



# Implementation of Advanced PID Control Algorithm for SDOF System

<sup>1</sup> Abdullah Turan , <sup>2</sup> Huseyin Aggumus 

<sup>1,2</sup>Department of Mechanical and Metal Technologies, Sirtak University, Sirtak, 73000 TURKEY

Corresponding author: First A. Author (e-mail: abdullahturan@sirtak.edu.tr).

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**ABSTRACT** The maximum performance to be obtained by applying the Proportion-Integral-Derivative (PID) controller on a system depends on the optimum adjustment of its parameters. This study aims to present a design method for tuning the PID control parameters. In this method, PID controller design is made based on the optimal proportional gain from the system to the desired settling time and overshoot. The infrastructure of the technique is based on obtaining the other PID controller parameters by adjusting the optimum proportional gain ( $k_p$ ), which minimizes the settling time in a stable loop and the error rate of the overshoot. Routh Hurwitz criterion is used to guarantee stability in the control system. The effectiveness of the proposed method is tested as an active control application of the PID controller on a single degree of freedom (SDOF) structural system. The efficiency of the PID controller designed with this method, which does not require the destruction of parameters and does not contains complex mathematical formulations, is proven by its successful suppression of SDOF structural system responses.

**KEYWORDS:** Active control; PID controller; SDOF system; structural control.

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## 1. INTRODUCTION

Control of structural systems is a current and vital issue. Damage to any part of a structure under the influence of external disturbances, such as crack formation in its beams [1,2], may damage the entire system. For this reason, control applications are one of the most effective methods to protect the structures from damage and the safety of the occupants. Although there are many control applications in the process control industry, PID control is widely used and maintains its importance due to its low cost, simple structure, and wide application area [3]. Therefore, improvements in the design methods of the PID controller will increase the performance of this controller. In general, the PID controller, which is obtained with an efficient design method, should respond optimally to the design features and be robust against uncertainties. For the PID controller's performance on the applied system to be at the maximum level, its parameters must be set correctly. The Ziegler and Nichols method is the oldest method used to set PID control parameters [4]. However, in the performance of the PID control obtained with this method, negative effects such as high settling time and overshoot arise. Therefore, the obtained answers need to be improved. In another study [5], in which a different method was proposed for the proportional gain value, it was determined that negative responses were obtained for time delay systems. Other standard methods in the literature include; gain and phase margin method [6,7], Astrom and Haggland method [8], optimization method [9,10], direct synthesis method [11], the weighted geometric center method [12,13]. In addition, many studies suggest new control methods [14-18]. For example, PID controller design using stability limit position matching [19], near-optimal PID controller design

with interval arithmetic approach [20], PID controller design with numerical optimization approach [21] are suggested methods.

In this study, a new approach is discussed to determine the optimum PID parameters. In the proposed method, first of all, the desired settling time and overshoot values are defined in the closed-loop system response, then the proportional gain  $k_p$ , which will minimize the error rates of the expected settling time and overshoot values from the system, is determined with a loop formed in the stable area. Based on the obtained  $k_p$  value, the other parameters of the PID controller,  $k_i$  and  $k_d$ , are calculated. This proposed simple adjustment method has some advantages over other methods in the literature. The solution processes are simple as there is no need for complex mathematical equations, and there is a high chance of getting maximum efficiency as optimization occurs. The efficiency of the PID controller obtained with the proposed method in this study is tested on an SDOF structural system.

## 2. EQUATION OF MOTION

Control of structural systems is an area that remains popular today. As a result, both single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) system models have been frequently used in studies. In this study, the SDOF structural system used to test the designed control performance is shown in Figure 1.

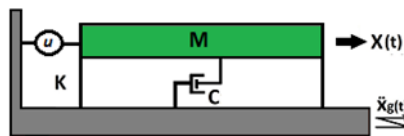


Figure 1. SDOF system

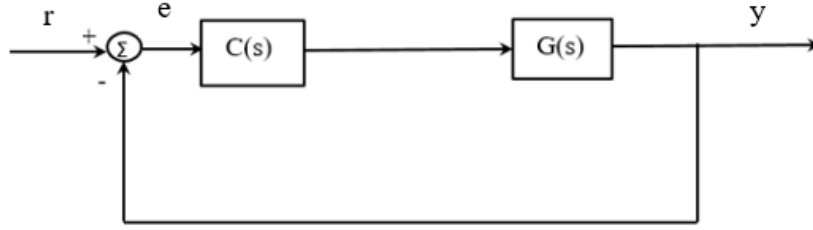
The control force acts on the system from outside. The equation of motion of the system is seen in Eq. (1).

