The Impact of Technology on High School Mathematics Curriculum

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Abstract

The infusion of technology into school mathematics has intensified in the last two decades. This article discusses the effects of this infusion on the mathematics curriculum. After a review of the different roles technology plays in mathematics and the diversity of the tools and their functions in teaching and learning mathematics, an epistemological perspective is offered to understand how technology could affect our cognition and perception while doing mathematics. With this background, specific examples are offered for the ways in which our curricular goals are re-prioritized in algebra and geometry. The paper is concluded with a discussion of teachers' proficiency as a factor to promote effective use of technology in the high school mathematics curriculum based on Beaudin &Bowers' (1997) PURIA model.

Key Words: Technology, mathematics curriculum, algebra, geometry, high school

1. Introduction

Man has been using technology in mathematics for thousands of years, starting with own fingers and stones for counters. He then progressed to using the stones in an Abacus, which is still used for complex arithmetic computations by some in Japan (and perhaps in other countries). The Slide rule was invented in the 17th century, and is credited as the tool of computation used for the Apollo moon missions in the 1950's and 60's (Oughtred Society, 2011). Various mathematicians, using the slide rule and other tools) then laboriously did millions of calculations to formulate logarithmic and trigonometric tables for all to use. However, these fell into disuse in the mid-1970's, when the first hand held scientific calculators were used by students. Nowadays, graphing calculators are common in many mathematics classrooms in western countries. Most graphing calculators also include a cable for data transfer from probes to calculators or from calculators to computers.

Since the calculator and computer have become household items in the last two decades, the number and types of electronic tools for mathematics classrooms have. These tools of technology typically serve to *do* and *learn* mathematics. However, some are primarily used to *teach* mathematics, while others are for *publishing* mathematics content (Usiskin, 2011).

Technological tools used in *doing* and *learning mathematics* abound. The most common examples are dynamic geometry systems (e.g., Jackiw, 2001; Hohenwarter, 2002), computer algebra systems (e.g., Maplesoft, 2005), graphing calculators (e.g., Texas Instruments, 2001), spreadsheets, electronic virtual manipulatives (e.g., National Library of Virtual Manipulatives, 2001), internet applets (e.g., Shodor, 1994), and special micro-worlds (e.g., SimCalc, 2003).

Special tools to facilitate *teaching mathematics* include interactive whiteboards, and tablet computers. Other examples of technology that help in teaching are machine scorable tests, e-mail facilities to message parents and students, and instant student response systems (SRS) (e.g., Turningpoint, 2012) for formative assessment.

These tools typically help to push further our ability or alleviate our human limitations for information processing in *computations* and *visualization*. They also help to *graph* mathematical functions, *simulate* complicated mathematical processes and *manipulate mathematical symbols* accurately and efficiently. In general, we can now do numerical computations much faster, visualize mathematical relationships more easily, even perform symbolic manipulations more accurately - sometimes all at the same time.

Astronomers can see further by using a telescope, but they still have to interpret what they see to understand astronomical phenomena. Similarly, students can compute and visualize mathematical relationships using technology more quickly, and more of their mental resources are freed to ask new questions, interpret mathematical information and solve more difficult mathematical problems.

So exactly what role does technology play in the natural evolution of school mathematics curriculum? What skills are still critical? What skills have become less important? In the remainder of this paper, we will attempt to answer these questions by using insights from pertinent literature. By providing a review of different roles technology play in teaching and learning mathematics in high schools, this article will be of interest to teachers, curriculum planners and researchers in Turkey and abroad.

