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**Investigation Of Conditions For Sphere Flotation (I)**

by

**Mustafa ÖZCAN**

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Faculté des Sciences de l'Université d'Ankara  
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## Investigation Of Conditions For Sphere Flotation (I)

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### ABSTRACT

This work involves the determination of the critical size which we can define as the largest particle which can float, and the critical density, which is the densest particle which can float at specific radius and contact angle, it also includes calculation of height maxima and maximum force acting on the particle under the same conditions. The calculations require not only the limit of possible equilibrium configurations but also the stability of the particle. Also the critical densities maximum force acting on sphere, and maximum heights, along with the other forces acting on the particle were calculated for various contact angles and as reduced value of  $R$  equals to 0.5. There is a linear relationship between contact angle and critical density.

### INTRODUCTION

An understanding of the behaviour of a solid particle at a horizontal fluid/liquid interface in a gravity field is important in several areas of surface chemistry and physics; for example, flotation has long been used as an efficient method for the concentration of minerals and separation of particles. If the particle has higher or lower density than each of the two immiscible fluids forming the interface, it will either pass through the interface or float at it from restraint by capillary forces acting on the particle. It is therefore useful to know the largest particle or the critical density of the particle and the force acting on it, at specific radius and contact angle, which can float in equilibrium.

The studies of the equilibrium position of particles at fluids interface have traditionally been undertaken by means of force analysis which predict equilibrium when the net vertical force acting on the particle at the interface is zero. The theory of this force, which can be calculated from the capillary theory of flotation, originated with the work of Nutt<sup>1</sup> and was further developed by Scheludko et al<sup>2,3</sup>. The works of

Giffard and Scriven<sup>4</sup>, Maru et al<sup>5</sup>; Hardland<sup>6</sup>; Huh and Mason<sup>7</sup>; Princen<sup>8</sup>; Rapacchietta and Neumann<sup>9,10</sup>, and Boucher and Kent<sup>11</sup> may also be regarded as part of this theory. Recently Boucher and Kent have considered the mechanical manipulation of the sphere, and described the equilibrium configuration by using both numerical computation of meridian curves and associated quantities and formal thermodynamic analysis.

This work involves the determination of the critical size, which we can define as the largest particle which can float, and the critical density, which is the densest particle which can float, at specific radius and contact angle. It also includes calculation of height maxima and maximum force acting on the particle under the same conditions. The calculations require not only the limit of possible equilibrium configurations but also the stability of the particle. In the first part of this work critical densities, maximum force acting on sphere, and maximum heights, along with the other forces acting on the particle were calculated for various contact angles and as reduced value of  $R$  equals to 0.5.

#### The Equilibrium Position of Solid Particles (Spheres) at Horizontal Fluid Interfaces

When a solid particle, a sphere (which is assumed to be rigid and not deformed in any way by the interface), under the influence of gravity, may approach an interfacial region (interface) between two liquids, or between a gas and a liquid, it may either take up an equilibrium position at the interface or pass through the interface, depending on the size of the particle, the density of the particle and the fluid phases, the interfacial tension and the contact angle at the fluid/fluid/solid boundary. Figure 1 illustrates one possible configuration of a system consisting of a solid particle (sphere) at a fluid interface. The radius of the sphere is  $R$  and its density is  $\rho^s$ . The densities of the upper and the lower fluids are  $\rho^\alpha$  and  $\rho^\beta$  respectively.  $Z^\theta$  is the distance of the three-phase confluence from  $z = 0$  level,  $\theta$  is the contact angle measured through fluid  $\beta$  between tangents of surface  $s\beta$  and  $\beta\alpha$  at the three phase line. The angle  $\psi$  is a position coordinate which locates the three-phase line on the solid surface,  $\psi$  is the angle made between the vertical axis of rotational symmetry and the position of the three-phase confluence with respect to the center of sphere. The three-phase confluence is at the position  $(X^\theta, Z^\theta)$

where the sphere of radius  $R$  makes a contact angle with the fluid /fluid interface. The angle  $\varepsilon$  is the parameter of the meniscus it is the angle between the horizontal and the tangent of the meniscus at any point on the profile.

If the density of the solid sphere ( $\rho^s$ ) is intermediate in magnitude between that of the lower fluid ( $\rho^\beta$ ), usually a liquid, and that of the upper fluid ( $\rho^\alpha$ ), often a gas, the solid particle will always take up an equilibrium position and float, regardless of its size and shape, the interfacial tension and the contact angle. In this case when  $\rho^\alpha > \rho^s > \rho^\beta$  buoyancy effects are sufficient to ensure equilibrium at the interface. This is only true when  $\rho^s$  is intermediate between  $\rho^\beta$  and  $\rho^\alpha$ , because, as shown in Figure 1, the case  $\rho^\alpha > \rho^s > \rho^\beta >$  is not physically realisable. Such a system would be hydrodynamically unstable. However, when the particle is denser than both fluids  $\rho^s > \rho^\beta > \rho^\alpha$  buoyancy effects are not sufficient to ensure that the particle be supported at the interface. In this case surface forces will begin to have an effect on the possible equilibrium of the particle in the interfacial region. If the interfacial properties of the system are such that the surface forces contribute positively to support the particle, then at least there is a possibility that an appropriate particle will not pass through interface ( $\alpha \beta$ ) into fluid ( $\beta$ ). Systems of this kind are extremely important in the floatation of minerals and emulsions stabilised by solid particles. Particularly in floatation problems the critical density and size of particles play an essential role.

