$$E^2_{JPCZ}(A)_{\lambda=0.05}=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{2-\left|\mu_A^L(x_i)-\nu_A^L(x_i)+\mu_A^U(x_i)-\nu_A^U(x_i)\right|+\pi_A^L(x_i)+\pi_A^U(x_i)}{2+\left|\mu_A^L(x_i)-\nu_A^L(x_i)+\mu_A^U(x_i)-\nu_A^U(x_i)\right|+\pi_A^L(x_i)+\pi_A^U(x_i)}\right\} \quad (28)$$

- Entropy measure developed by Xu & Shen [44]:

$$E_{XS}(A)=-\frac{1}{n\ln 2}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\mu_A^L(x_i)+pW_{\mu_A}(x_i)\right)\\ \times\ln\left(\mu_A^L(x_i)+pW_{\mu_A}(x_i)\right)\\ +\left(\nu_A^L(x_i)+qW_{\nu_A}(x_i)\right)\\ \times\ln\left(\nu_A^L(x_i)+qW_{\nu_A}(x_i)\right)\\ -\left(\mu_A^L(x_i)+pW_{\mu_A}(x_i)+\nu_A^L(x_i)+qW_{\nu_A}(x_i)\right)\\ \times\ln\left(\mu_A^L(x_i)+pW_{\mu_A}(x_i)+\nu_A^L(x_i)+qW_{\nu_A}(x_i)\right)\\ -\left(1-\mu_A^L(x_i)-pW_{\mu_A}(x_i)-\nu_A^L(x_i)-qW_{\nu_A}(x_i)\right)\\ \times\ln 2\end{array}\right] \quad (29)$$

where $W_{\mu_A}(x_i)=\mu_A^U(x_i)-\mu_A^L(x_i), W_{\nu_A}(x_i)=\nu_A^U(x_i)-\nu_A^L(x_i)$ *and* $p,q\in[0,1]$

Table 1. Comparison of the fuzziness under A

	E_{LZX}	E_Y	E^1_{ZJ}	E^2_{ZJ}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E^1_{WWZ}	E^2_{WWZ}	E_{SL}
$A^{(1)}$	0.3294	0.4423	0.4200	0.4300	0.2819	0.1547	0.3796	0.3796	0.6869
$A^{(3/2)}$	0.2978	0.4055	0.3697	0.3800	0.2036	0.1693	0.3223	0.3223	0.6495
$A^{(2)}$	0.2886	0.3766	0.3540	0.3690	0.2103	0.1771	0.3185	0.3185	0.6289
$A^{(5/2)}$	0.2882	0.3587	0.3551	0.3679	0.2477	0.1828	0.3316	0.3316	0.6169
$A^{(3)}$	0.2700	0.3468	0.3242	0.3423	0.2102	0.1875	0.3061	0.3061	0.5898
			E^b_{CYWY}	E_{GS}	E^1_{ZXLYT}	E^2_{ZXLYT}	E^3_{ZXLYT}	$E^1_{JPCZ(\lambda=0.5)}$	$E^2_{JPCZ(\lambda=0.5)}$
$A^{(1)}$	0.4792	0.4276	0.3909	0.2655	0.3203	0.4100	0.3200	0.3796	0.3796
$A^{(3/2)}$	0.4412	0.3793	0.3808	0.2376	0.3006	0.3761	0.2841	0.3223	0.3223
$A^{(2)}$	0.4231	0.3693	0.3521	0.2310	0.2726	0.3390	0.2360	0.3185	0.3185
$A^{(5/2)}$	0.4137	0.3721	0.3230	0.2334	0.2469	0.3092	0.2041	0.3316	0.3367
$A^{(3)}$	0.3869	0.3455	0.2981	0.2193	0.2250	0.2941	0.1876	0.3061	0.3061
	$E_{XS(p=q=0.5)}$	E^1_{ZX}	E^2_{ZX}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}	E_{WZ}
$A^{(1)}$	UNDEF	0.3990	0.2513	0.5772	0.4000	0.2513	0.2650	0.5760	0.6000
$A^{(3/2)}$	UNDEF	0.3955	0.2276	0.5571	0.3618	0.2276	0.2438	0.5557	0.5848
$A^{(2)}$	UNDEF	0.3785	0.2246	0.5227	0.3320	0.2246	0.2295	0.5214	0.5498
$A^{(5/2)}$	UNDEF	0.3602	0.2295	0.4900	0.3038	0.2295	0.2288	0.4888	0.5143
$A^{(3)}$	UNDEF	0.3438	0.2169	0.4618	0.3000	0.2169	0.2028	0.4608	0.4837

Table 2. Comparison of the fuzziness under B

	E_{LZX}	E_{Y}	E_{ZJ}^{1}	E_{ZJ}^{2}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^{1}	E_{WWZ}^{2}	E_{SL}
$B^{(1)}$	0.3504	0.4509	0.4400	0.4530	0.2905	0.1619	0.3796	0.3952	0.6869
$B^{(3/2)}$	0.3273	0.4234	0.3994	0.4171	0.2169	0.1800	0.3223	0.3457	0.6495
$B^{(2)}$	0.3268	0.4058	0.3930	0.4134	0.2285	0.1911	0.3185	0.3497	0.6289
$B^{(5/2)}$	0.3351	0.4007	0.4033	0.4225	0.2712	0.2000	0.3316	0.3705	0.6169
$B^{(3)}$	0.3241	0.4023	0.3813	0.4067	0.2392	0.2076	0.3061	0.3528	0.5898
	E_{J}	E_{CYWY}^{a}	E_{CYWY}^{b}	E_{GS}	E_{ZXLYT}^{1}	E_{ZXLYT}^{2}	E_{ZXLYT}^{3}	$E_{JPCZ(\lambda=0.5)}^{1}$	$E_{JPCZ(\lambda=0.5)}^{2}$
$B^{(1)}$	0.5082	0.4504	0.4372	0.2774	0.3526	0.4330	0.3360	0.3952	0.3952
$B^{(3/2)}$	0.4831	0.4125	0.4455	0.2555	0.3456	0.4100	0.3078	0.3457	0.3457
$B^{(2)}$	0.4771	0.4123	0.4299	0.2546	0.3277	0.3834	0.2673	0.3497	0.3497
$B^{(5/2)}$	0.4789	0.4245	0.4105	0.2628	0.3103	0.3637	0.2428	0.3705	0.3757
$B^{(3)}$	0.4627	0.4068	0.3935	0.2544	0.2955	0.3585	0.2336	0.3528	0.3528
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^{1}	E_{ZX}^{2}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$B^{(1)}$	0.6331	0.6451	0.4274	0.2632	0.6222	0.4260	0.2632	0.2770	0.6206
$B^{(3/2)}$	0.6289	0.6494	0.4361	0.2454	0.6212	0.4000	0.2454	0.2616	0.6193
$B^{(2)}$	0.6105	0.6321	0.4301	0.2482	0.6040	0.3818	0.2482	0.2530	0.6021
$B^{(5/2)}$	0.5928	0.6123	0.4218	0.2589	0.5866	0.3648	0.2589	0.2579	0.5848
$B^{(3)}$	0.5778	0.5958	0.4143	0.2521	0.5722	0.3717	0.2521	0.2372	0.5706