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -f_u(t) - M\ddot{x}_g \quad (1)$$

Here,  $M$ ,  $C$ , and  $K$  denote the system's mass, damping, and stiffness values, respectively.  $f_u$  is the control force and  $\ddot{x}_g$  is the acceleration excitation of the system. Parameter of the system are  $M= 107.5$  kg,  $K= 145152$  N/m,  $C= 6.7646$  N.s/m.

## 3. DETERMINATION of PID PARAMETERS and CONTROLLER DESIGN

Both semi-active [22, 23] and active control [24] applications are widely studied in structural systems. Active control is an indispensable choice in obtaining the best performance from these systems. In this study, the active control of the SDOF structural system is investigated by obtaining the optimum parameters of the PID controller with a new approach. The PID control method remains essential because it has been widely used and can be easily applied to systems [25]. In the most general case, the block diagram of the unit feedback control system is shown in Figure 2.



**Figure 2.** Feedback control system

Here,  $C(s)$  given in Eq. (2) and  $G(s)$  shown in Eq. (3) represent the transfer function of the PID controller and SDOF system, respectively. Here  $r$ ,  $y$  and  $e$  denote the input, output of the system and error rate, respectively.

$$C(s) = k_p + \frac{k_i}{s} + k_d s \quad (2)$$

$$G_s = \frac{G_N(s)}{G_D(s)} \quad (3)$$

By arranging Eq. (2), the controller equation, in general, is obtained as follows.

$$C(s) = \frac{(k_d s^2 + k_p s + k_i)}{s} \quad (4)$$

The closed-loop system  $T(s)$  is obtained as in Eq. (5) using Eq. (4).

$$T(s) = \frac{C(s)G(s)}{1+C(s)G(s)} \quad (5)$$

$T(s)$  is obtained by substituting Eq. (3) and Eq. (4) in Eq. (5) as follows.

$$T(s) = \frac{G_N(s)(k_d s^2 + k_p s + k_i)}{G_D(s)s + G_N(s)(k_d s^2 + k_p s + k_i)} = \frac{T_N(s)}{T_D(s)} \quad (6)$$

$T_D(s)$  is the characteristic equation of the system, and its degree is determined. Then, settling time  $t_s$  and overshoot  $M_p$  value are determined according to system performance. The damping rate of the system is calculated by Eq. (7), and its natural frequency is calculated by Eq. (8).

$$\zeta = \frac{-\log(M_p)}{\sqrt{\pi^2 + \log(M_p)^2}} \quad (7)$$

$$w_n = \frac{4}{\zeta t_s} \quad (8)$$

The target polynomial equation of the closed-loop system is as follows.

$$\Delta(s) = s^2 + 2\zeta w_n s + w_n^2 \quad (9)$$

Here, a residue polynomial ( $R(s)$ ) must be defined since  $\Delta(s)$  is of order 2. Also, the variable number of  $R(s)$  should be equal to the degree difference ( $m$ ) between  $T_D(s)$  and  $\Delta(s)$ .

$$R(s) = \begin{cases} s + a, & m = 1 \\ s^2 + a_1s + a_2, & m = 2 \\ s^3 + a_1s^2 + a_2s + a_3, & m = 3 \\ s^n + a_1s^{n-1} + a_2s^{n-2} + \dots + a_n s^{n-m}, & m = n \end{cases} \quad (10)$$

The fact that the variables expressed in Eq. (10) are  $a_1, a_2, a_3, \dots, a_n \in \mathbb{R}$  allows it to cover all solutions and keep the system free of complexity. The condition to be satisfied here is that the coefficients of the product of  $\Delta(s)$  and  $R(s)$  are equal to the system's characteristic equation coefficients, as seen in Eq. (11).

$$(\Delta(s)R(s))_{\text{coeff}} \equiv T_D(s)_{\text{coeff}} \quad (11)$$

In Eq. (11), it is seen that the number of variables is one more than the number of significant equations. Therefore, to equalize the number of equations, the number of variables is reduced by one by subtracting  $k_p$  from the variables ( $k_p, k_i, k_d, a_1, a_2, \dots, a_n$ ). Then, by solving Eq. (11) again, it is ensured that the variables contain terms with  $k_p$ . As a result, the existence of a multiple-choice solution emerges. Initially,  $\Delta(s)$  is chosen as stable. Also, the  $R(s)$  polynomial must also check for stability. For stability, it is sufficient that the variables ( $a_1, a_2, \dots, a_n$ ) in  $R(s)$  are positive. As a result, the flowchart in Figure 2 is followed to set the PID parameters optimally.