This work investigates the equilibrium position of a spherical particle at the interface region when  $\rho^s > \rho^\beta > \rho^\alpha$ , that is, when surface phenomena are necessary to ensure the support of the particle at the interface.

### SPHERE FLOTATION

Laplace's equation relates the pressure difference across the interface with the shape of the interface which is given in terms of the two principle radii of curvature  $r_1$  and  $r_2$

$$\Delta P = \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \quad (1)$$

where  $\gamma$  is the surface tension.

From Pascal's laws, the variation of pressure with displacement for a holm is given by

$$\Delta P = \mp Z^0 \Delta \rho g \quad (2)$$

where  $Z^0$  is the elevation of the fluid/fluid interface from the level  $z = 0$ ;  $z = 0$  is where  $P = 0$ . From equations (1) and (2) we obtain

$$\gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) = \mp \Delta \rho g z \quad (3)$$

In reduced terms<sup>12</sup> equation (3) gives

$$\frac{1}{R_1} + \frac{1}{R_2} = \mp 2Z \quad (4)$$

where quantities of dimension  $L^v$  have been divided by  $a^v$ ;  $a^2 = 2\gamma / \Delta \rho g$ . For rotationally symmetric holms

$$R_1 = \frac{dS}{d\Phi} \text{ and } R_2 = \frac{X}{\sin \Phi}$$

where  $S$  is the meridian arc length and  $\Phi$  is the angle the meridian makes with the horizontal;  $\Phi = 180 + \epsilon$  (see Fig. 1) substituting the above value of  $R_1$  and  $R_2$  into equation (4) we obtain

$$\frac{d\Phi}{dS} + \frac{\sin \Phi}{X} = \mp 2Z \quad (5)$$

Equation (5) which is a first order differential equation, cannot be solved in closed analytic form, but must be solved by numerical methods. The most efficient and accurate method is to use Meon's version<sup>12</sup> of the Runge-Kuffa (fourth order) procedure .

The solution by the numerical integration, gives a meridian curve  $Z = Z(X)$  with the arc length  $S$  as the independent variable. The meridian angle  $\Phi$  is also calculated i.e. the output gives  $S$ ,  $\Phi$ ,  $X$  and  $Z$ . For sphere floatation only  $\Phi$ ,  $X$  and  $Z$  are required. By direct numerical methods the analysis of the conditions for sphere floatation would require extensive numerical computation, and it seems to be for just this reason that such a study has not been made.

There is however, an alternative approach which uses approximate methods of known accuracy. One such approximation, termed a first integral approximation gives  $Z$  as a function of  $X$  and  $\Phi$ . The approximation is<sup>11</sup>

$$Z = \frac{K_0(\sqrt{2X})}{K_1(\sqrt{2X})} (1 + \text{Cos}\Phi)^{1/2} \quad (6)$$

where  $K$  is the meridian angle,  $X$  is the contact radius, and  $K_1$  and  $K_1$  are the modified Bessel functions of the second time of zero and first order. At the three phase confluence

$$Z^0 = \frac{K_0(\sqrt{2X^0})}{K_1(\sqrt{2X^0})} (1 + \text{Cos}\Phi)^{1/2} \quad (7)$$

The range of applicability and accuracy of the approximation given in equation (6) have been studied in detail by comparison with accurate numerical computation<sup>13</sup>. In this work the above approximation will be applied to sphere flotation problems.

#### Force Analysis

When a solid particle of density  $\rho^s$  is retained at equilibrium in the interface, the net force acting on it must be zero. For the case of a sphere to float unaided, the net force on the sphere must be zero, i.e., the body forces of the sphere in the two fluids must be balanced by the surface forces whose origin is the interfacial (surface) tension  $\gamma$ .