## 2. An Epistemological Perspective on Mathematical Experience

One important idea in understanding how we relate to mathematics is the construct of *mathematical objects*. Mathematics in a sense is a system of knowledge and it is a collection of abtract ideas that we call mathematical objects. Examples of mathematical objects include the concepts of set, integer, rate, ratio, equation, function, binomial expression, or circle, or square. We access these abstract ideas only by creating and using their external representations, or *signifiers* (Duval, 2000). When we do mathematics or solve problems, we function cognitively within a *system of signifiers*. A system of signifiers include tools of *representation*, *transforming* the same representation from one tool to another of the same kind, and *converting* one signifier to a different kind of signifier. For example, as seen in Figure 1 below, we can 'represent' a mathematical object, binomial expression using symbolic signifier $(a + b)^2$. We can 'transform' this signifier by using

certain rules into $a^2 + 2ab + b^2$.   We can also 'convert' this symbolic register or signifier into its diagrammatic signifier.
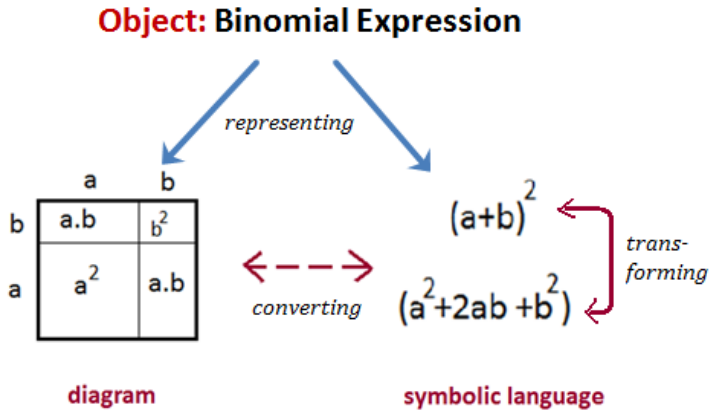

Figure 1. A system of signifiers for binomial expression.

An important part of mathematical problem solving is the ability to move between settings and representations. Mathematical objects and their signifiers, plus the rules of operations on these signifiers make up a *setting* (see Figure 2). For example, equations and allowed rules of operations accessed on the symbols reside in the setting of algebra. In solving problems, a student under the guidance of a teacher starts with a signifier in which the problem is given and the setting in which the problem is situated. When finding the solution with the tools of the initial setting is not possible, the student moves on to another signifier and/or setting (Douady, 1985).
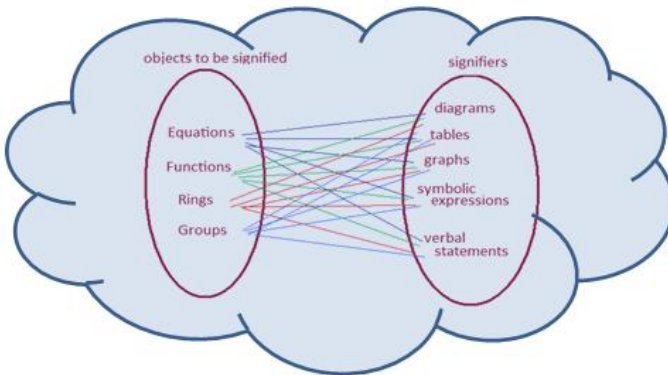

