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Soliton solutions for perturbed Radhakrishnan-Kundu-Lakshmanan equation

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Abstract

To find some soliton solutions of the equation, the perturbed Radhakrishnan-Kundu-Lakshmanan (RKL) equation has been considered. For this purpose, GKM (generalized Kudryashov method), which is one of the solution methods of nonlinear evolution equations (NLEEs), *has been applied to the perturbed RKL equation. First, considered the nonlinear partial differential equation, is reduced to an ordinary differential equation with the help of the traveling wave transformation. Afterward, obtained the algebraic equation system through the balance principle was solved with the help of Wolfram Mathematica 12. Thus, some new soliton solutions of the discussed equation have been obtained. Both 2D and 3D graphics have been drawn with the help of Wolfram Mathematica 12 by giving some values to obtained these new solutions.*

Keywords: *GKM, perturbed RKL equation, soliton solutions.*

Perturbe edilmiş Radhakrishnan-Kundu-Lakshmanan denklemi için soliton çözümler

Öz

Denklemin bazı soliton çözümlerini bulmak için perturbe edilmiş Radhakrishnan-Kundu-Lakshmanan (RKL) denklemi ele alınmıştır. Bu amaç için lineer olmayan evrim denklemleri (NLEEs)'nin çözüm yöntemlerinden biri olan GKM (genelleştirilmiş Kudryashov metodu), perturbe edilmiş RKL denklemine uygulanmıştır. İlk olarak ele alınan lineer olmayan kısmi diferansiyel denklem, hareketli dalga dönüşümü yardımıyla bir adi diferansiyel denkleme indirgenmiştir. Daha sonra denge prensibi ile elde edilen

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cebirsel denklem sistemi Wolfram Mathematica 12 yardımıyla çözülmüştür. Böylece ele alınmış olan denklemin bazı yeni soliton çözümleri elde edilmiştir. Elde edilen bu yeni çözümlere birtakım değerler verilerek Wolfram Mathematica 12 yardımıyla hem 2 boyutlu hem de 3 boyutlu grafiklerin çizimleri yapılmıştır.

Anahtar Kelimeler: *GKM, perturbe edilmiş RKL denklemi, soliton çözümler.*

1. Introduction

NLEEs are handled in very important scientific areas such as plasma physics, mathematical physics, mathematical chemistry, optical fibers, medicine, hydrodynamics, fluid dynamics, geochemistry, control theory, meteorology, optics, mechanics, chemical kinematics, biology biophysics, biogenetics and so on. With the developing world, NLEEs emerges as having more difficult and complex solutions. Solving these equations and creating new methods forms a very important field of study. For this purpose, various solution methods have been brought to the literature by some scientists. For example: modified extended tanh-function method [1], modified simple equation method [2], $\left(m+1/G'\right)$-Expansion method [3], extended modified Kudryashov method [4], extended Jacobi's elliptic function expansion scheme[5], tanh-method [6], modified extended direct algebraic method [7], Darboux transformation [8], sine–Gordon expansion method [9], F-expansion method [10], $\left(1/G'\right)$-expansion method, finite difference method and Laplace perturbation method [11], modified sub-equation method [12], modified Kudryashov methods [13].

Perturbed RKL equation is given as [14-16]:

$$iq_t + aq_{xx} + b|q|^2 q = i\varepsilon q_x + i\lambda\left(|q|^2 q\right)_x + i\mu\left(|q|^2\right)_x q - i\gamma q_{xxx}, \tag{1}$$

where $q(x,t)$ is a complex-valued dependent function and x is spatial variables and t is independent variables representing temporal variables. The first term on the left side of Eq. (1) specifies the temporal variation of the nonlinear wave, parameter a represents the group velocity distribution, and b symbolizes the nonlinearity coefficient. The coefficients on the right side of equality in the equation, ε represent the intermodal distribution, λ represent the rise coefficient for short pulses, μ represent the higher-order distribution coefficient, and γ represent the third-order distribution term [14-16].

The perturbed RKL equation has been studied by some authors recently. Biswas used traveling wave hypothesis for the perturbed RKL equation [14]. Biswas et al. used modified simple equation and trial equation methods for the perturbed RKL equation [15]. Ghanbari and Gomez-Aguilar used generalized exponential rational function method for the RKL equation [16].

Our aim in this study is to find some new soliton solutions of the perturbed RKL equation using GKM [17-20]. First, the definition of GKM was made. Afterwards, GKM was applied to the discussed equation and some new soliton solutions were obtained by using the Wolfram Mathematica 12 package program.