- Entropy measure developed by Wei & Zhang [29]:

$$E_{WZ}(A)=\frac{1}{n}\sum_{i=1}^{n}\cos\left\{\frac{\left|\mu_A^L(x_i)-\nu_A^L(x_i)\right|+\left|\mu_A^U(x_i)-\nu_A^U(x_i)\right|}{2(2+\pi_A^L(x_i)+\pi_A^U(x_i))}\pi\right\} \tag{30}$$

- Entropy measure developed by Zhao & Xu [45]:

$$E_{ZX}^{1}(A)=1-\frac{1}{n}\sum_{i=1}^{n}\frac{\left|\mu_A^L(x_i)-\nu_A^L(x_i)\right|^2+\left|\mu_A^U(x_i)-\nu_A^U(x_i)\right|^2+\left(1-\pi_A^L(x_i)\right)^2+\left(1-\pi_A^U(x_i)\right)^2}{4} \tag{31}$$

$$E_{ZX}^{2}(A)=\frac{1}{n}\sum_{i=1}^{n}\frac{\left(2-\left|\mu_A^L(x_i)-\nu_A^L(x_i)\right|-\left|\mu_A^U(x_i)-\nu_A^U(x_i)\right|\right)\left(2+\pi_A^L(x_i)+\pi_A^U(x_i)\right)}{8} \tag{32}$$

- Entropy measure developed by Rani et al. [46]:

$$E_{RJH}(A)=\frac{1}{n\left(\sqrt{e}-1\right)}\sum_{i=1}^{n}\left\{\left(\frac{(\mu_A^L(x_i)+\mu_A^U(x_i))+2-(\nu_A^L(x_i)+\nu_A^U(x_i))}{4}\right)e^{\left(\frac{(\nu_A^L(x_i)+\nu_A^U(x_i))+2-(\mu_A^L(x_i)+\mu_A^U(x_i))}{4}\right)}+\left(\frac{(\nu_A^L(x_i)+\nu_A^U(x_i))+2-(\mu_A^L(x_i)+\mu_A^U(x_i))}{4}\right)e^{\left(\frac{(\mu_A^L(x_i)+\mu_A^U(x_i))+2-(\nu_A^L(x_i)+\nu_A^U(x_i))}{4}\right)}-1\right\} \tag{33}$$

Table 3. Comparison of the fuzziness under C

	E_{LZX}	E_{Y}	E_{ZJ}^{1}	E_{ZJ}^{2}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^{1}	E_{WWZ}^{2}	E_{SL}
$C^{(1)}$	0.3974	0.5693	0.5200	0.5450	0.3869	0.0961	0.3880	0.4130	0.7050
$C^{(3/2)}$	0.3871	0.5975	0.5111	0.5399	0.3525	0.0961	0.3370	0.3839	0.6745
$C^{(2)}$	0.4003	0.6111	0.5320	0.5603	0.3998	0.0927	0.3410	0.4187	0.6598
$C^{(5/2)}$	0.4224	0.6155	0.5658	0.5880	0.4760	0.0891	0.3632	0.4831	0.6529
$C^{(3)}$	0.4140	0.6099	0.5640	0.5834	0.4721	0.0855	0.3484	0.5115	0.6300
	E_{J}	E_{CYWY}^{a}	E_{CYWY}^{b}	E_{GS}	E_{ZXLYT}^{1}	E_{ZXLYT}^{2}	E_{ZXLYT}^{3}	$E_{JPCZ(\lambda=0.5)}^{1}$	$E_{JPCZ(\lambda=0.5)}^{2}$
$C^{(1)}$	0.5357	0.4729	0.4905	0.3071	0.4437	0.5250	0.4200	0.4130	0.4130
$C^{(3/2)}$	0.5354	0.4569	0.5275	0.2880	0.4402	0.5036	0.4129	0.3839	0.3839
$C^{(2)}$	0.5593	0.4861	0.5251	0.2870	0.4137	0.4783	0.3610	0.4187	0.4187
$C^{(5/2)}$	0.5938	0.5360	0.5053	0.2936	0.3839	0.4467	0.3303	0.4831	0.4883
$C^{(3)}$	0.6022	0.5544	0.4789	0.2822	0.3562	0.4323	0.3183	0.5115	0.5252
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^{1}	E_{ZX}^{2}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$C^{(1)}$	0.7882	0.6853	0.4003	0.2928	0.7542	0.5300	0.2928	0.2525	0.7524
$C^{(3/2)}$	0.7871	0.7198	0.3847	0.2780	0.7687	0.4887	0.2780	0.2045	0.7671
$C^{(2)}$	0.7610	0.7296	0.3549	0.2806	0.7515	0.4875	0.2806	0.1754	0.7502
$C^{(5/2)}$	0.7302	0.7257	0.3248	0.2897	0.7252	0.4562	0.2897	0.1681	0.7241
$C^{(3)}$	0.6994	0.7065	0.2979	0.2798	0.6965	0.4478	0.2798	0.1409	0.6957

- Entropy measure developed by Rashid et al. [21]:

$$E_{RFZ}(A)=1-2d_{FZ}(A,F_X) \quad (34)$$

where $d_{FZ}(A,F_X)=\frac{1}{2}(\underline{d}+\overline{d})$

$$\underline{d}=\frac{1}{n}\sum_{i=1}^{n}\min\left\{\left|\mu_A^L(x_i)-\frac{1}{2}\right|,\left|\nu_A^L(x_i)-\frac{1}{2}\right|,\left|\mu_A^U(x_i)-\frac{1}{2}\right|,\left|\nu_A^U(x_i)-\frac{1}{2}\right|\right\} \quad (35)$$

$$\overline{d}=\frac{1}{n}\sum_{i=1}^{n}\max\left\{\left|\mu_A^L(x_i)-\frac{1}{2}\right|,\left|\nu_A^L(x_i)-\frac{1}{2}\right|,\left|\mu_A^U(x_i)-\frac{1}{2}\right|,\left|\nu_A^U(x_i)-\frac{1}{2}\right|\right\} \quad (36)$$