Figure 2. Algebraic setting

The symbolic expressions (or representations) favors analytical reasoning, the pictorial (diagrammatic) representation cognitively supports gestaltist (holistic) reasoning, and verbal representations support sequential reasoning (Hollebrands, Laborde & Sträβer, 2008). Mathematicians use various signifiers not only to communicate a mathematical object, but also to process it in problem solving and while doing mathematics. Multiple signifiers of a mathematical object bridge the difference between a geometric figure and its drawing. A figure is an idealized shape, where as a drawing is an imperfect representation of the figure. Certain types of signifiers with the associated rules of operation on them come together to make broad *settings*, which we call an arithmetical setting, or algebraic setting (see Figure 2 and 3), or geometric setting (see Figure 4). A mathematical object may be primarily situated within a setting, e.g., a circle being in a geometric setting, but it may have signifiers in a different setting as well (e.g., algebraic, $x^2 + y^2 = r$).
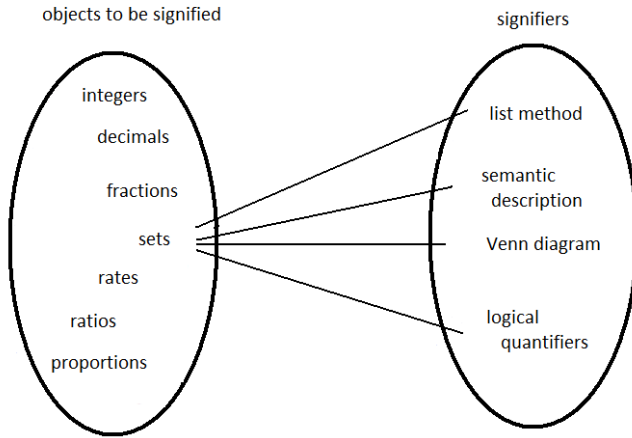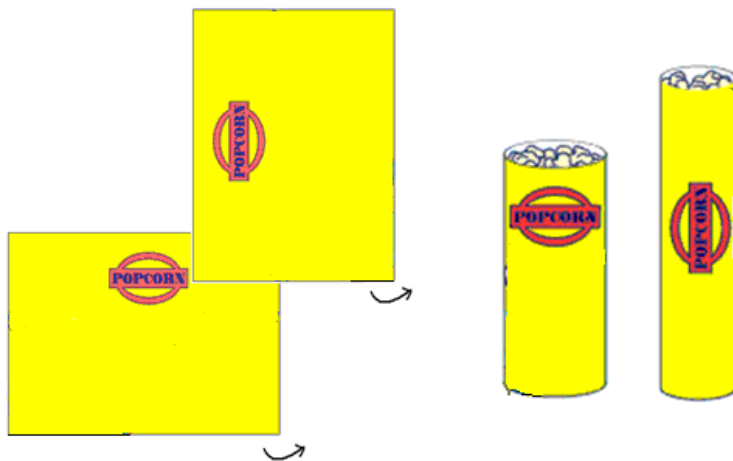

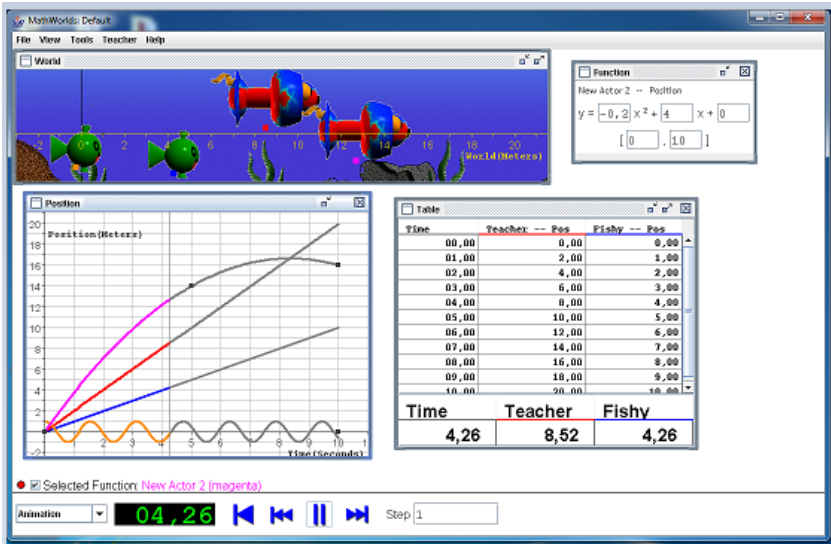**Figure 3**. Arithmetical setting
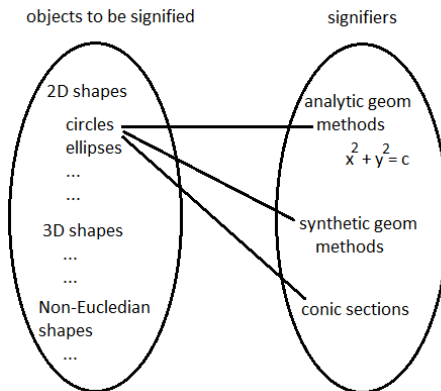
Figure 4. Geometric setting

To illustrate problem solving within and between settings, lets consider the problem (NCTM, 2005) illustrated in Figure 5.