2. Structure of GKM

We consider a general nonlinear partial differential equation for a function q of two different variables in the following form:

$$R\left(q, q_t, q_x, q_{xx}, \ldots\right) = 0. \tag{2}$$

Step 1: We regard travelling wave transform as in the following equation;

$$q\left(x,t\right) = q\left(\eta\right), \quad \eta = x - vt. \tag{3}$$

Using Eq. (3), Eq. (2) is transformed into an ordinary differential equation:

$$L\left(t, x, q, q', q'', \ldots\right) = 0, \tag{4}$$

where superscripts demonstrate ordinary derivatives according η .

Step 2. Suppose that we imagine the solutions of Eq. (4) as Eq. (5):

$$q\left(\eta\right) = \frac{\sum_{i=0}^{N} a_i Q^i\left(\eta\right)}{\sum_{j=0}^{M} b_j Q^j\left(\eta\right)} = \frac{P\left[Q(\eta)\right]}{S\left[Q(\eta)\right]}, \tag{5}$$

where Q is $\frac{1}{1 \pm e^{\eta}}$. We must specify that Q ,

$$Q_\eta = Q^2 - Q, \tag{6}$$

is a solution to the Eq. (6). Using Eq. (5), the following derivatives are obtained,

$$q'(\eta) = \frac{P'Q'S - PS'Q'}{S^2} = Q'\left[\frac{P'S - PS'}{S^2}\right] = \left(Q^2 - Q\right)\left[\frac{P'S - PS'}{S^2}\right], \tag{7}$$

$$q''(\eta) = \frac{Q^2 - Q}{S^2}\left[(2Q-1)(P'S-PS') + \frac{Q^2-Q}{S}\left[S(P''S-PS'') - 2S'P'S + 2P(S')^2\right]\right]. \tag{8}$$

Step 3. The solution of the nonlinear ordinary differential equation given by Eq. (4) is sought according to the GKM as follows:

$$q\left(\eta\right) = \frac{a_0 + a_1 Q + a_2 Q^2 + \cdots + a_N Q^N}{b_0 + b_1 Q + b_2 Q^2 + \cdots + b_M Q^M}. \tag{9}$$

We use the homogeneous balance principle to find the values of M and N in Eq. (5). For this purpose, we balance between the highest order derivative and highest order nonlinear term in Eq. (4).

Step 4. We put in Eq. (5) into Eq. (4). Thus we get a polynomial $R(Q)$ of Q. Then equalizing the overall coefficients of $R(Q)$ to zero, we find an algebraic equation system. By solving the resulting system, we determine c and the variable coefficients of $a_0, a_1, a_2, \cdots, a_N, b_0, b_1, b_2, \cdots, b_M$ Finally we can have the exact solutions of Eq. (4).

3. Practice of GKM to the perturbed RKL equation

To get some soliton solutions of Eq. (1) we take into account the following equality:

$$q(x,t) = u(\eta)e^{iP(x,t)}, \eta = x - vt, P(x,t) = -kx + wt. \tag{10}$$

Replacing Eq. (2) into Eq. (1), we find the following real and imaginary equations respectively,

$$(a+3\gamma k)u'' - \left(w + ak^2 + \gamma k^3 + \varepsilon k\right)u + \left(b - k\lambda\right)u^3 = 0, \tag{11}$$

and

$$3\gamma u'' - 3\left(v + \varepsilon + 2ak + 3\gamma k^2\right)u - \left(3\lambda + 2\mu\right)u^3 = 0. \tag{12}$$

Following equality is obtained by applying the balance principle of u'' and u^3 in Eqs. (11) and (12).

$$N = M + 1. \tag{13}$$

If we choose $M = 1$ and $N = 2$ we ascertain the following solution constant,

$$u(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}, \tag{14}$$

$$u'(\eta) = \left(Q^2 - Q\right)\left[\frac{(a_1 + 2a_2 Q)(b_0 + b_1 Q) - b_1\left(a_0 + a_1 Q + a_2 Q^2\right)}{(b_0 + b_1 Q)^2}\right], \tag{15}$$

$$\begin{aligned} u''(\eta) &= \frac{Q^2 - Q}{(b_0 + b_1 Q)^2}(2Q - 1)\left[(a_1 + 2a_2 Q)(b_0 + b_1 Q) - b_1\left(a_0 + a_1 Q + a_2 Q^2\right)\right] \\ &+ \frac{\left(Q^2 - Q\right)^2}{(b_0 + b_1 Q)^3}\left[2a_2\left(b_0 + b_1 Q\right)^2 - 2b_1\left(a_1 + 2a_2 Q\right)\left(b_0 + b_1 Q\right) + 2b_1^2\left(a_0 + a_1 Q + a_2 Q^2\right)\right]. \end{aligned} \tag{16}$$

Eqs. (11) and (12) were solved together and solutions of Eq. (1) were found in the following different cases.

