

An Analysis of Prospective Mathematics Teachers' Views on the Use of Computer Algebra Systems in Algebra Instruction in Turkey and in the United States*

S. Asli Özgün-Koca¹

Abstract

This study investigated the views of Turkish and U.S. prospective mathematics teachers on the use of advanced calculators with Computer Algebra Systems (CAS) in algebra instruction. The possible roles for CAS suggested by Heid and Edwards (2001), along with the black and white box dichotomy and Technological, Pedagogical, and Content Knowledge model were used as conceptual frameworks. An open-ended questionnaire and group interviews revealed participants' views and beliefs about why, when, and how they prefer to use CAS. Results revealed the similarities and differences in Turkish and U.S. participants' views regarding the use of CAS when teaching and learning of algebraic manipulation.

Key Words: Mathematics education, teachers' views, technology

1. Introduction

Studies in the literature point out that teachers' beliefs and views about teaching and learning their subject area influence their pedagogical choices (Ball, Lubienski, & Mewborn, 2001; Kendal & Stacey, 2001; NCTM, 1991; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984, 1992). The decision whether to use technology or not, and if yes, how to use those chosen technologies in mathematics teaching and learning is among the decisions teachers must make (Özgün-Koca, 2009). In 2005 and 2007, at the Computer Algebra in Mathematics Education Symposiums, the need for obtaining and studying teachers' views of Computer Algebra Systems (CAS), was emphasized.

The current study investigated the views of Turkish and U.S. prospective mathematics teachers on the utilization of advanced calculators with CAS in algebra instruction before and after having a brief experience with CAS.

The research questions of this study were:

- What are Turkish and U.S. teachers' views of the use of CAS in algebra instruction?
- What are their views of two possible ways of using CAS in algebra instruction—black box and white box (including Symbolic Math Guide)?

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¹ Assoc. Prof. Dr., Wayne State University, College of Education, Teacher Education, aokoca@wayne.edu

- What are the similarities and differences in Turkish and U.S. participants' views regarding CAS in algebra instruction?

Background Information on CAS

As NCTM (2000) stated, “Technology not only influences how mathematics is taught and learned but also affects what is taught and when a topic appears in the curriculum” (p. 26). Using CAS, with a few key presses, students can simplify, factor, and expand very complex expressions, solve equations, obtain the quotient and remainder from the division of two polynomials, and find the common denominator for rational expressions (see Figure 1). These powerful capabilities and potential different uses of CAS may cause a cautious approach from some teachers.

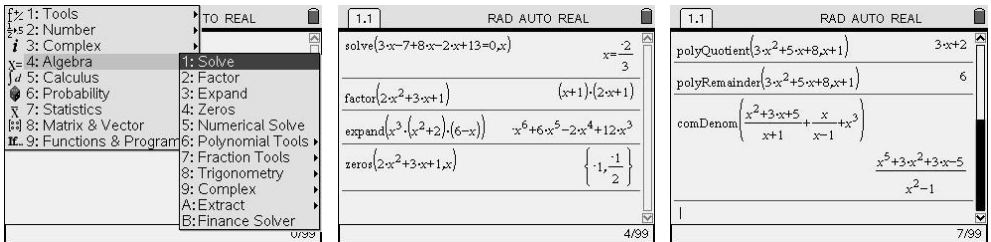


Figure 1. Some possible algebraic calculations with CAS (TI-Nspire CAS was used to create the screen shots)

Without question, the capabilities of CAS force teachers to reconsider and reflect on the mathematics that they are teaching and their instructional strategies (Lumb, Monaghan & Mulligan, 2000). Since the secondary Algebra curriculum looks like just a few key strokes away with CAS, Kutzler (2000) discusses the importance of the balance between curricular tasks requiring the use of CAS technology with the aim of emphasizing problem solving strategies and the traditional tasks.

If CAS can do many algebraic manipulations easily and efficiently, one can question the role of symbolic manipulation in algebra teaching and learning (Edwards, 2002; Heid, 2005; Mahoney, 2002; Pierce & Stacey, 2004; Schneider, n.d.). Many researchers suggested that students need to focus on a bigger picture which includes symbolic sense, where conceptual understanding of algebraic manipulations and transformations is emphasized via the help of CAS (Drijvers, 2004; Edwards, 2002, Forgasz & Griffith, 2006, Heid & Edwards, 2001; Mahoney, 2002; Pierce & Stacey, 2004). Instead of repetitive paper-and-pencil activities for algebraic computation, students could use CAS to do discovery and problem solving in algebra activities (Edwards, 2003; Mahoney, 2002). Kutzler (2000) and Pierce and Stacey (2004) also suggest that CAS could play a supportive scaffolding role where CAS could execute algebraic calculations, so students who lack those lower-level skills could focus on the higher-level skills. Peschek (2007) carries this discussion further and argues that CAS could be responsible for all the calculations, so the

students' job would be to interpret the results. However, Kieran (2007) argues that "it is quite unrealistic to expect students to be able to interpret and assess the answers produced by the CAS if their general mathematical education does not include provision for developing operative knowledge and skills" (p. 106). Kieran's worry may result from the use of CAS as a black box. As a black box, CAS could carry out "complex procedures in a way that is not transparent to them [students]" (Drijvers, 2004, p. 80).