Figure 5. The popcorn problem (from www.figurethis.org)

This problem is about the volume enclosed by two cylinders made by folding two same sized rectangular sheets of papers, one vertically and one horizantally (Figure 5). The question is whether they enclose the same or different amounts of volume. Solving this problem is not easy if we stay in the original setting in which it is given, that is the geometric setting, -because it does not readily provide a useful path for solution. The problem needs to be translated into a setting in which it is easier to look into it analytically. When we do that, we will find out that the volumes of the cylinders can be expressed as, $V_{tall} = h_t. \pi r_t^2$ and $V_{short} = h_s. \pi r_s^2$. The interested reader will see that it is the radii of the base of cylinders that makes a bigger difference in creating a volume, rather than the height of the cylinders. The heights will add to volume linearly, where as the radii will add to it quadratically because they are squared. To solve this problem, we had to change the setting in which the problem was given and move to an algebraic setting. The new setting afforded us to think in a new way which is more condusive to a solution.

A computer environment offers a set of objects and tools in doing mathematics (such as those in the problem above). Interplay betwen settings can be exemplified in a microworld such as SimCalc (SimCalc, 2003) as shown in Figure 6.

Figure 6. Multiple mathematical settings in one computer platform in SimCalc.

In microworlds such as SimCalc, a given mathematical function can be represented by using multiple signifiers; algebraic (symbolic) representation, arithmetical (tabular) representation, graphical representation, and pictorial representation. What is more, the effects of changing one parameter in any of these representations can be seen immediately and simultaneously in the other representations. Running animations of the "story" with moving "fish" can be traced through these four different representations simultaneously. Observing and reflecting about the interplay between these multiple settings can afford students to see the conceptual links among different conceptual facets and components of mathematical functions. Seeing these links is much more difficult in a paper and pencil platform (Hollebrands, Laborde & Sträβer, 2008).

3. The Shift in Algebra and Technology

Algebra is one of the biggest strands of school mathematics at high school level (MEB, 2005; NCTM, 2000). Historically, computational routines and symbol manipulation have dominated algebra instruction. For example, the picture in Figure 7 shows a page from an entrance examination to a high School in 1885 (Kelly, 2003). Some items required recall of algebraic nomenclature in this test, while most were about manipulating symbols using prescribed algebraic rules.
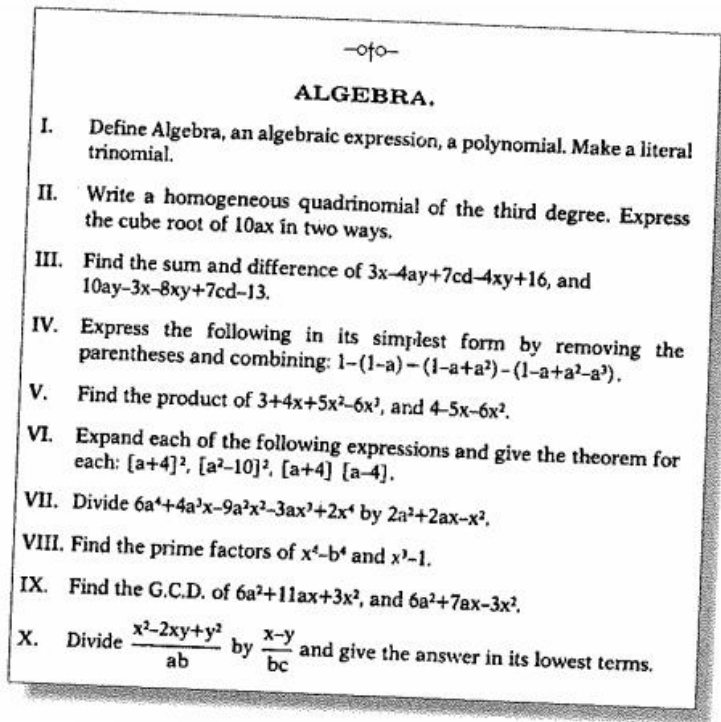
Figure 7. An algebra examination at a high school in the US in 1885 (Kelly, 2003, p. 1040).