Case 1:

$$a_0=-\frac{i\sqrt{a+3(\gamma+\gamma k)}b_0}{\sqrt{2b-2\lambda(k+3)-4\mu}},a_1=-\frac{a_2}{2}+\frac{i\sqrt{2}\sqrt{a+3(\gamma+\gamma k)}b_0}{\sqrt{b-\lambda(k+3)-2\mu}},$$
$$b_1=-\frac{i\sqrt{b-\lambda(k+3)-2\mu}a_2}{\sqrt{2a+6\gamma(1+k)}}, \tag{17}$$
$$w=\frac{1}{2}\left(-a-2ak(6+ak)-6v-3\gamma-\gamma k(3+2\gamma k(9+\gamma k))-2(3\varepsilon+\varepsilon k)\right).$$

We get the trigonometric solutions of Eq. (1) by placing the obtained Eqs. (17) in Eq. (14)

$$q_1(x,t)=-\frac{e^{i(-kx+wt)}\sqrt{a+3(\gamma+\gamma k)}\tan\left[\frac{ix-ivt}{2}\right]}{\sqrt{2}\sqrt{b-\lambda(k+3)-2\mu}}, \tag{18}$$

$$q_2(x,t)=-\frac{e^{i(-kx+wt)}\sqrt{a+3(\gamma+\gamma k)}\cot\left[\frac{ix-ivt}{2}\right]}{\sqrt{2}\sqrt{b-\lambda(k+3)-2\mu}}, \tag{19}$$

where $w=\frac{1}{2}\left(-a-2ak(6+ak)-6v-3\gamma-\gamma k(3+2\gamma k(9+\gamma k))-2(3\varepsilon+\varepsilon k)\right)$. Both 2D and 3D graphics of solution (18) are shown in Figure 1.

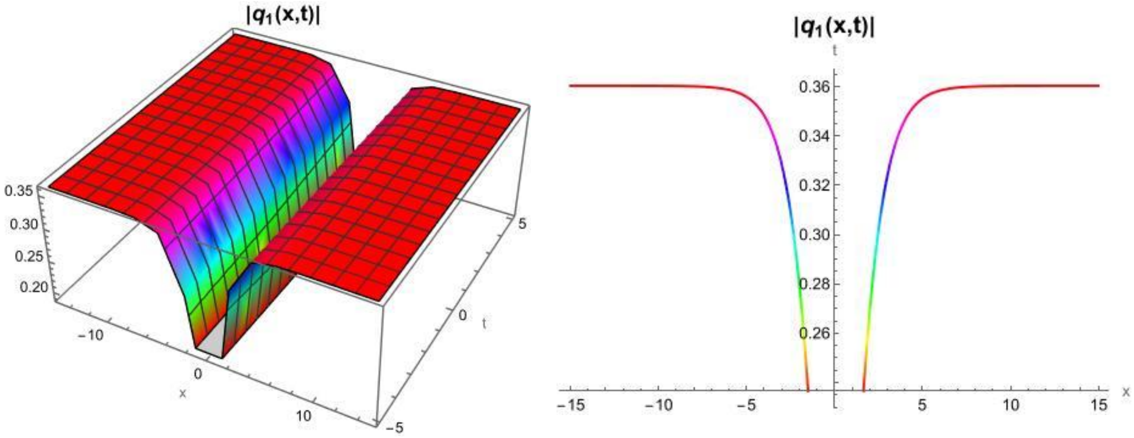

Figure 1. 3D plot of solution (18) for $a=5, v=0.05, k=4, \gamma=0.1, b=3, \lambda=4, \mu=0.01$, $w=0.5, -15<x<15$ values with $-5<t<5$ range and 2D plot of solution for $t=1.5$ with these values.