The surface forces consist of two parts; the vertical component of the surface tension and the pressure difference across the interface, both acting at the three phase confluence. Since the pressure in phase  $\beta$  is greater than that in phase  $\alpha$ , both the resolved component of the surface tension  $\gamma$  and the pressure difference across the interface are acting to support the particle i.e., the forces are acting in the upward direction.

$$\text{Vertical component of surface tension} = 2\pi x^0 \gamma \sin \epsilon$$

$$\text{Force across interface due to pressure difference} = 4 (x^0)^2 z^0 \Delta\rho g$$

$$\text{Total upward force} = f_{\uparrow} = 2\pi x^0 \gamma \sin \epsilon + \pi (x^0)^2 z^0 \Delta\rho g \quad (8)$$

$$\text{or, in the reduced terms}^{12}, F_{\uparrow} = \pi X^0 \sin \epsilon + \pi (X^0)^2 Z^0 \quad (9)$$

now  $\epsilon = \Phi - 180^\circ = \theta + \psi - 180$  and  $\sin \epsilon = - \sin \Phi$

also  $Z^0$  is negative and hence  $F_{\uparrow}$  is negative. The body force of the sp-

here  $F$  (downward force) is taken as positive. The weight of the particle gives rise to the second force term:

Body force of the sphere =  $V^{s,\alpha} (\rho^s - \rho^\alpha) g + V^{s,\beta} (\rho^s - \rho^\beta) g$   
 where  $V^{s,\alpha}$ ,  $V^{s,\beta}$  are the volume of the solid sphere in phases  $\alpha$  and  $\beta$  respectively and  $\rho^s$  is the density of the solid material forming the sphere.

If  $V^s$  is the total volume of sphere ( $= \frac{4}{3} \pi R^3$ ) then we have

$$\begin{aligned} f_{\downarrow} &= (V^s - V^{s,\beta}) (\rho^s - \rho^\alpha) g + V^{s,\beta} (\rho^s - \rho^\beta) g & (10) \\ &= V^s (\rho^s - \rho^\alpha) g - V^{s,\beta} (\rho^\beta - \rho^\alpha) g \\ &= V^s \Delta^s_{\alpha} \rho g - V^{s,\beta} \Delta \rho g \end{aligned}$$

in reduced terms<sup>11</sup>, noting  $k^\alpha = (\rho^s - \rho^\alpha) / (\rho^\beta - \rho^\alpha) = \Delta^s_{\alpha} \rho / \Delta \rho$

$$F_{\downarrow} = V^s k^\alpha - V^{s,\beta} \quad (11)$$

$$\text{and} \quad V^{s,\beta} = \frac{1}{3} \pi R^3 (1 - \text{Cos}\psi) \quad (12)$$

In order to sphere float sum of upward and downward forces must be zero that is

$$F_{\text{abs}}^{\text{ext}} = F_{\uparrow} + F_{\downarrow} = 0 \quad (13)$$

substituting the values of forces from equation (9) and (11) into equation (13)

$$\pi X^\theta \sin \Phi + \pi (X^\theta)^2 Z^\theta + V^s k^\alpha - V^{s,\beta} = 0 \quad (14)$$

more generally if there is a net force on the sphere such that it must be supported externally<sup>11,12</sup>, then if

$$F_{\text{abs}}^{\text{ext}} = F_{\uparrow} + F_{\downarrow} \quad (15)$$

$$F_{\text{abs}}^{\text{ext}} = \pi X^\theta \sin \Phi + \pi (X^\theta)^2 Z^\theta + V^s K^\alpha - V^{s,\beta} \quad (16)$$

a volume, or more strictly a force,  $V^{\alpha,\theta}$  is defined such that

$$V^{\alpha,\theta} = \pi X^\theta \sin \Phi + \pi (X^\theta)^2 Z^\theta$$

so that equation (16) becomes



$$F_{abs}^{ext} = V^{\alpha, \theta} + V^s K^\alpha - V^{s, \beta} \quad (17)$$

The above equations give the general theory of the forces on spheres at interfaces. Flotation is the special case when the net force  $F_{abs}^{ext} = 0$ .

The approximation given in equation (6 or 7) may now be applied to the specific problem of sphere flotation.

Equation (7) gives  $Z^\theta$  values, and hence if  $X^\theta$  and  $\Phi$  are known, then  $Z^\theta$  may be calculated from equation (7) and consequently  $V^{\alpha, \theta}$  is known. A sphere at an interface is characterised by three quantities: the radius of the sphere  $R$ , the contact angle  $\theta$  and the reduced density  $K^\alpha$ ,

$$k^\alpha = \frac{\rho^s - \rho^\alpha}{\rho^\beta - \rho^\alpha} \quad (18)$$

These three values must be chosen for a particular system.

From Figure I.

$$\Phi = \psi + \theta \quad (19)$$

$$X^\theta = R \sin \psi \quad (20)$$

$Z^\theta$  is given by equation (7), and  $V^{s, \beta}$  is given by equation (12) which depends only on  $R$  and  $\psi$ . Hence  $\psi$  may be used as the independent variable for the movement of the sphere through the interface.  $\psi$  must be chosen

by trial and error until the value when  $F_{abs}^{ext} = 0$  is determined. The position of the sphere is defined by the angle  $\psi$  (see Fig. 1).

An important quantity also obtainable from the approximate treatment is the distance of the center of the sphere above or below the  $Z = 0$  level. Denoted by  $Z^x$ , this distance is given by

$$Z^x = Z^\theta + R \cos \psi \quad (21)$$

which again depends only on  $R$  and  $\psi$ .