At the beginning of the 20th century, the function concept entered school curricula, and textbooks and tests gradually adopted the change (Heid & Blume, 2008). Especially in the last 3 decades, function has gained a central place in mathematics curriculum as an organizing construct, as many types of quantitative relationships in real life could be modeled by functions in the form of, for example; linear, quadratic, exponantial, logarithmic, and trigonometric functions. This shift was facilitated in part by the increasing relevance of technology to mathematics to compute for these type sof relationships: The first mainframe computer was available in 1942, the first four-function calculator in 1967, the first microcomputer in 1978, and the first graphing calculator in 1985 (Kelly, 2003).

Technology can contribute to learning algebra in a number of different ways. Spreadsheets for example can help *conceptualize* the construct of variable by demonstrating assignment of a series of values to something that can vary. Computer algebra systems (CAS) can faciliate students in seeing a function as an 'object,' in addition to the more common view of it being a computational rule or a process.

Technology can also help students consider simultaneously *multiple representations* of algebraic objects such as function. Microworlds (e.g., SimCalc, 2003) can link symbolic,

graphical and tabular representation of functions to enable visualization of important concepts such as rate of change, local maxima, minima, and optimal values, and monotonocity of functions. These concepts are often difficult to 'see' in a purely symbolic setting.

Generalization is another goal that technology can help materialize in instruction. Spreadsheets can carry out large number of numerical computations and generate numerical values both by a using rule and on a random basis. This capability can, for example afford simulations of the Law of Large Numbers and the Central Limit Theorem (Heid & Blume, 2003).

Functions coming to the forefront in algebra in the last centry has brought the possibilities with them of different types of functions *modeling* real life situations or scientific phenomena. Technological tools such as videos, microcomputers, calculator-based laboratory devices, spreadsheets and microworlds can now serve as tools to talk about many quantitative relationships. When for example, a teacher is introducing trigonometric functions, it is often wise to use a physical phenomena. A trigonometric function can help students understand important parameters of such functions. The problem shown in Figure 8 is one such task. Solving this problem requires coming up with a function, in which input values would be months and output values would be light intensity in *btu/ft^2* (British thermal unit / foot squared). However, it is difficult to foresee the shape of this function before entering them into a spreadsheet and graphing the values to see how they change each month. Figure 9 shows this changing relationship.
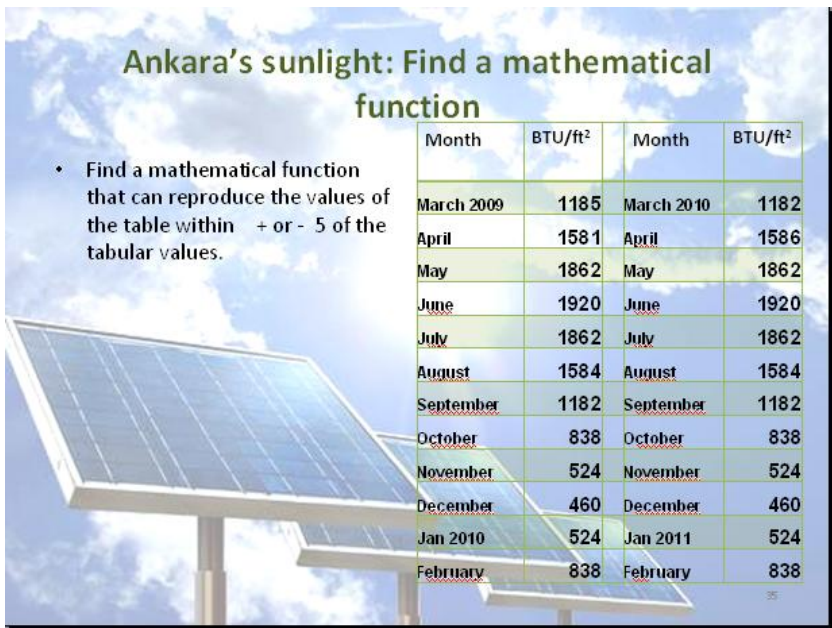
Figure 8. Sunlight received in Ankara: A problem of periodicity
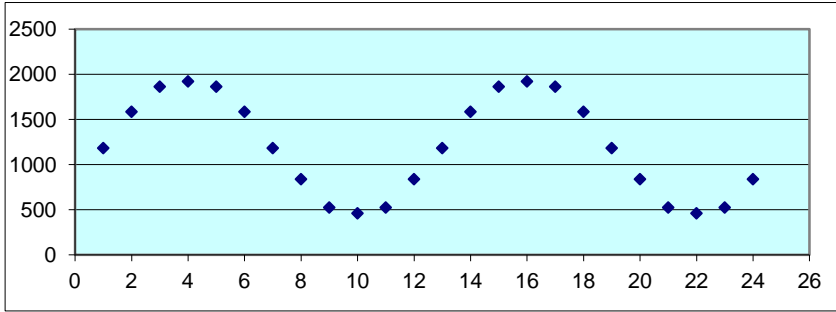
Figure 9. A scatter plot of the sunlight data from Ankara