Conceptual Framework

The White Box and Black Box dichotomy as different approaches to using CAS in a teaching and learning environment is well established in the mathematics education literature. Buchberger (1990) explains the black box approach as when CAS produces algebraic manipulations without showing inner workings. In the black box phase, algebraic manipulations involving more than one step can be done all at once using symbolic computation software systems with intermediate steps hidden from the user. On the other hand, in the white box phase, algorithms must be studied thoroughly in the CAS environment and intermediate steps are not hidden (Buchberger, 1990). Specific examples of the white box and black box approaches are provided in the next section.

The second part of this framework is adapted from Heid and Edwards' (2001) list of four possible roles for CAS in mathematics education:

1. CAS as the primary producer of symbolic results in which CAS makes computations in order for the user to focus on the concepts,
2. CAS as an assistance for students to generate many examples in order to search for symbolic patterns,
3. CAS as a generator of results for problems posed in abstract form, and
4. CAS as a pedagogical tool, which creates and generates symbolic procedures to assist students in constructing conceptual understandings (pp.130-132).

In order to create a conceptual framework regarding CAS for this study, the White Box and Black Box dichotomy with Heid and Edwards's possible roles for CAS were integrated in order to analyze prospective teachers' views of the use advanced calculators to teach algebra. This framework is represented in Figure 2.

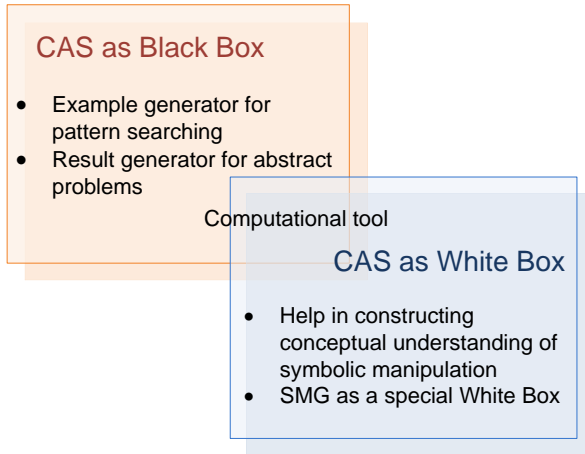


Figure 2. Conceptual framework for CAS

CAS as a computational/transformational tool, the first role defined by Heid and Edwards (2001), could be used “to liberate students ‘from the technical aspects of paper-and-pencil computing’ while encouraging them ‘to keep sight of the main goal’” (Lagrange, 2005b, p. 118). Similarly, according to Drijvers (2003, 2004), when CAS is used as a computational tool, it could free students from focusing on algebraic procedures and allow them to focus on concept development and problem solving strategies. So, CAS could transform the nature of the teaching and learning environment. This role could be part of both white and black box approaches; hence it was placed in the intersection. CAS as an example generator could be used for pattern searching. The main focus would be on input and output not on the intermediate steps (Drijvers, 2004; Lagrange, 2005a, 2005b; Stacey, Kendal, & Pierce, 2002; Schneider, 2000). Students can use CAS as a result generator when they check their hand-calculated responses or want to have a quick result for a complicated algebraic manipulation. Here the focus is not on how to do the algebraic manipulation. That is why these two roles were categorized as a black box. CAS, when used as a white box, can help students develop conceptual understanding of symbolic algebraic manipulation. In this study, Symbolic Math Guide (SMG) is used as a special version of the white box approach which provides more guidance to users.

Mishra and Koehler (2006) and Niess et al. (2009) introduced the concept of the teachers’ Technological Pedagogical Content Knowledge (TPACK) in the beginning of the 21st century (see Figure 3). TPACK involves content knowledge, pedagogical knowledge and technological knowledge. The intersections in the Venn diagram display the possible combinations. Shulman (1986) established the importance of Pedagogical Content Knowledge (PCK) in the 1980s. PCK focuses on the mutual relationship between content and pedagogy. The technological knowledge circle includes knowledge such as knowing how to operate a specific technology and two other combinations, Technological

Pedagogical Knowledge (TPK) and Technological Content Knowledge (TCK). TPK is “knowledge of the existence, components, and capabilities of various technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as the result of using particular technologies” (Mishra & Koehler, 2006, p. 1028). In this study, pedagogical knowledge with specific attention to the TPK served as the main framework for data analysis and interpretation.