A small computer programme is used to calculate  $\Phi$ ,  $X^\theta$ ,  $Z^\theta$ ,  $Z^x$  and  $F_{abs}^{ext}$  for chosen values of  $R$ ,  $\theta$  and  $K^\alpha$  with  $\psi$  as the independent variable.

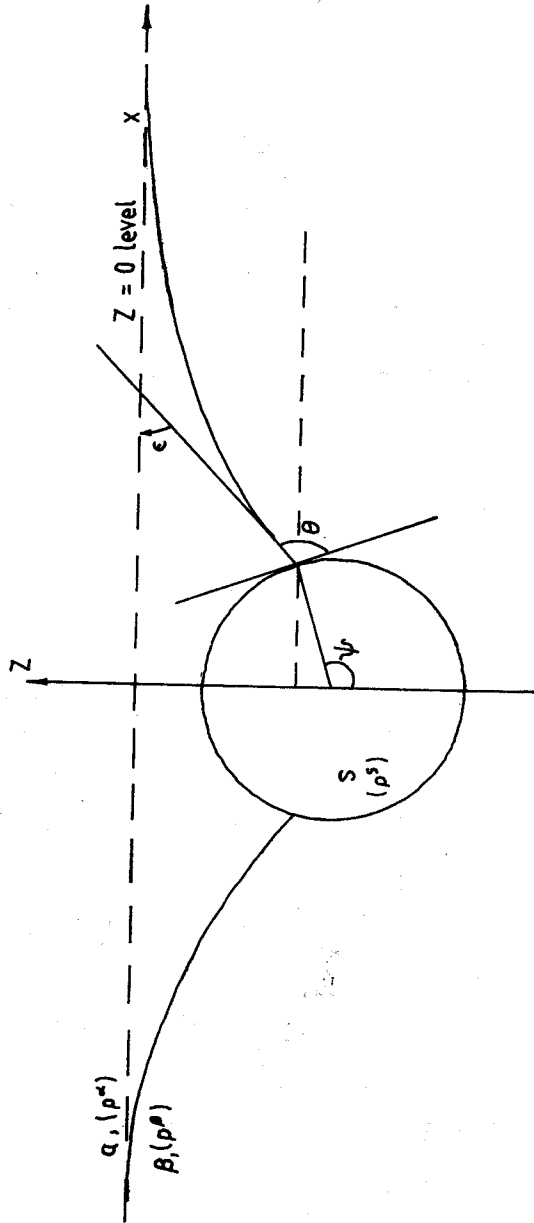


Fig. 1 Stationary States of a Small solid sphere at a fluid interface.

*Results and Discussion*

This investigation involves the calculation of  $\Phi$ ,  $X^\theta$ ,  $Z^\theta$ ,  $Z^x$  and  $F_{abs}^{ext}$  for chosen values of  $R$ ,  $\theta$  and  $k^\alpha$ , with  $\psi$  as the independent variable. The approximation is applied to calculate the results. Table I, contains the calculated values of  $\Phi$ ,  $X^\theta$ ,  $Z^\theta$ ,  $Z^x$  and  $F_{abs}^{ext}$  for the spe-

Table I. Calculated value of  $X^\theta$ ,  $Z^\theta$ ,  $Z^x$  and  $F_{abs}^{ext}$  for

Sphere Radius = 0.50000000  
 Contact Angle = 130.00000000  
 Red. Density of Sphere in Less Dense Phase = 3.83390000

DEG	X	Z	Z <sup>x</sup>	FORCE
160.00000	0.25000	0.11882	0.55183	2.29264
165.00000	0.28679	0.09473	0.50431	2.25302
170.00000	0.32139	0.06627	0.44929	2.18444
175.00000	0.35355	0.03443	0.38798	2.08735
180.00000	0.38302	0.00000	0.32140	1.96328
185.00000	0.40958	-0.03636	0.25043	1.81486
190.00000	0.43301	-0.07409	0.17591	1.64575
195.00000	0.45315	-0.11271	0.09860	1.46053
200.00000	0.46985	-0.15180	0.01921	1.26458
205.00000	0.48296	-0.19095	-0.06154	1.06384
210.00000	0.49240	-0.22980	-0.14298	0.86462
215.00000	0.49810	-0.26800	-0.22442	0.67334
220.00000	0.50000	-0.30520	-0.30520	0.49624
225.00000	0.49810	-0.34106	-0.38463	0.33916
230.00000	0.49240	-0.37523	-0.46206	0.20728
235.00000	0.48296	-0.40737	-0.53678	0.10487
240.00000	0.46985	-0.43708	-0.60809	0.03511
245.00000	0.45315	-0.46397	-0.67528	-0.00001
250.00000	0.43301	-0.48757	-0.73757	0.00012
255.00000	0.40958	-0.50739	-0.79418	0.03481
260.00000	0.38302	-0.52283	-0.84423	0.10198
265.00000	0.35355	-0.53320	-0.88675	0.19834
270.00000	0.32139	-0.53765	-0.92067	0.31949
275.00000	0.28679	-0.53510	-0.94468	0.46009
280.00000	0.25000	-0.52417	-0.95718	0.61417
285.00000	0.21131	-0.50297	-0.95612	0.77538
290.00000	0.17101	-0.46878	-0.93863	0.93731
295.00000	0.12941	-0.41742	-0.90039	1.09385
300.00000	0.08682	-0.34166	-0.83407	1.23960
305.00000	0.04358	-0.22614	-0.72424	1.37034
309.00000	0.00873	-0.07109	-0.57101	1.46250