The shape of the graph in Figure 9 would immediately remind a student of a *sine* function for reference. By considering the component parameters of the sine function, one can then deduce the modeling function for this data set as shown in Table 1.

Table 1. Summary of deduction of the parameters of the modeling function

Reference Function	Modeling Function
y = a. sin(bx) + c	*y = 730.sin[(π/6)x] + 1185*
a: amplitude [-1, 1]	**a**: (max-min)/2 $\rightarrow$ (1420 – 460)/2
b: 360^0 or 2π	**b**: $2\pi/2 = \pi/6$
c: starting value (0)	**c**: 1185

Taking advantage of the capabilities of technology, modeling real life or scientific phenomena like the one discussed above can be used to teach elementary algebraic functions in high school mathematics. These modeling experiences will help students understand the structure of functions. In addition, there are motivational aspects as students experience the application and use of mathematics in real life.

4. Proof in Geometry with Technology

Historically, since the beginning of 20th century in the West, proofsare an important instructional goal in geometry as a way of establishing reasons for patterns and truths (Herbst, 2002). Proofs are–still an important goal in school mathematics emphasizing students' need to rely on their own logic, rather than on an external authority, to determine the soundness of a geometric argument (NCTM, 2000). The recent emphasis on letting students invest personally in the conjecture to be proved or discover a pattern on their own, and to plan and complete a proof, all point to the need for providing motivation for the process. When students do not observe a pattern or develop a conjecture on their own, it is difficult to motivate them to prove somebody else's argument. The process thus generally becomes an exercise in rote learning for many.

Dynamic geometry systems (DGS) has brought much ease and power to represent geometric objects on the computer screen. DGS can also automatically measure features of figures such as length and area, while the user is dynamically changing its size and position

on the screen (Hollebrands, Laborde and Sträβer, 2008). This can let students to experiment and come up with conjectures of their own. They can then attempt to prove these conjectures on their own.