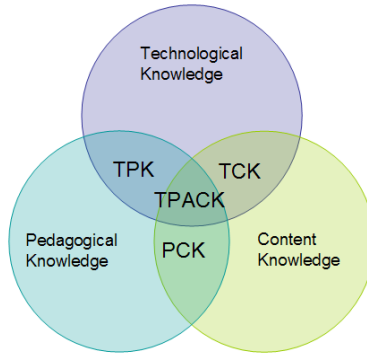


Figure 3. Re-creation of Mishra and Koehler’s TPACK model

2. Methodology

Setting and Participants

The setting in Turkey** was located in a 5-year teacher education program ($n=27$) and the other was in a Midwestern university in the United States ($n=22$). Both samples were selected as a result of convenience typical case sampling (Miles & Huberman, 1994). Turkish participants were in an integrated program. This program accepted its students as undergraduates, and subject area courses were completed in seven semesters. Courses related to professional education and electives were offered in the three remaining semesters, where the last year was categorized as the graduate part of the program. When students complete the program, they graduate with a master’s degree and a teaching certificate. Turkish participants had no prior experience with advanced graphing calculators in their K-12 education, but they experienced some graphing activities with the advanced calculators in their mathematics methods courses. The U.S. participants represented a range of backgrounds in general, education, experiences, and technological expertise regarding graphing calculators. Most of the U.S. participants categorized themselves as intermediate level users of graphing calculators. Many of them used graphing calculators when they were in secondary education or at college when learning mathematics.

Looking at the TIMSS 2007 data, we saw that the U.S. and Turkey represent the two ends of the spectrum when it comes to calculator use (Martin, Mullis, & Foy, 2008). While 41% of the Turkish eighth grade students are not permitted to use calculators, this

**A manuscript involving only Turkish data has been published (Özgün-Koca, 2010)

percentage is only 11% for the U.S. population (the international average is 25%). The calculators were used for solving complex problems (U.S.: 57%, TR: 5%, International Average: 31%), checking answers (U.S.: 45%, TR: 6%, International Average: 26%), performing routine computations (U.S.: 43%, TR: 3%, International Average: 25%), and exploring number concepts (U.S.: 43%, TR: 2%, International Average: 16%) (Martin, Mullis, & Foy, 2008). So, the access to technology varies greatly in these two countries. Even though the selection of these two countries was a result of convenience sampling, comparing the naïve views provided by Turkish prospective teachers to those of their more experienced American counterparts resulted in interesting results in spite of the different experience levels with graphing calculators.

Data Collection Methods and Analysis

First, participants were presented with an activity for solving linear equations in which Buchberger's black box and white box ideas were introduced along with Symbolic Math Guide (SMG), a special case of white box. These activities were executed in order to help participants to see the basic principles underlying different uses of CAS and lasted approximately one 3-hour blocked class period. After these introductory activities, participants were asked to respond in writing to the following questions:

- Discuss the utilization of (advanced/graphing) calculators with CAS capability in algebra instruction and please take the following points into consideration:
 - The advantages and disadvantages of black box, white box, and SMG.
 - The effects on the teaching and learning environment and process.
 - When black box, white box, and SMG would be more beneficial, (i.e. under what conditions, when, and how they should be used).
- Did your views on the use of calculators in algebra instruction change? If yes, how?

Then, all participants participated in group interviews which were taped for further analysis. The same questions from the writing portion were asked in semi-structured group interviews. Due to the nature of qualitative data, the codes and categories were formed to study the emerging themes throughout. Checklist matrices (Miles & Huberman, 1994) were created with the aim of uncovering complementary and opposing views of the participants on the advantages, disadvantages, and possible utilizations of black box and white box (including SMG). While creating the checklist matrices, first the advantages and disadvantages that participants assigned for black box and white box were taken into consideration using the CAS framework. Next, the TPACK framework was utilized to re-interpret the data related to the participants' views regarding the potential effects of CAS on algebra teaching and learning. The tallies in the checklist matrices made it possible to calculate the percentages of participants sharing similar views. Taking these codes and patterns into account, the group interviews were analyzed. Some of the decision rules followed for data entry and selecting quotes included: (1) the results of a counting method were used in reporting the results; (2) rival explanations were considered, even though they

were mentioned by a minority of the participants; and (3) participants' quotes were selected when they were typical, reflective, and/or communicative.

Data triangulation and member checks were used to ensure the trustworthiness of this study. Although writings guaranteed that the participants had sufficient private time to respond without pressure, interviews were added to provide an environment where group communication and interaction could be observed. Document analysis is strengthened when augmented with interviews. Thus, the two forms of data collection with different encounter levels provided rich sources of data for this study. The synthesis of three conceptual frameworks also strengthened the data analysis and interpretation of the data. Member check questions during the group interviews were used in order to ensure correct understanding of what the participants meant.