- Entropy measure developed by Xian et al. [22]:

$$E_{XDLJ}(A)=\frac{1}{2n}\sum_{i=1}^{n}\frac{\left(1-\frac{\left|\mu_A^L(x_i)-\nu_A^L(x_i)\right|+\left|\mu_A^U(x_i)-\nu_A^U(x_i)\right|}{2}\right)}{\left(1+\frac{\left|\pi_A^L(x_i)+\pi_A^U(x_i)\right|}{2}\right)} \quad (37)$$

- Entropy measure developed by Tiwari & Gupta [23]:

$$E_{TG}(A)=1-\left(\frac{3}{4n}\sum_{i=1}^{n}\left[\begin{array}{c}\max\left\{\left|\mu_A^L(x_i)-\frac{1}{3}\right|,\left|\mu_A^U(x_i)-\frac{1}{3}\right|\right\}\\+\max\left\{\left|\nu_A^L(x_i)-\frac{1}{3}\right|,\left|\nu_A^U(x_i)-\frac{1}{3}\right|\right\}+\\\max\left\{\left|\pi_A^L(x_i)-\frac{1}{3}\right|,\left|\pi_A^U(x_i)-\frac{1}{3}\right|\right\}\end{array}\right]\right) \quad (38)$$

- Entropy measure developed by Mishra et al. [24]:

$$E_{MRPMSP}(A) = \frac{1}{n\sqrt{e}\left(\sqrt{e}-1\right)} \sum_{i=1}^{n} \left[\begin{array}{l} e - \left(\frac{\left(\begin{array}{c} \left(\nu_A^L(x_i)+\nu_A^U(x_i)\right)+2 \\ -\left(\mu_A^L(x_i)+\mu_A^U(x_i)\right) \end{array} \right)}{4} \right) \\ \times e^{\left(\frac{\left(\left(\nu_A^L(x_i)+\nu_A^U(x_i)\right)+2-\left(\mu_A^L(x_i)+\mu_A^U(x_i)\right)\right)}{4} \right)} \\ - \left(\frac{\left(\begin{array}{c} \left(\mu_A^L(x_i)+\mu_A^U(x_i)\right)+2 \\ -\left(\nu_A^L(x_i)+\nu_A^U(x_i)\right) \end{array} \right)}{4} \right) \\ \times e^{\left(\frac{\left(\left(\mu_A^L(x_i)+\mu_A^U(x_i)\right)+2-\left(\nu_A^L(x_i)+\nu_A^U(x_i)\right)\right)}{4} \right)} \end{array} \right] \quad (39)$$

COMPARISON OF ENTROPY MEASURES

In this section, to understand whether a variation of fuzziness degree of one or more elements of IVIFSs changes the ranking results, ten different tests are conducted.

Test 1. Suppose that $A = \left\{ \left(x, \left[\mu_A^L(x_i), \mu_A^U(x_i) \right], \left[\nu_A^L(x_i), \nu_A^U(x_i) \right] \middle| x \in X \right) \right\}$ an IVIFS in $X = \{6,7,8,9,10\}$ that is defined by [20].

$$A = \left\{ \begin{array}{l} \langle 6,[0.1,0.2],[0.6,0.7]\rangle, \langle 7,[0.3,0.5],[0.4,0.5]\rangle, \\ \langle 8,[0.6,0.7],[0.1,0.2]\rangle, \langle 9,[0.8,0.9],[0.0,0.1]\rangle, \\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle \end{array} \right\}$$

In order to state the linguistic variables' characterization, A as "High" in X. By using the exponentiation operator presented in Eq. (8), the following IVIFS pertaining to A can be calculated. In terms of mathematical operations, the entropy of these IVIFSs should have the below ranking:

$$E(A) > E(A^{1.5}) > E(A^2) > E(A^{2.5}) > E(A^3).$$

$A^{1.5}$ can be considered as "Medium High";
A^2 can be considered as "Very High";
$A^{2.5}$ can be considered as "Quite Very High";
A^3 can be considered as "Absolutely High".

$$A^{1.5} = \left\{ \begin{array}{l} \langle 6,[0.032,0.089],[0.747,0.836]\rangle, \\ \langle 7,[0.164,0.354],[0.535,0.646]\rangle, \\ \langle 8,[0.465,0.586],[0.146,0.284]\rangle \\ ,\langle 9,[0.716,0.854],[0.0,0.146]\rangle, \\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle \end{array} \right\}$$

$$A^{2} = \left\{ \begin{array}{l} \langle 6,[0.01,0.04],[0.84,0.91]\rangle, \\ \langle 7,[0.09,0.25],[0.64,0.75]\rangle, \\ \langle 8,[0.36,0.49],[0.19,0.36]\rangle, \\ \langle 9,[0.64,0.81],[0.0,0.19]\rangle, \\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle \end{array} \right\}$$

$$A^{2.5} = \left\{ \begin{array}{l} \langle 6,[0.003,0.018],[0.899,0.951]\rangle, \\ \langle 7,[0.049,0.177],[0.721,0.823]\rangle, \\ \langle 8,[0.279,0.410],[0.232,0.428]\rangle, \\ \langle 9,[0.572,0.768],[0.0,0.232]\rangle, \\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle \end{array} \right\}$$

$$A^{3} = \left\{ \begin{array}{l} \langle 6,[0.001,0.008],[0.936,0.973]\rangle, \\ \langle 7,[0.027,0.125],[0.784,0.875]\rangle, \\ \langle 8,[0.216,0.343],[0.271,0.488]\rangle, \\ \langle 9,[0.512,0.729],[0.0,0.271]\rangle, \\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle \end{array} \right\}$$

Test 2. In order to understand whether a variation of fuzziness degree of the one element of IVIFS change the ranking results, the degree of fuzziness of the element '10' which is the last point, is decreased to a lower level**.** The modified IVIFS $B = \left\{ \left(x, \left[\mu_B^L(x_i), \mu_B^U(x_i) \right], \left[\nu_B^L(x_i), \nu_B^U(x_i) \right] \middle| x \in X \right) \right\}$ in $X = \{6,7,8,9,10\}$ is denoted as follow.

$$B = \left\{ \begin{array}{l} \langle 6,[0.1,0.2],[0.6,0.7]\rangle, \langle 7,[0.3,0.5],[0.4,0.5]\rangle, \\ \langle 8,[0.6,0.7],[0.1,0.2]\rangle, \langle 9,[0.8,0.9],[0.0,0.1]\rangle, \\ \langle 10,[0.90,0.95],[0.03,0.05]\rangle \end{array} \right\}$$

Test 3. In order to understand whether a variation of fuzziness degree of the two elements of IVIFSs change the ranking results, the degrees of fuzziness of the elements '9' and '10' are changed**.** The modified IVIFS $C = \left\{ \left(x, \left[\mu_C^L(x_i), \mu_C^U(x_i) \right], \left[\nu_C^L(x_i), \nu_C^U(x_i) \right] \middle| x \in X \right) \right\}$ in $X = \{6,7,8,9,10\}$ is denoted as follow.