here of density  $k\alpha = 3.8339$  g and radius = 0.5 at a contact angle of  $130^\circ$ . The results in table 1 have been chosen from a large number of data that have been computed to give an idea about the work in progress. Table 2 gives the reduced critical densities of solid spheres at different contact angles. Table 3 contains  $\psi$  values for specific critical densities and contact angles. Table 4 gives the  $F_{\max}$  and  $Z_{\max}^x$  for specified contact angles and reduced densities. Again results in tables 2,3 and 4 were chosen as an example to show the nature of the investigation in progress.

Table 2. Contact angles and reduced critical densities for  $R = 0.5$ 

$\theta =$ Contact angle	$k\alpha =$ Reduced Critical Density
$90^\circ$	2.6605
$100^\circ$	2.9605
$110^\circ$	3.27505
$120^\circ$	3.5655
$130^\circ$	3.8339
$140^\circ$	4.0705
$150^\circ$	4.2505
$160^\circ$	4.4025

Table 3.  $\psi$  values at  $F_{\text{abs}}^{\text{ext}} = 0$  for each critical density ( $k\alpha$ ) and contact angle ( $\theta$ )

$\theta$	$k\alpha$	$\psi$
90	2.6605	135
100	2.9605	130
110	3.27505	125
120	3.5655	120
130	3.8339	115
140	4.0705	115
150	4.2505	110
160	4.4025	105

Table 4.  $F_{\max}^x$  and  $Z_{\max}^x$  for each critical density ( $k\alpha$ ) and contact angles ( $\theta$ )

$\theta$	$k\alpha$	$F_{\max}$	$Z_{\max}^x$
90	2.6605	2.13399	-0.79809
100	2.9605	2.02155	-0.83910
110	3.2750	2.25885	-0.88146
120	3.5655	2.28737	-0.92057
130	3.8339	2.9350	-0.95612
140	4.0705	2.27241	-0.99352
150	4.2505	2.21856	-1.02560
160	4.4025	2.14901	-1.05335

Figures 2,3 and 4 gives the forces acting on the sphere as a function of  $Z^x$  at contact angles  $100^\circ$ ,  $130^\circ$  and  $150^\circ$  respectively. Each figure contains more than one curve and each curve represents a different density  $k^\alpha$ . As can be seen from the figures only one of the curves for given  $R$  and  $\theta$  has a position where  $F_{abs}^{ext}$  is zero. This curve corresponds to the critical density at specified radius and contact angle. Critical-density curves show that there is only one equilibrium point (where  $F_{abs}^{ext} = 0$ ) where a sphere can float. Curves above the critical density

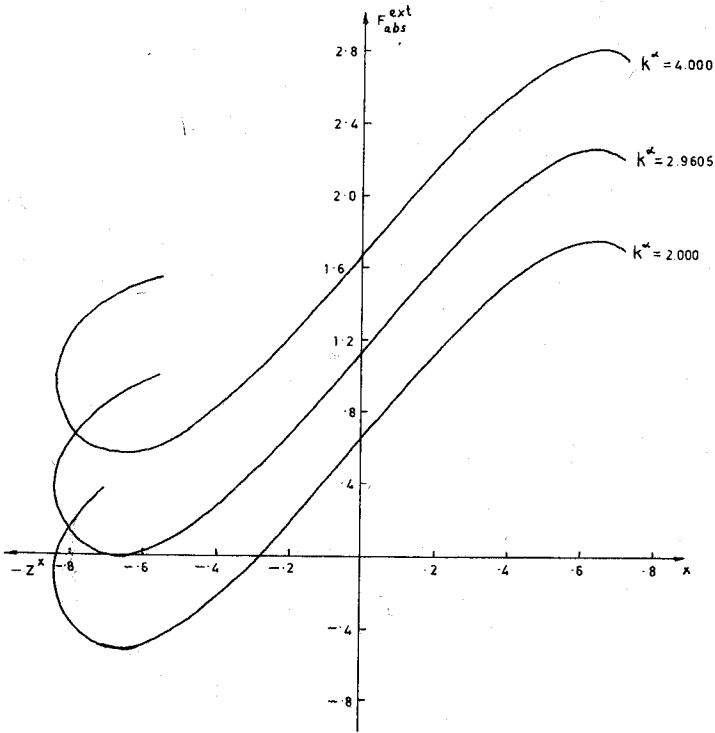


Fig. 2 Dependence of  $F_{abs}^{ext}$  on  $Z^x$  for  $R = 0.5$ ;