Case 2:

$$a_0 = \frac{(3i-3)\sqrt{(a+3\gamma(1+k))}b_0}{2\sqrt{2(b-\lambda(k+3)-2\mu)}},$$

$$a_1 = -\frac{2i\sqrt{2}\sqrt{a+3\gamma(1+k)}b_0}{\sqrt{(b-\lambda(k+3)-2\mu)}}, a_2 = \frac{2i\sqrt{2}\sqrt{a+3\gamma(1+k)}b_0}{\sqrt{(b-\lambda(k+3)-2\mu)}}, b_0 = -2b_0, \tag{20}$$

$$w = -\frac{5a}{4} - ak(6+ak) - 3v - \frac{15\gamma}{4} - \frac{\gamma k}{4}(15+4\gamma k(9+\gamma k)) - 3\varepsilon - \varepsilon k - \frac{3i(a+3\gamma(1+k))}{\sqrt{-16}}.$$

We get the hyperbolic function solutions of Eq. (1) by placing the obtained Eqs. (20) in Eq. (14)

$$q_3(x,t) = \frac{-ie^{i(-kx+wt)}\sqrt{a+3\gamma(1+k)}\left((-1+3\cosh[vt-x])\operatorname{csch}[vt-x] - \coth\left[\frac{x-vt}{2}\right]\right)}{2\sqrt{2}\sqrt{b-\lambda(k+3)-2\mu}}, \tag{21}$$

where
$$w = -\frac{5a}{4} - ak(6+ak) - 3v - \frac{15\gamma}{4} - \frac{\gamma k}{4}(15+4\gamma k(9+\gamma k)) - 3\varepsilon - \varepsilon k - \frac{3i(a+3\gamma(1+k))}{\sqrt{-16}}.$$
Both 2D and 3D graphics of solution (21) are shown in Figure 2.

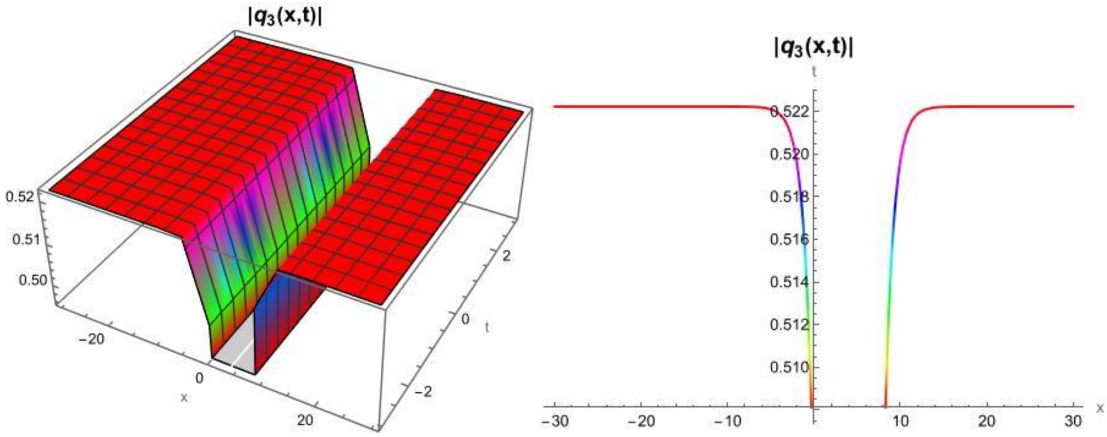

Figure 2. 3D plot of solution (21) for $a=-3, v=2, k=2, \gamma=1, b=3, \lambda=-2,$ $\mu=1, w=3, -30<x<30$ values with $-3<t<3$ range and 2D plot of solution for $t=2$ with these values.

Case 3:

$$a_0 = \frac{1}{4}a_1\left(-2-\frac{\sqrt{2}\left(a+3(\gamma+\gamma k)\right)b_1}{\sqrt{-\left(a+3(\gamma+\gamma k)\right)\left(b-\lambda\left(k+3\right)-2\mu\right)a_1^2}}\right),$$

$$a_2 = \frac{\sqrt{2}\left(a+3(\gamma+\gamma k)\right)b_1}{\sqrt{-\left(a+3(\gamma+\gamma k)\right)\left(b-\lambda\left(k+3\right)-2\mu\right)}}, \tag{22}$$

$$b_0 = \frac{\sqrt{-\left(a+3(\gamma+\gamma k)\right)\left(b-\lambda\left(k+3\right)-2\mu\right)a_1^2}}{\sqrt{2}\left(a+3(\gamma+\gamma k)\right)}+\frac{b_1}{2},$$

$$w=\frac{1}{2}\left(-a-2ak(6+ak)-6v-3\gamma-\gamma k(3+2\gamma k(9+\gamma k))-2(3\varepsilon+\varepsilon k)\right).$$