Table 4. Comparison of the fuzziness under D

	E_{LZX}	E_{Y}	E^1_{ZJ}	E^2_{ZJ}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E^1_{WWZ}	E^2_{WWZ}	E_{SL}
$D^{(1)}$	0.3159	0.4581	0.4080	0.4390	0.3067	0.0888	0.3626	0.3626	0.6853
$D^{(3/2)}$	0.2893	0.4334	0.3769	0.4056	0.2641	0.0883	0.3238	0.3238	0.6534
$D^{(2)}$	0.2750	0.4101	0.3793	0.3894	0.2718	0.0837	0.3203	0.3203	0.6304
$D^{(5/2)}$	0.2577	0.3891	0.3459	0.3692	0.2611	0.0785	0.3057	0.3057	0.6053
$D^{(3)}$	0.2340	0.3683	0.3186	0.3395	0.2293	0.0733	0.2775	0.2775	0.5750
	E_{J}	E^a_{CYWY}	E^b_{CYWY}	E_{GS}	E^1_{ZXLYT}	E^2_{ZXLYT}	E^3_{ZXLYT}	$E^1_{JPCZ(\lambda=0.5)}$	$E^2_{JPCZ(\lambda=0.5)}$
$D^{(1)}$	0.4665	0.4127	0.3824	0.2548	0.3475	0.4310	0.3620	0.3626	0.3626
$D^{(3/2)}$	0.4352	0.3773	0.3643	0.2331	0.3274	0.3986	0.3331	0.3238	0.3238
$D^{(2)}$	0.4154	0.3664	0.3341	0.2220	0.2997	0.3749	0.3025	0.3203	0.3439
$D^{(5/2)}$	0.3926	0.3476	0.3059	0.2089	0.2740	0.3547	0.2860	0.3057	0.3057
$D^{(3)}$	0.3631	0.3183	0.2817	0.1902	0.2515	0.3371	0.2697	0.2775	0.2775
	$E_{XS(p=q=0.5)}$	E_{WZ}	E^1_{ZX}	E^2_{ZX}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$D^{(1)}$	UNDEF	0.5922	0.3661	0.2437	0.5972	0.4200	0.2437	0.2650	0.5959
$D^{(3/2)}$	UNDEF	0.5693	0.3521	0.2257	0.5713	0.3874	0.2257	0.2542	0.5700
$D^{(2)}$	UNDEF	0.5304	0.3287	0.2176	0.5327	0.3763	0.2176	0.2309	0.5316
$D^{(5/2)}$	UNDEF	0.4963	0.3054	0.2064	0.4955	0.3482	0.2064	0.2184	0.4945
$D^{(3)}$	UNDEF	0.4642	0.2840	0.1888	0.4617	0.3351	0.1888	0.1980	0.4608

$$C=\left\{\begin{array}{l}\langle 6,[0.1,0.2],[0.6,0.7]\rangle,\langle 7,[0.3,0.5],[0.4,0.5]\rangle,\\ \langle 8,[0.6,0.7],[0.1,0.2]\rangle,\langle 9,[0.8,0.85],[0.1,0.15]\rangle,\\ \langle 10,[0.9,0.95],[0.03,0.05]\rangle\end{array}\right\}$$

Test 4. In order to understand whether an increase of non-membership degrees of the more than one element of IVIFS change the ranking results, non-membership degrees of the elements '6', '7', '8' and '9' are increased to a higher level. The modified IVIFS $D=\left\{\left(x,\left[\mu_D^L(x_i),\mu_D^U(x_i)\right],\left[\nu_D^L(x_i),\nu_D^U(x_i)\right]\middle| x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

$$D=\left\{\begin{array}{l}\langle 6,[0.1,0.2],[0.7,0.75]\rangle,\langle 7,[0.3,0.5],[0.46,0.5]\rangle,\\ \langle 8,[0.6,0.7],[0.2,0.25]\rangle,\langle 9,[0.8,0.9],[0.1,0.15]\rangle,\\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle\end{array}\right\}$$

Test 5. In order to understand whether a decrease of non-membership degrees of the more than one element of IVIFS change the ranking results, non-membership degrees of the elements '6', '7' and '8' are decreased to a lower level. The modified IVIFS $F=\left\{\left(x,\left[\mu_F^L(x_i),\mu_F^U(x_i)\right],\left[\nu_F^L(x_i),\nu_F^U(x_i)\right]\middle| x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

$$F=\left\{\begin{array}{l}\langle 6,[0.1,0.2],[0.45,0.7]\rangle,\langle 7,[0.3,0.5],[0.37,0.5]\rangle,\\ \langle 8,[0.6,0.7],[0.08,0.2]\rangle,\langle 9,[0.8,0.9],[0.0,0.1]\rangle,\\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle\end{array}\right\}$$

Test 6. In order to understand whether an increase of membership degrees of the more than one element of IVIFS change the ranking results, mem-