$\theta = 100^\circ$  and  $k^\alpha =$   
 2.0000  
 2.9605  
 4.0000

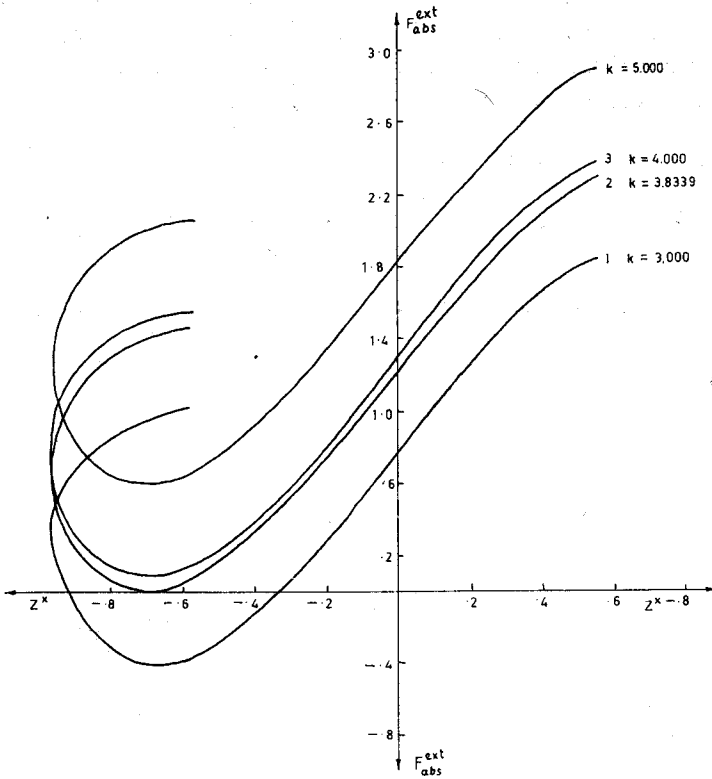


Fig. 3 Dependence of  $F_{abs}^{ext}$  on  $Z^x$  for  $R = 0.5$ ;

$$\theta = 130^\circ \text{ and } k^\alpha = \begin{matrix} 3.0000 \\ 3.8339 \\ 4.0000 \\ 5.0000 \end{matrix}$$

curve do not have any point that  $F_{abs}^{ext} = 0$ . This means that at specified contact angles and radius, spheres with densities greater than the reduced critical density cannot float at all. Curves below the critical-density curve have two points where  $F_{abs}^{ext}$  is equal to zero. At these points equilibrium is reached and the sphere can float, that is, at spe-

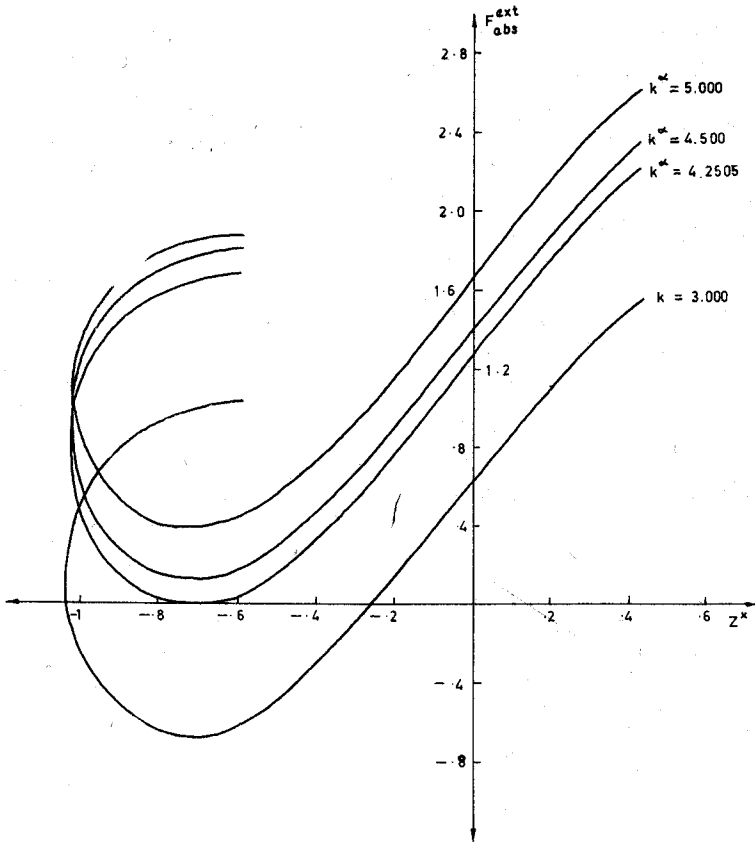


Fig. 4 Dependence of  $F_{abs}^{ext}$  on  $Z^x$  for  $R = 0.5$ ;