The Activity-Three Different Uses of CAS in Algebra

A black box environment applies more than one step in order to simplify an algebraic expression automatically where intermediate steps are hidden and produces algebraic manipulations such as expand or solve which involve more than one step where intermediate steps are hidden (see Figure 4a). Still the appropriate use of black box could enable users to search for patterns and make conjectures (see Figure 4b).

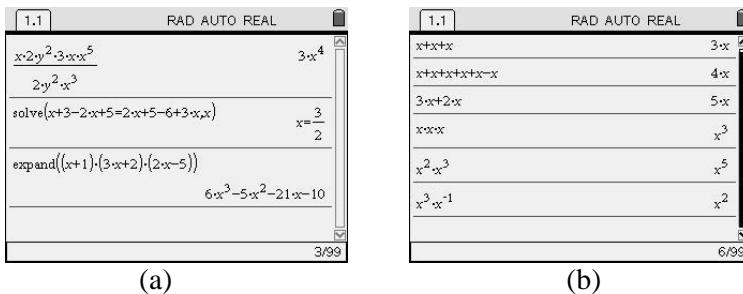


Figure 4. CAS as black box

Solving a linear equation, $3x - 6 = 9 - 2x$, is executed in a CAS environment in a white box fashion in Figure 5. The intermediate steps are not hidden in this environment, but students remain free from algebraic and arithmetic computations and can focus on the algebraic transformations needed to solve the equation. Once the equation is entered, it is displayed as it is on the calculator screen (see Figure 5a). Different operations could be applied to the whole equation in order to attempt to solve it such as adding $2x$ to both sides by just hitting $+2x$ (see Figure 5a, 5b, 5c, and 5d). Students could subtract 5 from in an attempt to get rid of the coefficient of x in Figure 5e. The white box environment does not provide any messages for the students. Users must judge whether the applied operation is helpful to solve an equation (see Figure 5f). They can cancel an applied operation after deciding that the transformation was not efficient and continue to solve the equation (see Figure 5g and 5h).

Edwards (2001) suggested that the use of white box before the use of black box could lead to optimum learning: “Buchberger's call for a ‘white box first’ teaching approach, one in which step-by-step algebraic processes are well understood before resorting to the use of the calculator as black box, are better suited for the conceptual level of understanding of the typical secondary school mathematics student” (p. 306). However, first using black box and then white box could “elicit curiosity and can lead to interesting discoveries” (Drijvers, 2000, p. 190).

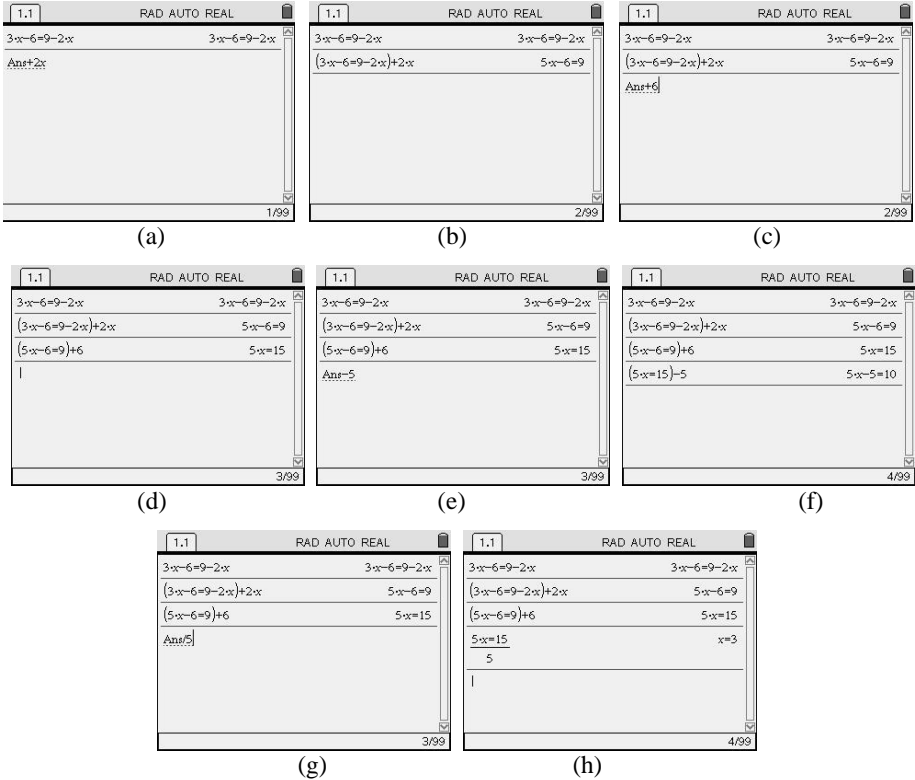


Figure 5. CAS as white box

Symbolic Math Guide (SMG) as a special white box aims to help students with symbolic and algebraic transformations by providing step-by-step transformations. A linear equation is solved in the SMG environment in Figure 6. After entering an equation to solve (see Figure 6a), users are offered all possible transformations (not only the most effective but also the least effective ones) that students could choose from, in order to do an algebraic manipulation (see Figure 6b). After students make a choice, their transformation is displayed without any simplification or calculation (see Figure 6c). This pause may help