Table 5. Comparison of the fuzziness under F

	E_{LZX}	E_{Y}	E_{ZJ}^{1}	E_{ZJ}^{2}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^{1}	E_{WWZ}^{2}	E_{SL}
$F^{(1)}$	0.3498	0.4619	0.4220	0.4460	0.2909	0.1852	0.4013	0.4013	0.7035
$F^{(3/2)}$	0.3184	0.4281	0.3711	0.3993	0.2025	0.2010	0.3414	0.3414	0.6712
$F^{(2)}$	0.3076	0.3946	0.3541	0.3833	0.2035	0.2070	0.3337	0.3337	0.6522
$F^{(5/2)}$	0.3046	0.3724	0.3515	0.3795	0.2363	0.2104	0.3422	0.3422	0.6391
$F^{(3)}$	0.2906	0.3580	0.3256	0.3609	0.2210	0.2129	0.3269	0.3269	0.6164
	E_{J}	E_{CYWY}^{a}	E_{CYWY}^{b}	E_{GS}	E_{ZXLYT}^{1}	E_{ZXLYT}^{2}	E_{ZXLYT}^{3}	$E_{JPCZ(\lambda=0.5)}^{1}$	$E_{JPCZ(\lambda=0.5)}^{2}$
$F^{(1)}$	0.4976	0.4474	0.4030	0.2829	0.3180	0.4100	0.3100	0.4013	0.4013
$F^{(3/2)}$	0.4641	0.3998	0.4062	0.2531	0.3053	0.3923	0.2784	0.3414	0.3414
$F^{(2)}$	0.4469	0.3883	0.3843	0.2437	0.2812	0.3533	0.2287	0.3337	0.3337
$F^{(5/2)}$	0.4360	0.3882	0.3562	0.2436	0.2566	0.3208	0.1954	0.3422	0.3471
$F^{(3)}$	0.4139	0.3696	0.3292	0.2337	0.2343	0.3028	0.1777	0.3269	0.3269
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^{1}	E_{ZX}^{2}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$F^{(1)}$	UNDEF	0.6138	0.4174	0.2670	0.5890	0.4060	0.2670	0.2620	0.5878
$F^{(3/2)}$	UNDEF	0.6121	0.4206	0.2416	0.5790	0.3941	0.2416	0.2395	0.5776
$F^{(2)}$	UNDEF	0.5844	0.4062	0.2361	0.5489	0.3606	0.2361	0.2240	0.5475
$F^{(5/2)}$	UNDEF	0.5508	0.3878	0.2388	0.5168	0.3270	0.2388	0.2180	0.5154
$F^{(3)}$	UNDEF	0.5189	0.3695	0.2307	0.4873	0.3273	0.2307	0.1953	0.4861

bership degrees of the elements '6', '7', '8' and '9' are increased to a higher level. The modified IVIFS $G=\left\{\left(x,\left[\mu_G^L(x_i),\mu_G^U(x_i)\right],\left[\nu_G^L(x_i),\nu_G^U(x_i)\right]\right|x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

$$G=\left\{\begin{array}{l}\langle 6,[0.15,0.2],[0.6,0.7]\rangle,\langle 7,[0.37,0.5],[0.4,0.5]\rangle,\\ \langle 8,[0.7,0.75],[0.1,0.2]\rangle,\langle 9,[0.87,0.9],[0.0,0.1]\rangle,\\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle\end{array}\right\}$$

Test 7. In order to understand whether a decrease of membership degrees of the more than one element of IVIFS change the ranking results, membership degrees of the elements '6', '7', '8', '9' and '10' are decreased to a lower level. The modified IVIFS $H=\left\{\left(x,\left[\mu_H^L(x_i),\mu_H^U(x_i)\right],\left[\nu_H^L(x_i),\nu_H^U(x_i)\right]\right|x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

$$H=\left\{\begin{array}{l}\langle 6,[0.06,0.2],[0.6,0.7]\rangle,\langle 7,[0.27,0.5],[0.4,0.5]\rangle,\\ \langle 8,[0.54,0.7],[0.1,0.2]\rangle,\langle 9,[0.77,0.9],[0.0,0.1]\rangle,\\ \langle 10,[0.90,1.0],[0.0,0.0]\rangle\end{array}\right\}$$

Test 8. In order to examine the situation of membership and non-membership degrees have non-zero values, the membership degrees and non-membership degrees of the elements '9' and '10' are changed with the non-zero values. The modified IVIFS $I=\left\{\left(x,\left[\mu_I^L(x_i),\mu_I^U(x_i)\right],\left[\nu_I^L(x_i),\nu_I^U(x_i)\right]\right|x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

Table 6. Comparison of the fuzziness under G

	E_{LZX}	E_{Y}	E_{ZJ}^{1}	E_{ZJ}^{2}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^{1}	E_{WWZ}^{2}	E_{SL}
$G^{(1)}$	0.3079	0.4205	0.4000	0.4200	0.2999	0.1097	0.3685	0.3685	0.6670
$G^{(3/2)}$	0.2685	0.3800	0.3385	0.3638	0.2039	0.1181	0.2938	0.2938	0.6241
$G^{(2)}$	0.2543	0.3515	0.3140	0.3430	0.1916	0.1215	0.2800	0.2800	0.6007
$G^{(5/2)}$	0.2533	0.3378	0.3073	0.3412	0.2149	0.1242	0.2916	0.2916	0.5902
$G^{(3)}$	0.2501	0.3317	0.3109	0.3364	0.2282	0.1272	0.2963	0.2963	0.5785
	E_{J}	E_{CYWY}^{a}	E_{CYWY}^{b}	E_{GS}	E_{ZXLYT}^{1}	E_{ZXLYT}^{2}	E_{ZXLYT}^{3}	$E_{JPCZ(\lambda=0.5)}^{1}$	$E_{JPCZ(\lambda=0.5)}^{2}$
$G^{(1)}$	0.4574	0.4119	0.3624	0.2482	0.3146	0.4000	0.3340	0.3685	0.3685
$G^{(3/2)}$	0.4119	0.3508	0.3565	0.2147	0.2991	0.3638	0.2888	0.2938	0.2938
$G^{(2)}$	0.3908	0.3339	0.3323	0.2028	0.2751	0.3410	0.2634	0.2800	0.2800
$G^{(5/2)}$	0.3835	0.3367	0.3066	0.2033	0.2531	0.3206	0.2382	0.2916	0.2916
$G^{(3)}$	0.3740	0.3346	0.2845	0.2024	0.2345	0.3026	0.2182	0.2963	0.3164
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^{1}	E_{ZX}^{2}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$G^{(1)}$	UNDEF	0.5665	0.3611	0.2369	0.5550	0.3900	0.2369	0.2575	0.5537
$G^{(3/2)}$	UNDEF	0.5554	0.3576	0.2063	0.5378	0.3500	0.2063	0.2377	0.5363
$G^{(2)}$	UNDEF	0.5254	0.3424	0.1972	0.5065	0.3205	0.1972	0.2167	0.5050
$G^{(5/2)}$	UNDEF	0.4950	0.3268	0.1998	0.4773	0.2970	0.1998	0.2116	0.4760
$G^{(3)}$	UNDEF	0.4681	0.3133	0.2003	0.4528	0.2869	0.2003	0.2014	0.4517

$$I=\left\{\begin{array}{l}\langle 6,[0.1,0.2],[0.6,0.7]\rangle,\langle 7,[0.3,0.5],[0.4,0.5]\rangle,\\ \langle 8,[0.6,0.7],[0.1,0.2]\rangle,\langle 9,[0.8,0.9],[0.05,0.1]\rangle,\\ \langle 10,[0.9,0.92],[0.05,0.08]\rangle\end{array}\right\}$$