$\theta = 150^\circ$  and  $k^\alpha =$   
 5.0000  
 4.5000  
 4.2505  
 3.0000

cified contact angle and radius, the sphere with a lower density than the reduced critical density can always float. But it is not certain that, at this point, which equilibrium position is stable. The force analysis predicts the equilibrium states of the system; however this analysis does not easily clarify the stability of such states. Rapacchietto and Neumann<sup>9</sup> reported that secondary equilibrium states predicted by force analysis is unstable because these states correspond to the equilibrium

states for which the free energy of the system is a maximum. Each curve of figures 2, 3 and 4 also gives the height maximum and the force maximum for specified contact angle and density. As is seen from the curves, the force changes as density changes, the greater the density the greater the maximum force. Figure 5 gives the plot of reduced critical density

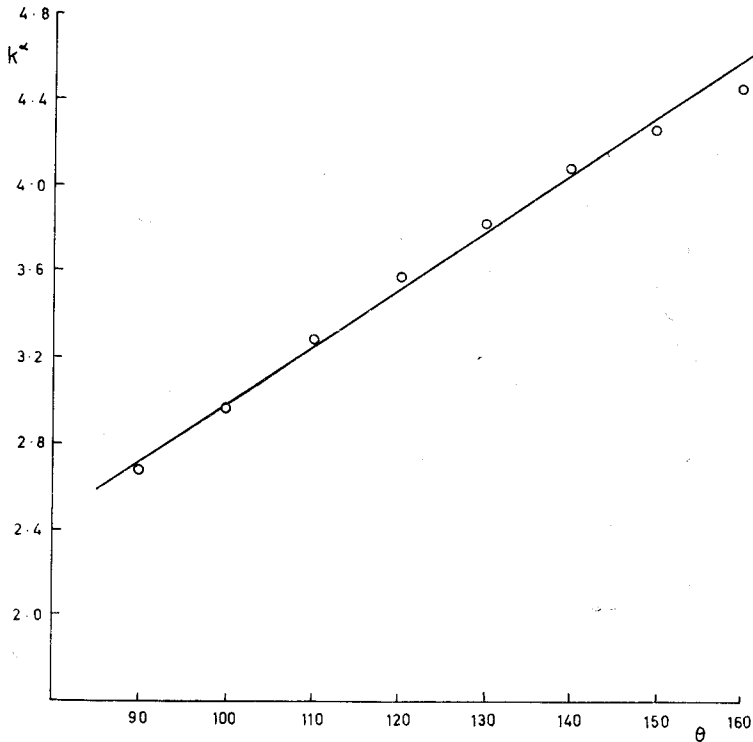


Fig. 5 Plot of reduced critical density ( $k^\alpha$ ) vs. Contact angle ( $\theta$ ) for  $R = 0.5$

( $k^\alpha$ ) v.s. the contact angle. From this figure a linear relationship between contact angle and critical density can be obtained. At the higher contact angles some deviations occur from linearity. Spheres with density above this line will not float at all. Spheres with density below this line will float at the corresponding contact angle and radius. From the  $k^\alpha$  v.s.  $\theta$  plot one can now predict the critical density for given contact



angle and vice versa in this range. As a result of this investigation we can predict the critical density, in the flotation problem, for a given contact angle and vice versa, for a large range of contact angles.

Figure 6 shows the plot of contact angle versus  $\psi$ , when  $R = 0.5$  and for critical values of  $k^\alpha$ . Figure 7 is a plot of contact angle versus  $\psi$ . For both plots  $\psi$  values are such that they correspond to when  $F_{abs}^{ext}$  is equal to zero. Both figures show that the change of  $\psi$  is small with respect to changes of  $k^\alpha$  and contact angles. For a large range of contact angle and critical density  $\psi$  changes are only of the order of  $30^\circ$ .