After the variables are determined as positive, values are assigned to  $k_p$  in a specific interval and incremental cycle, and the variables ( $k_p, k_i, k_d, a_1, a_2, \dots, a_n$ ) and the  $t_s$  and  $M_p$  values of the system are determined. The aim here is to obtain a value as close as possible to the expected (desired) value of  $t_s$  and  $M_p$ . Therefore, the error rates of  $t_s$  and  $M_p$ , which need to be normalized, are assigned the variables  $e_1$  and  $e_2$ , respectively.

$$e_1 = \frac{M_p - M_{p_{\text{ans}}}}{M_p} \quad (12)$$

$$e_2 = \frac{t_s - t_{s_{\text{ans}}}}{t_s} \quad (13)$$

By combining Eq. (12) and Eq. (13), Eq. (14) is obtained to show a single error.

$$\text{err} = xe_1 + ye_2 \quad (14)$$

Here,  $x, y$  are the coefficients affecting the total error, and  $x$  and  $y$  values are selected according to the importance expected from the system and  $x+y=1$ . The  $\text{err}$  value obtained is also added to the loop and determined according to  $k_p$ . Finally, PID controller parameters  $k_p, k_i$  and  $k_d$  are accepted according to the  $\text{err}_{\text{min}}$  value specified due to the loop.

### 3.1 PID CONTROLLER DESIGN

To determine the effectiveness of the method proposed in Section 3, a comparison is made with the PID controller tuning using the Matlab-Simulink Toolbox. In simulations, settling time, overshoot, peak time, and rise time are considered evaluation criteria. The transfer function of the model of the system in Figure 1 is as follows.

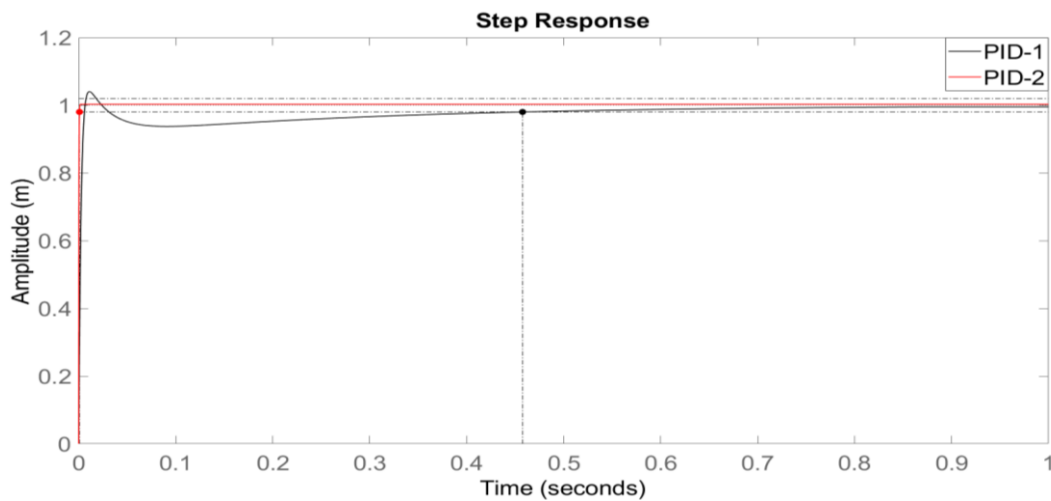
$$G(s) = \frac{0.0093023}{s^2 + 0.062927s + 1350.2512} \quad (15)$$

First of all, 0.01 % overshoot and 0.2 s settling time are expected performance values from the system. The residual polynomial is treated as in Eq. (17) ( $m=1$ ).