Test 9. In order to understand whether a decrease of hesitation degrees of the more than one element of IVIFS change the ranking results, hesitation degrees of the elements '6', '7', '8', and '9' are decreased to a lower level. The modified IVIFS $J=\left\{\left(x,\left[\mu_J^L(x_i),\mu_J^U(x_i)\right],\left[\nu_J^L(x_i),\nu_J^U(x_i)\right]\middle|x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

$$J=\left\{\begin{array}{l}\langle 6,[0.1,0.2],[0.75,0.75]\rangle,\langle 7,[0.3,0.5],[0.45,0.5]\rangle,\\ \langle 8,[0.6,0.7],[0.2,0.25]\rangle,\langle 9,[0.8,0.9],[0.05,0.1]\rangle,\\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle\end{array}\right\}$$

Test 10. In order to understand whether an increase of hesitation degrees of the more than one element of IVIFS change the ranking results, hesitation degrees of the elements '6', '7', '8', and '9' are increased to a higher level. The modified IVIFS $K=\left\{\left(x,\left[\mu_J^L(x_i),\mu_J^U(x_i)\right],\left[\nu_J^L(x_i),\nu_J^U(x_i)\right]\middle|x\in X\right)\right\}$ in $X=\{6,7,8,9,10\}$ is denoted as follow.

$$K=\left\{\begin{array}{l}\langle 6,[0.1,0.2],[0.55,0.65]\rangle,\langle 7,[0.3,0.5],[0.35,0.45]\rangle,\\ \langle 8,[0.6,0.7],[0.05,0.15]\rangle,\langle 9,[0.8,0.9],[0.0,0.05]\rangle,\\ \langle 10,[1.0,1.0],[0.0,0.0]\rangle\end{array}\right\}$$

DISCUSSION AND CONCLUSION

When studies comparing entropies [20,21,23, 47] are examined, it is seen that a comparison is not as comprehensive

Table 7. Comparison of the fuzziness under H

	E_{LZX}	E_{Y}	E_{ZJ}^{1}	E_{ZJ}^{2}	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^{1}	E_{WWZ}^{2}	E_{SL}
$H^{(1)}$	0.3489	0.4496	0.4060	0.4420	0.2715	0.1547	0.3796	0.3796	0.6869
$H^{(3/2)}$	0.3252	0.4170	0.3700	0.4045	0.2035	0.1693	0.3223	0.3223	0.6495
$H^{(2)}$	0.3222	0.3903	0.3626	0.3972	0.2193	0.1771	0.3185	0.3185	0.6289
$H^{(5/2)}$	0.3243	0.3737	0.3659	0.3979	0.2552	0.1828	0.3316	0.3316	0.6169
$H^{(3)}$	0.3039	0.3631	0.3335	0.3683	0.1959	0.1875	0.3061	0.3061	0.5898
	E_{J}	E_{CYWY}^{a}	E_{CYWY}^{b}	E_{GS}	E_{ZXLYT}^{1}	E_{ZXLYT}^{2}	E_{ZXLYT}^{3}	$E_{JPCZ(\lambda=0.5)}^{1}$	$E_{JPCZ(\lambda=0.5)}^{2}$
$H^{(1)}$	0.4792	0.4276	0.3909	0.2655	0.3203	0.4100	0.3200	0.3796	0.3796
$H^{(3/2)}$	0.4412	0.3793	0.3808	0.2376	0.3006	0.3761	0.2841	0.3223	0.3223
$H^{(2)}$	0.4231	0.3693	0.3521	0.2310	0.2726	0.3390	0.2360	0.3185	0.3185
$H^{(5/2)}$	0.4137	0.3721	0.3230	0.2334	0.2469	0.3092	0.2041	0.3316	0.3367
$H^{(3)}$	0.3869	0.3455	0.2981	0.2193	0.2250	0.2941	0.1876	0.3061	0.3061
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^{1}	E_{ZX}^{2}	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$H^{(1)}$	UNDEF	0.6000	0.3990	0.2513	0.5772	0.4000	0.2513	0.2650	0.5760
$H^{(3/2)}$	UNDEF	0.5848	0.3955	0.2276	0.5571	0.3618	0.2276	0.2438	0.5557
$H^{(2)}$	UNDEF	0.5498	0.3785	0.2246	0.5227	0.3320	0.2246	0.2295	0.5214
$H^{(5/2)}$	UNDEF	0.5143	0.3602	0.2295	0.4900	0.3038	0.2295	0.2288	0.4888
$H^{(3)}$	UNDEF	0.4837	0.3438	0.2169	0.4618	0.3000	0.2169	0.2028	0.4608

as in this study. Zhang et al. [20] compared the proposed entropy measures E_{ZXLYT}^{1}, E_{ZXLYT}^{2}, E_{ZXLYT}^{3}, E_{ZXLYT}^{4}, E_{ZXLYT}^{5} and E_{ZXLYT}^{6} with E_{LZX}, E_{Y}, $E_{WWZ(\delta=0.5)}$, $E_{ZMSZ(\delta=0.5)}$ and E_{ZJJL}. Zhang et al. [20] stated that the properties of E_{ZJJL}, E_{ZXLYT}^{1}, E_{ZXLYT}^{3}, E_{ZXLYT}^{4}, E_{ZXLYT}^{5} and E_{ZXLYT}^{6} were good, however, the behaviors of $E_{ZMSZ(\delta=0.5)}$, E_{LZX}, $E_{WWZ(\delta=0.5)}$ were very poor in terms of structured linguistic variables. In addition, they indicated that due to the difference between the entropy values of $E_{ZJJL}(A^2)$ and $E_{ZJJL}(A^3)$ was very small, and this can make distinguish difficulty. So, E_{ZXLYT}^{1}, E_{ZXLYT}^{3}, E_{ZXLYT}^{4}, E_{ZXLYT}^{5} and E_{ZXLYT}^{6} seem to be more reasonable than others. Rashid et al. [21] compared developed entropy measure (E_{RFZ}) with entropy measures considered in the study of Zhang et al. [20]. They stated E_{RFZ} seems rather reasonable as compared with other entropy measures. Tiwari and Gupta [23] compared the performance of some entropy measures. They interpreted the results as that E_{ZJJL}, $E_{WWZ(\delta=0.5)}$ and $E_{ZMSZ(\delta=0.5)}$ were unreasonable and their proposed entropies can distinguish the fuzziness of all the IVIFSs. Guo and Zang [47] compared E_{LZX}, E_{ZJJL}, $E_{WWZ(\delta=0.5)}$, E_{J}, E_{SL} and E_{WZ}. Their performance tests' results were shown that E_{LZX}, E_{J} and E_{SL} were outperformed the other ones throughout the process.