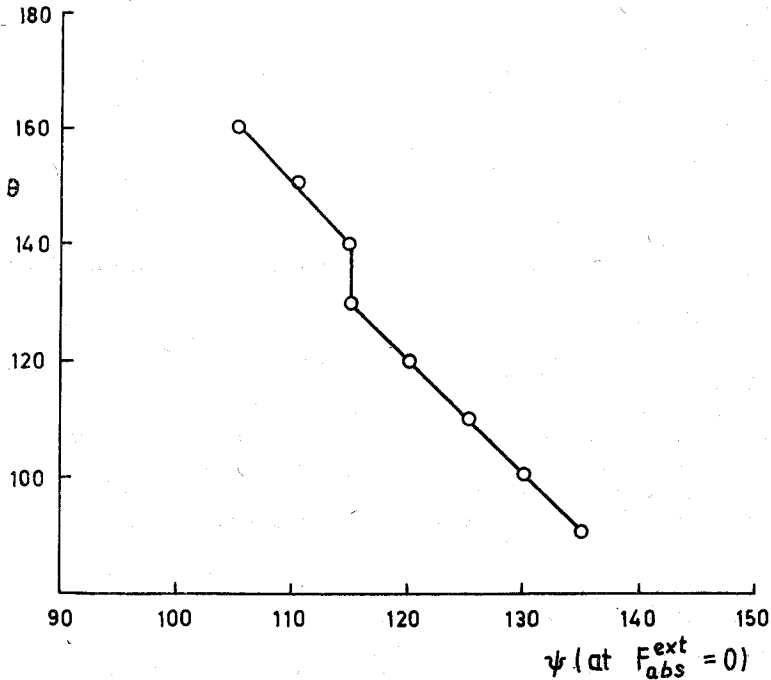


Fig. 6 Relationship between contact angle ( $\theta$ ) and position coordinate ( $\psi$ ) for  $R = 0.5$  and critical density,  $\psi$  values are corresponded to  $F_{abs}^{ext} = 0$ .

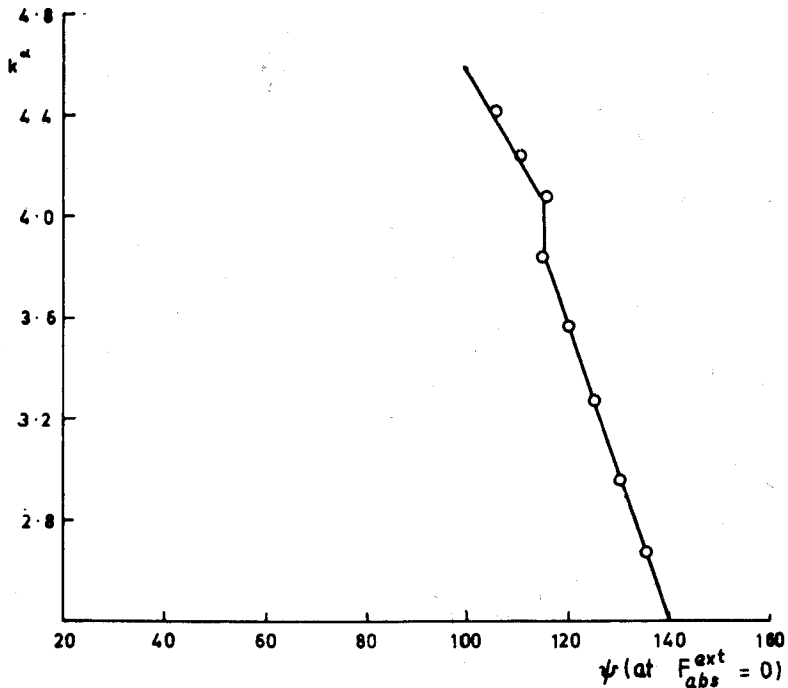


Fig. 7 Relationship between reduced critical density ( $k^\alpha$ ) and position coordinate ( $\psi$ ).  $\psi$  values are corresponded to  $F_{abs}^{ext} = 0$ .

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#### PRINCIPAL SYMBOLS

R	Radius of sphere
$\rho^s$	Density of sphere
$\rho^\alpha$	Density of upper fluids
$\rho^\beta$	Density of lower fluids
$Z^0$	Distance of the three-phase confluence from $Z=0$ level
$\theta$	Contact angle

$X^0, Z^0$	Donetes the position of the three-phase confluence
$\psi$	Position coordinate (angle) which locates the three-phase line on the solid surface
$r_1, r_2$	Principle radii of curvature
$\gamma$	Surface tension
$\Delta\rho$	Variation of pressure with displacement for a holm
$S$	Meridien arc length
$\Phi$	Angle the meridian makes with the horizontal
$X, Z$	Reduced coordinates
$K_0, K_1$	Modified Bessel Functions of the second time of zero and first order.
$f_{\uparrow}$	Total upward force
$F$	Upward force in the reduced term
$V^{s,\alpha}$	Volume of sphere in the phase $\alpha$
$V^{s,\beta}$	Volume of sphere in the phase $\beta$
$V^s$	Total valume of sphere
$f_{\downarrow}$	Total downward force
$F$	Downward force in the reduced term
$F_{abs}^{ext}$	Externall applied force
$K^\alpha$	Reduced density.

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**ÖZET**

Bu çalışma belirli yarıçap ve temas açısında yüzebilen en büyük parçacığın kritik büyüklüğü ve kritik yoğunluğunun tesbiti ile ilgili olup aynı koşullarda parçacığa etki eden maksimum kuvvet ve yükseklik maksimasının hesaplanmasını da içeriyor. Bu hesaplamalarda gerekli olan koşulun sadece parçacığın mümkün olan denge konumu değil aynı zamanda kararlı halidir. Çeşitli temas açılarında yarı çap  $R = 0,5$  alınarak kritik yoğunluk, küresel parçacık üzerine etki eden maksimum kuvvet, yükseklik maksiması ve parçacık üzerine etki eden diğer kuvvetler hesaplandı. Temas açısı ve kritik yoğunluk arasında doğrusal bir ilişki olduğu görüldü.

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