students to re-consider their choice of transformation. After hitting the “Enter” button, the equation is simplified (see Figure 6d). Users are offered again all possible transformations for the next steps (see Figure 6e). After the student verifies the chosen transformation, it is simplified (see Figure 6f). At this point, the list of the transformations is shortened due to the lesser number of possible transformations (see Figure 6g). Finally, the equation is solved after the last transformation (see Figure 6h). As in the white box method, while using SMG, students are less preoccupied with calculations – spending more time considering algebraic transformations and concepts of equation solving. However when compared to the white box method, SMG provides extra guidance by providing the list of permissible transformations from which users may choose.

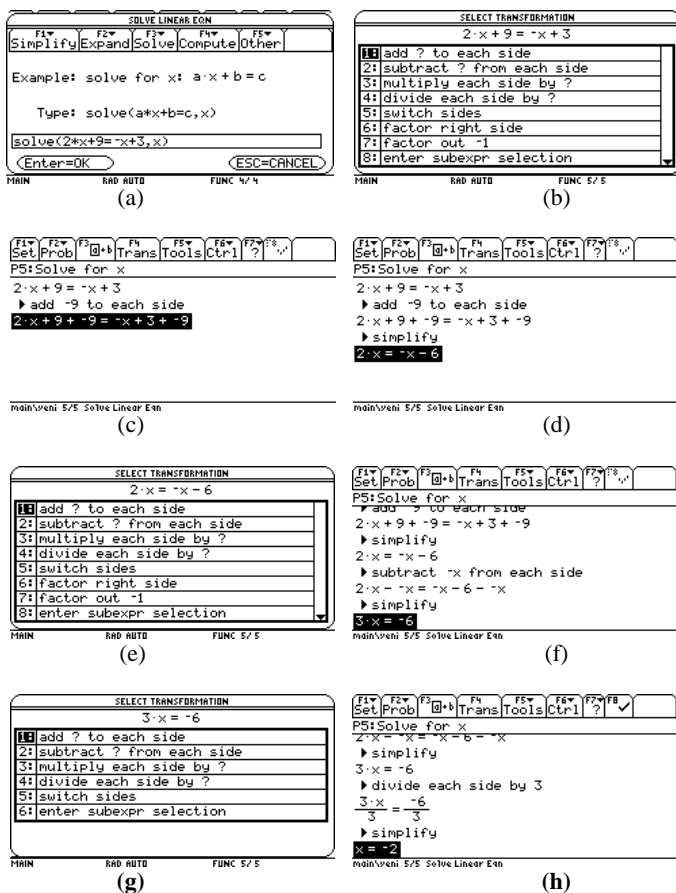


Figure 6. An example of solving a linear equation in SMG environment (A TI-92 was used to create the screen shots for the SMG demonstration)

3. Results

The main themes of the prospective teachers' views were:

- CAS could be an interesting tool to be used in the classroom. But some teachers preferred to use CAS after students have mastered skills, especially for the black box use.
- CAS as a white box was seen as more of a pedagogical tool compared to CAS as a black box.
- Participants' responses on the potential effects of white box on students' learning and how to use CAS effectively in teaching algebra differed slightly.

CAS as Teaching and Learning Tool

Learning with CAS. Participants did not assign a pedagogical role to black box use of CAS. Thirty-six percent of the U.S. and 52% of the Turkish participants mentioned the main weakness of this method would be hidden intermediate steps. As a result, 33% of the Turkish participants mentioned that the black box method would lead students to memorize without understanding.

Since it shows the result directly, it could hinder students' skills of doing manipulations, developing strategies, recognizing the procedural steps. Therefore, it leads the students to memorization and memorizing the result (Writing-Turkish Participant).

According to two Turkish and one U.S. prospective teachers, the only way of using CAS a learning tool would be when CAS is utilized as a "generator of many examples for pattern searching." In this environment, students are able to discover and explore:

Because the student sees many examples in that way, s/he can directly start discovering by seeing all of the examples at one time. If we had done that on the board one-by-one, it would have taken more time (Writing-Turkish Participant).

Another role assigned to CAS as black box by the participants was "generator of results for problems." In this use, students can obtain quick answers to algebraic problems after they have already mastered the subject and just need a fast result or to check their hand-calculated result: "Black box could also be helpful in double checking work and promoting a need for accuracy in students work habits" (Writing-U.S. Participant).