We get the dark soliton solutions of Eq. (1) by placing the obtained Eqs. (22) in Eq. (14)

$$q_4(x,t)=e^{i(-kx+wt)}$$

$$\frac{-a_1\left(2+\frac{\sqrt{2}Ab_1}{\sqrt{-ABa_1^2}}\right)-\left(2a_1-\frac{\sqrt{2}Aa_1b_1\left(1-\tanh\left[\frac{x-vt}{2}\right]\right)}{\sqrt{-ABa_1^2}}\right)\left(-1+\tanh\left[\frac{x-vt}{2}\right]\right)}{4\left(\frac{\sqrt{-ABa_1^2}}{\sqrt{2}A}+\frac{b_1}{2}\right)-2b_1\left(-1+\tanh\left[\frac{x-vt}{2}\right]\right)}, \tag{23}$$

$$q_5(x,t)=e^{i(-kx+wt)}$$

$$\frac{-a_1\left(2+\frac{\sqrt{2}Ab_1}{\sqrt{-ABa_1^2}}\right)-\left(2a_1-\frac{\sqrt{2}Aa_1b_1\left(1-\coth\left[\frac{x-vt}{2}\right]\right)}{\sqrt{-ABa_1^2}}\right)\left(-1+\coth\left[\frac{x-vt}{2}\right]\right)}{4\left(\frac{\sqrt{-ABa_1^2}}{\sqrt{2}A}+\frac{b_1}{2}\right)-2b_1\left(-1+\coth\left[\frac{x-vt}{2}\right]\right)}, \tag{24}$$

where $w=\frac{1}{2}\left(-a-2ak(6+ak)-6v-3\gamma-\gamma k(3+2\gamma k(9+\gamma k))-2(3\varepsilon+\varepsilon k)\right)$, $A=\left(a+3(\gamma+\gamma k)\right), B=\left(b-\lambda\left(k+3\right)-2\mu\right)$. Both 2D and 3D graphics of solution (23) are shown in Figure 3.

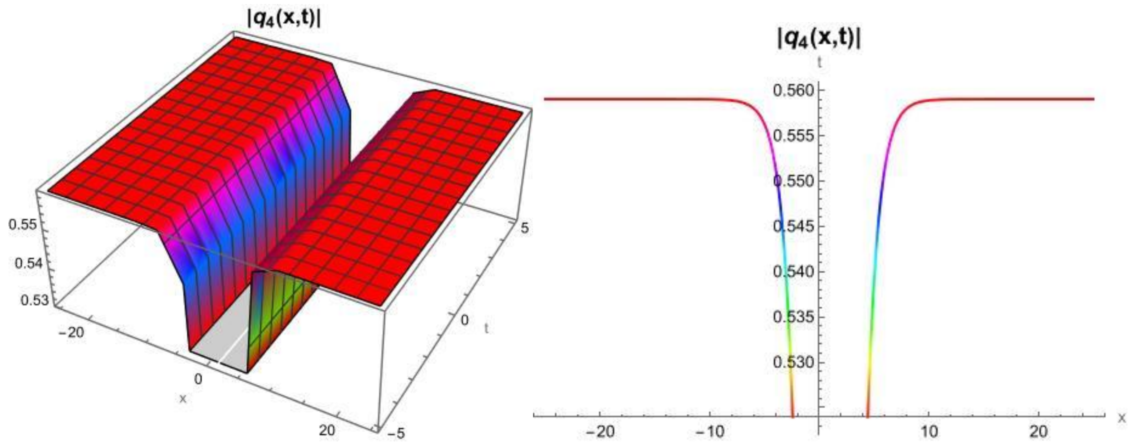

Figure 3. 3D plot of solution (23) for $a=1, a_1=2, k=2, v=1, \gamma=1, \lambda=3, \mu=1, b=1,$ $b_1=-2, w=2, -25<x<25$ values with $-5<t<5$ range and 2D plot of solution for $t=1$ with these values.