In this study, the results of 10 tests performed for comparing 27 entropies can be seen in Table 1, Table 2, Table 3, Table 4, Table 5, Table 6, Table 7, Table 8, Table 9 and Table 10. Performance rates of entropy measures on presenting the expected ranking are presented in Figure 1. From the entropy values in the above-mentioned tables, it is deduced that the closer the membership degree and the non-membership degree, or the higher the hesitation degree, the greater the entropy. From Figure 1, it can be easily

Table 8. Comparison of the fuzziness under I

	E_{LZX}	E_Y	E_{ZJ}^1	E_{ZJ}^2	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^1	E_{WWZ}^2	E_{SL}
$I^{(1)}$	0.3582	0.4631	0.4500	0.4660	0.3020	0.1539	0.3805	0.3997	0.6914
$I^{(3/2)}$	0.3380	0.4465	0.4154	0.4361	0.2351	0.1674	0.3243	0.3534	0.6559
$I^{(2)}$	0.3407	0.4401	0.4155	0.4382	0.2540	0.1736	0.3220	0.3613	0.6371
$I^{(5/2)}$	0.3525	0.4451	0.4326	0.4528	0.3047	0.1775	0.3370	0.3866	0.6268
$I^{(3)}$	0.3439	0.4551	0.4177	0.4422	0.2815	0.1800	0.3140	0.3742	0.6012
	E_J	E_{CYWY}^a	E_{CYWY}^b	E_{GS}	E_{ZXLYT}^1	E_{ZXLYT}^2	E_{ZXLYT}^3	$E_{JPCZ(\lambda=0.5)}^1$	$E_{JPCZ(\lambda=0.5)}^2$
$I^{(1)}$	0.5156	0.4564	0.4483	0.2836	0.3711	0.4460	0.3560	0.3997	0.3997
$I^{(3/2)}$	0.4946	0.4220	0.4614	0.2643	0.3713	0.4291	0.3372	0.3534	0.3534
$I^{(2)}$	0.4928	0.4257	0.4489	0.2658	0.3588	0.4082	0.3057	0.3613	0.3613
$I^{(5/2)}$	0.4989	0.4421	0.4315	0.2761	0.3456	0.3940	0.2899	0.3866	0.3918
$I^{(3)}$	0.4869	0.4290	0.4156	0.2695	0.3342	0.3940	0.2889	0.3742	0.3742
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^1	E_{ZX}^2	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$I^{(1)}$	0.6664	0.6562	0.4322	0.2693	0.6443	0.4400	0.2693	0.2920	0.6426
$I^{(3/2)}$	0.6687	0.6659	0.4422	0.2542	0.6504	0.4207	0.2542	0.2836	0.6484
$I^{(2)}$	0.6541	0.6531	0.4370	0.2594	0.6380	0.4090	0.2594	0.2818	0.6361
$I^{(5/2)}$	0.6380	0.6370	0.4289	0.2722	0.6238	0.3983	0.2722	0.2932	0.6221
$I^{(3)}$	0.6231	0.6230	0.4210	0.2672	0.6110	0.4113	0.2672	0.2787	0.6095

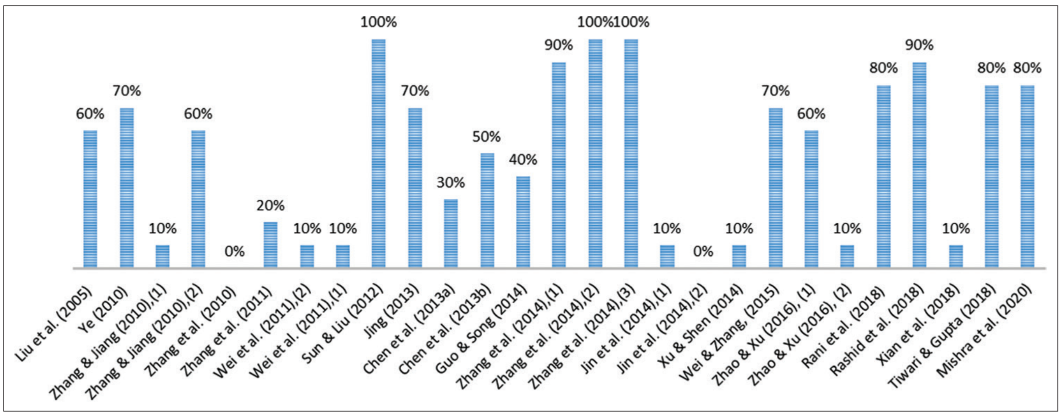

Figure 1. Comparison of the performance of entropy measures under all tests.

Table 9. Comparison of the fuzziness under J

	E_{LZX}	E_Y	E_{ZJ}^1	E_{ZJ}^2	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^1	E_{WWZ}^2	E_{SL}
$J^{(1)}$	0.3072	0.4381	0.3900	0.4250	0.2953	0.0949	0.3566	0.3566	0.6736
$J^{(3/2)}$	0.2812	0.4093	0.3572	0.3886	0.2464	0.1015	0.3169	0.3169	0.6393
$J^{(2)}$	0.2789	0.3825	0.3585	0.3853	0.2796	0.1034	0.3349	0.3349	0.6264
$J^{(5/2)}$	0.2505	0.3590	0.3245	0.3470	0.2294	0.1038	0.2931	0.2931	0.5886
$J^{(3)}$	0.2273	0.3383	0.2960	0.3149	0.1902	0.1035	0.2600	0.2600	0.5569