The main theme for the white box was that it could become a learning tool with the aim of helping students to comprehend the concepts as in the CAS conceptual framework. The importance of a step-by-step approach was mentioned by 36% of the U.S. prospective teachers: "While learning the essential steps in working out an algebra problem, students could easily see which steps would or would not work with the programs white box and SMG at their disposal" (Writing-U.S. Participant). Similar to CAS as black box, it was mentioned that CAS as white box could also be used as a checking tool where students

check their *process* of doing algebraic manipulation, not the *result*. Others mentioned that white box could provide an exploratory environment for students to reflect on the process:

The white box program is excellent for teaching students how to solve equations. The students can actually see what manipulating the terms does, and they can see their errors, and correct them (Writing-U.S. Participant).

In addition to, and as a part of, this exploratory process, 41% of Turkish and 27 % of U.S. participants emphasized the importance of the control that students have when using CAS as white box: "Since the options are not presented in white box [as in SMG], the student needs to decide what to do by herself or himself" (Writing-Turkish Participant). When students are in charge of what to do next and get immediate feedback, they can experience confirmation or disequilibrium during this process. Here, using their TPK, prospective teachers reflect on the importance of students deciding the next transformation on their own in an algebraic manipulation process. As a result of this conclusion, CAS as white box could provide opportunities for pattern searching which suggests a revision of the CAS conceptual framework to place CAS as an example generator for pattern searching/discovery at the intersection of CAS as Black Box and CAS as White Box.

A contradictory result about CAS as a white box concerned taking care of the basic calculations. Some participants mentioned that they liked this capability which allows students to focus on the transformations or algebraic steps. Others mentioned this as a disadvantage of the white box method, because students do not get to practice their basic skills. The participants who favored basic calculations being done by the white box allowing students to focus on the process emphasized the use of white box as a computational tool and a pedagogical tool in the CAS conceptual framework.

SMG was clearly favored over other uses of CAS by some of the participants. Sixty-three per cent of the Turkish and 55% of the U.S. participants discussed the importance for students' learning of seeing possible steps and different ways of doing algebraic manipulations:

I liked the way it offers choices. Because, if a student thinks of one or two ways of solving the problem, with these choices, the number of methods s/he thinks of would increase, which would broaden the student's horizon (Writing-Turkish Participant)

I think that the white box is a little harder than the SMG, but for kids having a hard time recalling or knowing what to do, it [SMG] gives them a choice. At least they can get started (U.S. Group Interview).

The existence of these steps was also seen as making it possible for students to have an exploratory experience:

Because of the steps, [SMG] is a great tool for discovery and exploration (Writing-U.S. Participant).

What happens when I choose an option? This could make the student question the process of solving an equation (Writing-Turkish Participant).

Moreover, as with the general white box, the nonjudgmental environment could make this exploratory environment more welcomed:

In discovery activities, we want the student to find things. When the student reaches a wrong result, we want her or him to find out why he or she did wrong...we are facilitators. This machine [SMG] is the same. It provides the options, it guides and says you can choose the one you want or like. You choose. 'It was not right. So I did something wrong. But where?' You can start over (Turkish Group Interview).

Again as with the general white box, participants thought that students could experience disequilibrium in the SMG environment when their chosen transformation does not have the intended effect. Hence learning opportunities for students would be created to re-interpret/assimilate this new knowledge, make necessary accommodations, if needed, and strengthen their schemes.

Participants also thought that the SMG environment could provide more guidance to those who do not know where to start or have difficulties during the process: "If a student is confused on what to do first, they can see the options and that may spark their memory" (Writing-U.S. Participant). A few warned that this could also become a guessing game.

Two U.S. participants thought that using words for those steps in SMG could be more beneficial for students' learning:

The choices are even written out in words, which sometimes makes it easier for the kids to understand. Sometimes looking at all the numbers and letters and symbols, they can get overwhelmed, and this clearly says "add a number to each side, multiply and set up" will help them get their grips on it (U.S. Group Interview).

Finally, the capability of SMG to create a new menu for each step was another benefit that some participants noted:

In solving equations, students being free to choose the operation and the calculator's limitations [to the legal operations] in some cases are good for finding the correct operation (Writing-Turkish Participant).

The good part about it is that the options they have decreases and that probably would trigger a little thinking why why...(U.S. Group Interview).

Some participants emphasized the motivational role of using CAS in the classroom. A few participants highlighted that CAS could be suitable for secondary or even college mathematics classrooms. Some participants shared their worries about the learning curve

involved with new technology, not only for students, but also for teachers. The teacher also should be knowledgeable about how to operate this new technology and how to use it in the classroom pedagogically. Here some participants were discussing the need not only for teachers' technological knowledge but also their technological pedagogical knowledge.