One example of this is related to a student's investigation on the well-known *Pythogeras' theorem* (Martinez-Cruz, McAlister, and Gannon, 2004). A student was captivated with a geometric pattern he observed while working on *Pythogeras'* triple in *Geometers' Sketchpad* (GSP) (Jackiw, 2001). The student combined the corners of adjacent squares to make three new triangles and measured and compared their areas. These triangles are shown yellow in Figure 10b.
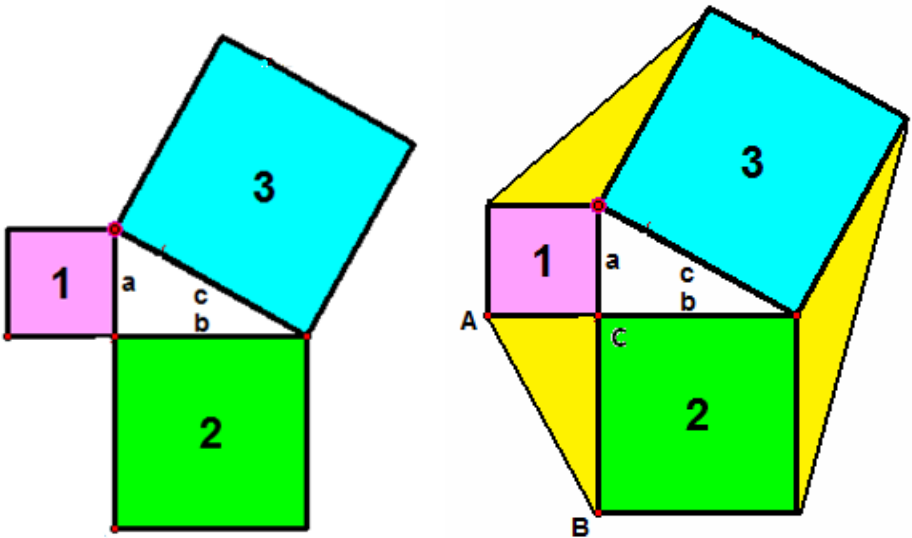

a: Pythogeras' triple   b: Pythogeras' triple with corner triagles

Figure 10. Comparing areas of the three corner triangles in Pythogera's theorem

The student could discover that the three new (yellow) triangles in Figure 10b have the same area as the inner triangle, using calculations or GSP. However, using GSP, they would discover that when the shape and size of the right triangle in the middle is changed by dragging, the areas of the other three triangles change equally to the area of the right triangle in the middle. The student could then be encouraged to come up with a deductive argument and 'prove' this observation.

It is easy to see that Δ (ABC), is congruent to the triangle in the middle due to the SAS rule. (We will leave it to the reader to explain why the remaining two triangles have the same area.) After explaining his observation through proof, the student could be encouraged to generalize it to <u>any</u> triangle in the middle (Jackiw, 2001). That is, would the same realationship hold for a triangle in the middle when it is not a right angled triangle?

After a quick contruction as in Figure 11, the student can empirically observes that the relationship holds for any triangle Δ (HIG) in the middle.
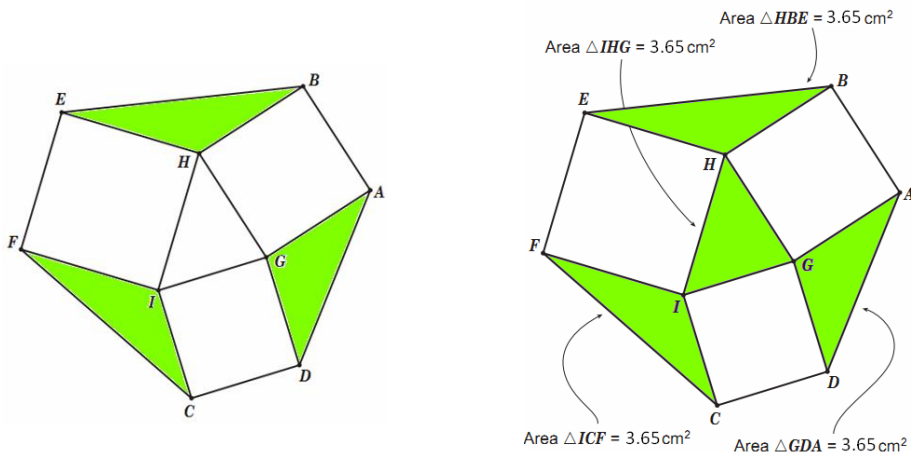
Figure 11. Generalizing the Pythogeras' triple conjecture to any triangle

He can test his conjecture once again by dynamically changing the shape of the triangle Δ (HIG), using GSP and observes that the areas of the four triangles in question are all the same regardles of the shape of Δ (HIG). He could then attempt to explain synthetically why this is the case.