Case 4:

$$a_0=0, a_2=-a_1, b_0=\frac{i\sqrt{\left(b-\lambda\left(k+3\right)-2\mu\right)}a_1}{2\sqrt{2}\sqrt{a+3\gamma(1+k)}}, b_1=\frac{-i\sqrt{\left(b-\lambda\left(k+3\right)-2\mu\right)}a_1}{\sqrt{2}\sqrt{a+3\gamma(1+k)}}, \tag{25}$$
$$w=a-ak(6+ak)-3v+3\gamma-\gamma k\left(-3+\gamma k(9+\gamma k)\right)-3\varepsilon-\varepsilon k.$$

We get the bright soliton solutions of Eq. (1) by placing the obtained Eqs. (25) in Eq. (14)

$$q_6(x,t)=\frac{-i\sqrt{2}e^{i(-kx+wt)}\sqrt{a+3\gamma(1+k)}\,\operatorname{csch}[x-vt]}{\sqrt{b-\lambda\left(k+3\right)-2\mu}}, \tag{26}$$

$$q_7(x,t)=\frac{-i\sqrt{2}e^{i(-kx+wt)}\sqrt{a+3\gamma(1+k)}\,\operatorname{sech}[x-vt]}{\sqrt{b-\lambda\left(k+3\right)-2\mu}}, \tag{27}$$

where $w=a-ak(6+ak)-3v+3\gamma-\gamma k\left(-3+\gamma k(9+\gamma k)\right)-3\varepsilon-\varepsilon k$. Both 2D and 3D graphics of solution (26) are shown in Figure 4.

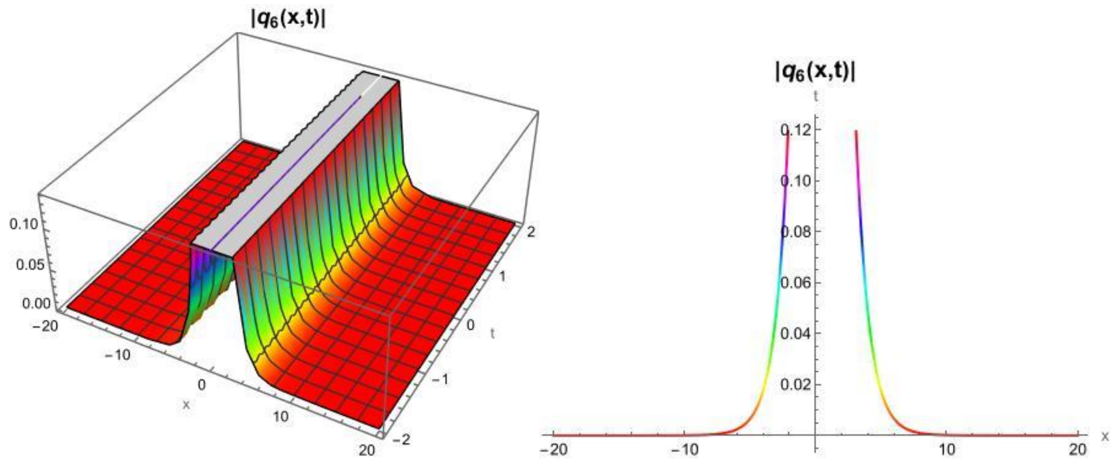

Figure 4. 3D plot of solution (26) for $v = 0.5, k = 2, \gamma = 1, b = 3, \lambda = 4,$ $\mu = 1, w = 3, -20 < x < 20$ values with $-2 < t < 2$ range and 2D plot of solution for $t = 1$ with these values.

4. Results and discussion

We obtained some soliton solutions of perturbed RKL equation by applying GKM. In addition, the accuracies of the obtained results are shown with some graphics drawn in 2D and 3D. When the results of previous studies on the perturbed RKL equation are compared with our results, our (25) solution is similar to the (64) solution given by Ghanbari and Gomez-Aguilar [16], our (27) solution is similar to the (13), (23) and (34) solutions given by Biswas et al. [15] and our (28) solution is similar to the (12), (22) and (33) solutions given by Biswas et al. [15] with the (11), (18), (30) and (34) solutions given by Biswas [14]. According to our research, other solutions are new.

5.. Conclusions

In this study, which was prepared on obtaining the solutions of NLEEs, GKM was discussed. GKM is implemented to the perturbed RKL equation and thus some trigonometric, hyperbolic function, dark soliton and bright soliton solutions of the equation are obtained. In addition, 3D and 2D graphics were made by giving certain values to the obtained solutions. Thus, the found solutions were verified through the graphs drawn. As a result of this study, it has been seen that GKM is reliable and giving precise results a method in finding the solutions of NLEEs.

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