All these discussions above about how the availability of a technological tool might influence the students' learning demonstrate participants' use of their TPACK entailing "knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face" (Mishra & Koehler, 2006, p. 1029). For instance, when a participant discussed that white box or SMG helps students focus on the algebraic transformations, s/he knows that many students could feel overwhelmed due the calculations which may prevent them learning a new concept. Or the technological capability of SMG reducing the list according to the available legal transformations might spark students' interest about those transformations. Moreover, some participants suggested that SMG could be used to provide scaffolding for those students who needed an extra push to move forward, while some warned about the possibility of abuse of SMG by some students aimlessly guessing. In these suggestions, the use of technology is viewed as an instructional tool enhancing students' learning.

Teaching with CAS. After obtaining participants' views on how learning might change with the availability of CAS, participants were asked to share their views about how they would use CAS in their teaching. Similar to Edwards's (2001) suggestion of a 'white box first' approach, some suggested that they would start with SMG first to teach algebraic concepts, and then they would move to the regular white box method. After students concretize their knowledge, the black box method could be used to check work or get quick results for complex algebraic problems.

I would definitely use the SMG to first expose algebra to students because it is a step-by-step process while giving the students the steps on the screen for them to review. After I would use the white box to get them to solve expressions on their own. Once they totally understand the process I would show them the black box as a short hand process for them to use (Writing-U.S. Participant).

However, one U.S. participant stated that s/he would start with using the black box for a discovery introduction and then use the SMG for students "go through step by step until the steps are memorized. Then we would use the white box to see practiced the steps one at a time" (Writing-U.S. Participant). This resembles Drijvers' (2000) suggestion of the use of the black box first to involve students' in a discovery activity. In these quotes, different aspects of the participants' technological pedagogical knowledge (TPK) come into play where they discuss "how teaching might change as the result of using particular technologies" (Mishra & Koehler, 2006, p. 1028). With the first approach, the prospective teacher values the guided practice at first. However, the second prospective teacher starts with a discovery activity which is followed by guided practice.

When considering this powerful tool at hand, 27% of the U.S. participants mentioned that it would be better if CAS is used “after teaching the material and when the students are proficient at doing equations” (Writing-U.S. Participant). So some shared their preference for by-hand skills over the use of technology (Kendal, Stacey & Pierce, 2005; Zbiek, 2002). Here these participants make use of their TPK and consider how the availability and appropriate, or inappropriate, use of the technology (in this case CAS) could be disruptive, or sometime helpful, in students’ learning of algebraic manipulation.

Following the black box, white box, SMG demonstration of CAS, approximately half of the Turkish and one third of the U.S. participants reconsidered their views on the use of CAS in algebra instruction:

I have always thought that the calculator is a tool that is based on black box. But, learning about the white box made me happy and changed my mind (Writing-Turkish Participant).

I think I am more open to the possibility that calculators can help learning, and not become crutches (Writing-U.S. Participant).

Some of the U.S. participants who emphasized that their views have not changed mentioned that they were already in favor of calculators. Teachers’ knowledge and belief systems have a complex and dynamic structure. Not only were dramatic changes accomplished, such as from “would not use” to “can consider now,” but also “I never knew this” suggests that some horizons were extended.

Comparison of Different Uses of CAS

Having considered the data and participants’ comments in light of the conceptual framework in the previous sections, in this section black box and white box uses of CAS will be directly compared.

Black box method. Prospective teachers mainly see black box as a “generator of results for problems” as in the CAS conceptual framework. Participants liked that the black box could provide fast accurate calculations when students know the subject and just need the result:

[You can use black box,] if you only want the result and you do not need to see the intermediate steps...for instance...when we are performing high school stuff at the college level...I already know [the intermediate steps]. If the only thing that I want is the result, then I use black box (Turkish Group Interview).

A few participants mentioned that the black box method would be appropriate to be used by higher grades in higher mathematics classes or even science or engineering classes. Participants emphasized the potential for fast calculation, but they did not mention that the time saved could be used for concept development or interpretation as stated in the literature (Drijvers, 2004; Heid & Edwards, 2001; Langrange, 2005a, 2005b). Following this use, participants also thought that CAS as black box could be used as a checking tool:

“An advantage is that you can check your answers a lot faster using this [black box] method” (Writing-U.S. Participant).

However, black box was the least advocated method as a teaching tool, due to the hidden intermediate steps and easy access to the solution. Participants highlighted the technological capability of the calculators, but not the pedagogical gains from this use.

White box method. When used as white box, CAS was perceived as pedagogical tool as in the CAS conceptual framework. In general, participants saw the white box method as more advantageous than the black box method: “White box is a more developed version of black box... at least [white box] makes it possible for students to experience the process” (Turkish Group Interview). Students have the authority and get immediate feedback about their actions to strengthen their newly constructed knowledge when using the general white box. However, students could have difficulties due to the lack of guidance. At this point participants discussed SMG’s capability of creating a list for algebraic transformations and guiding students into the possible and different directions without telling them what to do.