It is important to note here that empirical observations that come from the mesurement functions of GSP served as a step to develop a deductive argument. Developing a deductive argument is the valued essence of proof in mathematics. Investigations afforded by the dynamic geometry system here provided a way to personally invest into a conjecture, which then served as a meaningful context for a proof. This is obviously a more meaningful activity for a student than proving somebody else's theorem that is always true. However, Hollebrands et al. (2008) point out that "a positive evolution in proofs elaborated by students" result from the combination of software use, skillfully designed teaching/learning situations and tasks, the social organization of the classroom and the role of the teacher.

5. Development of Teachers' Proficiency in Using Technology

One factor that affects the realization of technology's potential in the classroom is the teachers' knowledge about and attitude toward technology. To implement a software package in teaching effectively, the teacher needs to become familiar with all aspects of the program. According to Beaudin and Bowers (1997) the teacher needs to proceed through a number of steps in a set order, to become a confident user and teacher with the software. They suggested the PURIA model of technology use, as stages through which a teacher must progress before they can teach mathematics with a given software package. Although

Beaudin and Bowers (1997) designed their "modes of use" for CAS, Zbiek and Hollebrands (2008) expanded it to include other software programs.

The PURIA model incorporates the steps of Play, Use, Recommend, Incorporate and Assess. The user needs to spend as much time as needed at each stage, with assistance where necessary, before moving onto the next stage. A brief description of each stage is given in Table 2 below.

Table 2. The PURIA model of development of teachers' proficiency with technology (Beaudin and Bowers, 1997 and extended by Zbiek and Hollebrands, 2008)

PURIA mode	Activity in each mode	Nature of activity
Play	User plays with the technology	No clear mathematical purpose in playing with the software.
Uses	Technology is used as a personal tool	Does mathematics of own design. May use it as a learner of mathematics but not in a formal classroom setting or with students
Recommends	Suggests to others that they investigate/use the technology	Recommends the use to a peer, individual or small group of students. Still not in a formal classroom setting or integrated part of teaching.
Incorporates	Uses the technology as part of classroom teaching	Starts using the technology in a formal teaching environment. Incorporates technology into lessons to varying degrees. (Positive experiences in Play and Recommends required)
Assesses	Uses the technology to assess students	Uses the technology to assess what the students are learning in terms of technology and mathematics.

In this model, the teacher progresses in the use of the technology package, until he or she is confident enough to go on to the next level. Levels can overlap and are negotiated at different speeds, depending on the capability of the user. However, most time is spent in the *Play* to *Recommend* modes. If a mode is skipped, the teacher may not develop far enough to allow students the freedom to explore for themselves, as he/she will be fearful of not being able to assist students or solve problems at the students' level.

During the modes of *Use, Recommend, Incorporate* and *Assess*, support in the form of a technical assistant, demonstrations, workshops and manuals guide the teacher and iron out any problems encountered. If technical assistance is available, teachers persevere longer and thus progress further. Without support, they may not even reach the incorporate mode.

The role of the teacher with technology and the questioning style of the teacher are influenced by the teachers' confidence of technology use (Zbiek and Hollebrands, 2008). As teachers gain confidence in the use of software, they change their questioning styles to incorporate higher order questions and allow students to play, explore, experiment, reason in mathematically valuable ways and thus enhance their learning experiences.

With much evidence that the teacher's attitude to technology influences students' attitude, it is important that the teacher himself or herself has a positive learning experience with technology. Thus a positive experience with technology, and especially in the first two PURIA modes are important if the teacher is to become a facilitator of "construction of deep learning" (Zbiek and Hollebrands, 2008).

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