	E_J	E_{CYWY}^a	E_{CYWY}^b	E_{GS}	E_{ZXLYT}^1	E_{ZXLYT}^2	E_{ZXLYT}^3	$E_{JPCZ(\lambda=0.5)}^1$	$E_{JPCZ(\lambda=0.5)}^2$
$J^{(1)}$	0.4578	0.4056	0.3738	0.2485	0.3318	0.4150	0.3500	0.3566	0.3566
$J^{(3/2)}$	0.4248	0.3688	0.3531	0.2270	0.3088	0.3815	0.3187	0.3169	0.3169
$J^{(2)}$	0.4168	0.3751	0.3237	0.2251	0.2812	0.3553	0.2840	0.3349	0.3349
$J^{(5/2)}$	0.3791	0.3347	0.2971	0.2031	0.2567	0.3325	0.2638	0.2931	0.2931
$J^{(3)}$	0.3474	0.3018	0.2746	0.1847	0.2357	0.3125	0.2440	0.2600	0.2600

	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^1	E_{ZX}^2	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$J^{(1)}$	UNDEF	0.5806	0.3618	0.2369	0.5765	0.4000	0.2369	0.2650	0.5752
$J^{(3/2)}$	UNDEF	0.5542	0.3509	0.2192	0.5476	0.3673	0.2192	0.2539	0.5463
$J^{(2)}$	UNDEF	0.5171	0.3319	0.2204	0.5099	0.3545	0.2204	0.2301	0.5087
$J^{(5/2)}$	UNDEF	0.4823	0.3132	0.2003	0.4750	0.3244	0.2003	0.2170	0.4739
$J^{(3)}$	UNDEF	0.4516	0.2961	0.1831	0.4443	0.3089	0.1831	0.1960	0.4433

seen that the performance of E_{ZXLYT}^1, E_{ZXLYT}^2, E_{ZXLYT}^3, E_{SL}, E_{RFZ} are more reasonable and have high performance than other existing entropy measures. E_{SL}, E_{ZXLYT}^2 and E_{ZXLYT}^3 seems to be overperform the other ones. However, the behaviors of E_{ZJ}^1 E_{ZJJL}, $E_{ZMSZ(\delta=0.5)}$, E_{WWZ}^1, E_{WWZ}^2,, $E_{JPCZ(\lambda=0.5)}^1$, $E_{JPCZ(\lambda=0.5)}^2$, E_{XS}, E_{ZX}^2, and E_{XDLJ} are very poor in terms of structured linguistic variables. Also, E_{ZJJL} and $E_{JPCZ(\lambda=0.5)}^2$ failed to perform the expected ranking for the tests conducted in this study. Finally, the entropy measure proposed by Xu and Shen [44] is undefined under $\mu_A^L(x_i)+\mu_A^U(x_i)=0$ or $\nu_A^L(x_i)+\nu_A^U(x_i)=0$ due to $\lim_{x\to 0^+}\ln(x)=-\infty$.

This study may assist researchers in choosing the entropy measure or in comparing them to select the best suitable entropy measure for their studies. This comparative analysis has limited itself to entropy measures for IVIF sets. The authors have tried their best to contain IVIF entropy measures to get a comprehensive analysis, but there is a minor possibility of some publications being left out. In future work, it is suggested that the entropy measures extended for fuzzy sets such as hesitant, spherical, Pythagorean in addition to IVIF sets can be examined.

AUTHORSHIP CONTRIBUTIONS

M.K and S.E. designed the study. M.K. developed the theoretical framework, processed the experimental data, performed the analysis. S.E. supervised the findings. M.K. drafted the manuscript and designed the figures.

Table 10. Comparison of the fuzziness under K

	E_{LZX}	E_Y	E_{ZJ}^1	E_{ZJ}^2	E_{ZJJL}	$E_{ZMSZ(\delta=0.5)}$	E_{WWZ}^1	E_{WWZ}^2	E_{SL}
$K^{(1)}$	0.3391	0.4377	0.4200	0.4250	0.2631	0.2056	0.3867	0.3867	0.6904
$K^{(3/2)}$	0.3136	0.4039	0.3629	0.3824	0.1779	0.2323	0.3357	0.3357	0.6616
$K^{(2)}$	0.3010	0.3740	0.3400	0.3613	0.1610	0.2491	0.3200	0.3200	0.6388
$K^{(5/2)}$	0.2989	0.3560	0.3347	0.3565	0.1837	0.2628	0.3245	0.3245	0.6252
$K^{(3)}$	0.2966	0.3463	0.3365	0.3521	0.2048	0.2746	0.3276	0.3276	0.6122
	E_J	E_{CYWY}^a	E_{CYWY}^b	E_{GS}	E_{ZXLYT}^1	E_{ZXLYT}^2	E_{ZXLYT}^3	$E_{JPCZ(\lambda=0.5)}^1$	$E_{JPCZ(\lambda=0.5)}^2$
$K^{(1)}$	0.4850	0.4339	0.3982	0.2753	0.3047	0.3950	0.3100	0.3867	0.4027
$K^{(3/2)}$	0.4556	0.3930	0.3992	0.2503	0.2887	0.3706	0.2891	0.3357	0.3357
$K^{(2)}$	0.4342	0.3750	0.3773	0.2392	0.2628	0.3313	0.2345	0.3200	0.3200
$K^{(5/2)}$	0.4221	0.3720	0.3508	0.2401	0.2379	0.2943	0.1960	0.3245	0.3245
$K^{(3)}$	0.4103	0.3682	0.3263	0.2413	0.2164	0.2725	0.1734	0.3276	0.3388
	$E_{XS(p=q=0.5)}$	E_{WZ}	E_{ZX}^1	E_{ZX}^2	E_{RJH}	E_{RFZ}	E_{XDLJ}	E_{TG}	E_{MRPMSP}
$K^{(1)}$	UNDEF	0.6039	0.4233	0.2587	0.5684	0.4000	0.2587	0.3025	0.5672
$K^{(3/2)}$	UNDEF	0.6002	0.4288	0.2373	0.5568	0.3594	0.2373	0.2831	0.5554
$K^{(2)}$	UNDEF	0.5724	0.4170	0.2303	0.5281	0.3345	0.2303	0.2511	0.5266
$K^{(5/2)}$	UNDEF	0.5405	0.4018	0.2344	0.4991	0.2974	0.2344	0.2356	0.4977
$K^{(3)}$	UNDEF	0.5108	0.3873	0.2377	0.4737	0.2821	0.2377	0.2145	0.4725

DATA AVAILABILITY STATEMENT

The published publication includes all graphics and data collected or developed during the study.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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