A few participants suggested that students could use white box as a checking tool to strengthen their newly constructed knowledge and reflect on the process of algebraic manipulation. When used as black box for just checking the result, participants mainly viewed the calculator as a technological tool that provides an accurate result. Here the technological use is ahead of the pedagogical use. However, when used as white box or SMG, participants saw pedagogical implications, even for checking purposes, where students focus on the steps instead of the calculations.

Some participants also pointed out that the white box and SMG could be used for deeper exploration and gives more autonomy to the students:

Because they have the choices it’s really helping them figure out, “What happens if I do this. Oh no,..., let’s go back. What happens if I do that, oh ok.” You know it kind of helps them ...I think it makes it user friendly. Sometimes when they’re working on their sheets of paper they are afraid to write the wrong answer. “Is this right, am I doing it right?” And this way they kind of feel like “Well I can actually chose here and make a decision and not really get really bad consequences, I can just kind of go back a step.” I think that kind of helps ease their nervousness (U.S. Interview).

Here, we can observe that the participant made use of his or her TPACK to discuss “how technologies can be used to build on existing knowledge” (Mishra & Koehler, 2006, p. 1029), where a student can use his or her prior knowledge to make some choices and extend his or her knowledge for solving a linear equation in an exploratory nature without feeling judged by an authority.

Comparison of the Views of U.S. and Turkish Participants

It is natural to question that why Turkish prospective teachers as novice users of graphing calculators are compared with more experienced U.S. counterparts. However, CAS was a novel instrument for both groups. Moreover, as it was mentioned at the beginning of this paper, teachers' views play a very important role in their pedagogical decisions. Some of those views are formed without much experience. At the end, Turkish and U.S. participants had some similar views regarding the use of CAS. For instance, they both preferred the white box methods to the black box method as a pedagogical tool.

On the other hand, there were differences in the viewpoints of Turkish and U.S. participants which may stem from the experience that U.S. participants had. The calculators have been used especially for graphing purposes in U.S. classrooms at least for two decades. Therefore, U.S. participants tended to have more experiences both as students and teachers or student teachers in the practical use of this technology and were able to observe some positive or negative consequences of the use of calculators in classrooms.

More U.S. participants offered that students should master skills first and then use CAS. On a similar note, some Turkish participants still shared the concern that students could become dependent on the calculators and can be led to memorizing instead of understanding if CAS is used inappropriately. Moreover, U.S. participants focused on more practical issues in their writings and discussions. Some U.S. participants mentioned that SMG providing the steps in words would help students more. Other U.S. participants discussed that SMG providing a better visual environment for students (similar to paper and pencil) was helpful. Moreover, some U.S. participants liked that SMG and white box would eliminate writing by hand. They believe that would help students. These ideas were not discussed in the Turkish writings or interviews. This could be the effect of U.S. participants having more experience such as knowing different visuals provided by the current and older calculators.

On the other hand, more Turkish participants than their U.S. counterparts mentioned that they liked white box, since it provided more autonomy for students, and they liked SMG due the list provided by it to guide students in their writings. Turkish participants approached the issue more theoretically and used their theoretical knowledge of pedagogy in order to justify their views.

4. Discussion

In 2007 at the Fifth Computer Algebra in Mathematics Education Symposium, Kadijevich put it succinctly:

It must be underlined that most mathematics teachers do not realize the full power of computer assisted learning of mathematics...which prevents them from using CAS or other able technological tool in a way profitable to their learning and teaching (p. 7).

In order to make informed decisions, teachers should be knowledgeable about the potentials and different uses of CAS. Having no knowledge or the knowledge of only powerful capabilities of CAS, teachers may reject adopting it in their classrooms. They could not reflect on how CAS could be utilized effectively to teach students algebra. Some of the participants acknowledge that they reconsidered their views after this experience. While making the decision to use CAS in their classroom or not, participants used their TPACK. Some of them used more of the pedagogical knowledge that they developed in their teacher education program to support their views. Some used more of the experiences formed in their classrooms on a daily basis. Even this limited experience with CAS affected participants' TPACK. Some mentioned that their knowledge was extended or their views were confirmed.

We, as teacher educators, may help teachers not only construct and develop their technological, pedagogical, and content knowledge separately but also consider the relationships among them (as in the TPACK framework). For instance, we can ask them to reflect on the characteristics of CAS when used as white box to help students reinforce their algebraic manipulation skills to solve a linear equation. Their connected knowledge would exceed the sum of their individual parts. Moreover, not only prospective teachers' knowledge, but also their views and beliefs are shaped during their teacher education program. While increasing their knowledge, we can bring them face-to-face with their own beliefs and views through reflective writings or discussions with the aim of forming and supporting their views with their knowledge and vice versa.

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