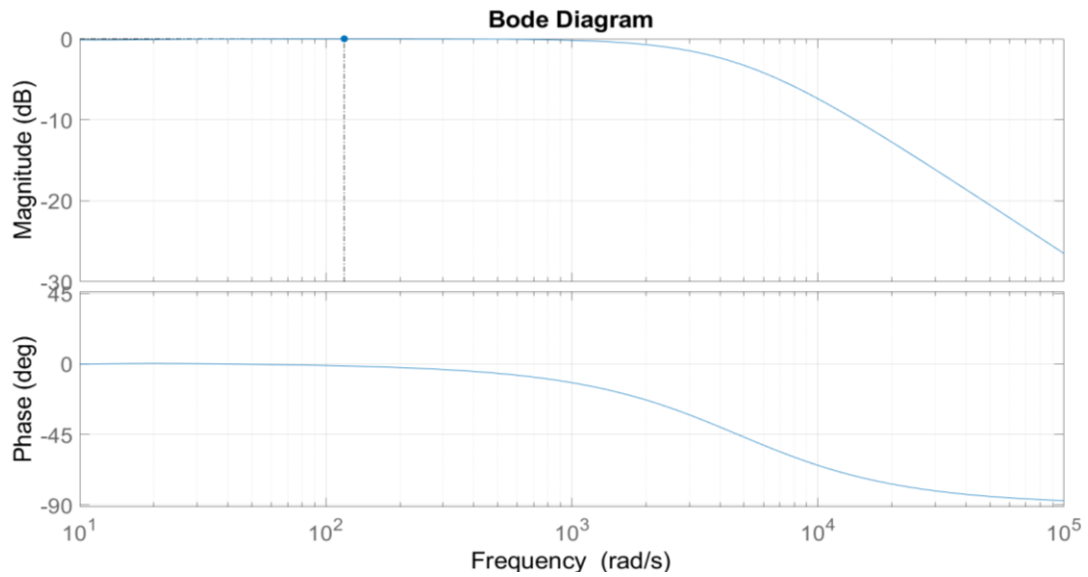
$$R(s) = s + a \quad (16)$$

Therefore, the expression  $a = 0.0004651 * k_p + 60.19$  is obtained with the proposed method, and it is sufficient for the variable  $a$  to be positive for the stability criterion. Therefore, for  $k_p$ , the interval value  $k_p = [-100000, 10000000]$  is selected in increments of 0.1. A loop is created by selecting the total error value as in Equation 14.

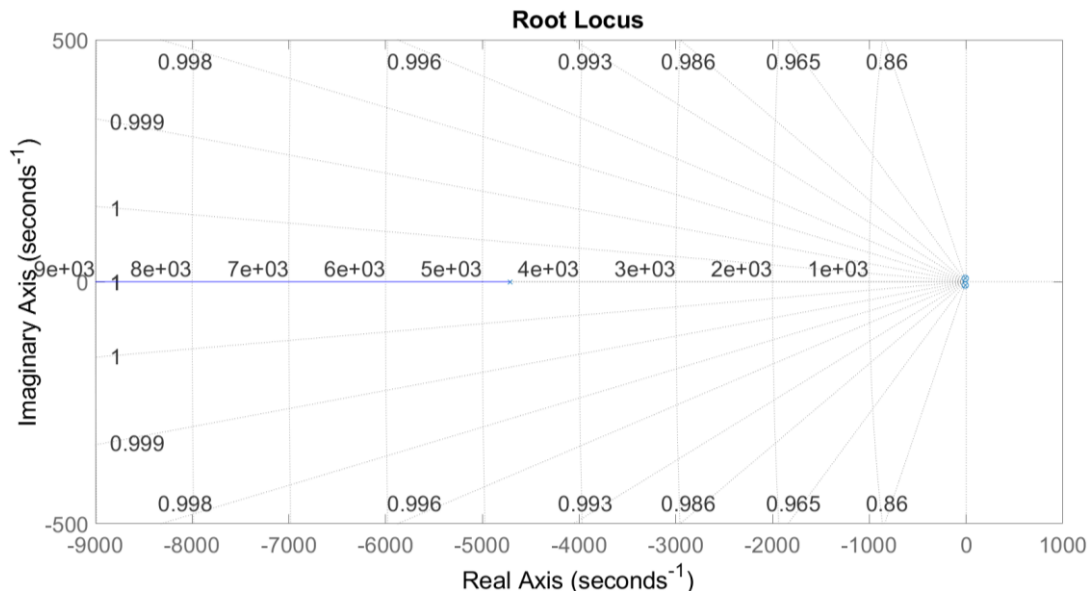
The parameters of the PID controller, from which the  $err_{min}$  value is obtained in the performed cycle, are given in Table 1. By applying the PID-1 control created in Matlab-Simulink Toolbox and the PID-2 control obtained by the proposed method to the system, unit step responses in the closed-loop are given in Figure 3. In addition, the bode and root locus graphs of the system with the PID-2 controller are shown in Figure 4 and Figure 5, respectively. In the characteristic equation for the whole closed-loop, the roots are in the left half of the 's' plane. Therefore, the system is stable. In the unit step response with the designed PID-2 controller, a better (%) overshoot value and peak time are obtained as well as much better settling time. Performance criteria of PID-1 and PID-2 controllers are given in Table 1. It can be clearly said that the proposed PID-2 controller performs better than PID-1.



**Figure 3.** Step responses for the system.



**Figure 4.** Bode response for the system



**Figure 5.** Root locus response for the system

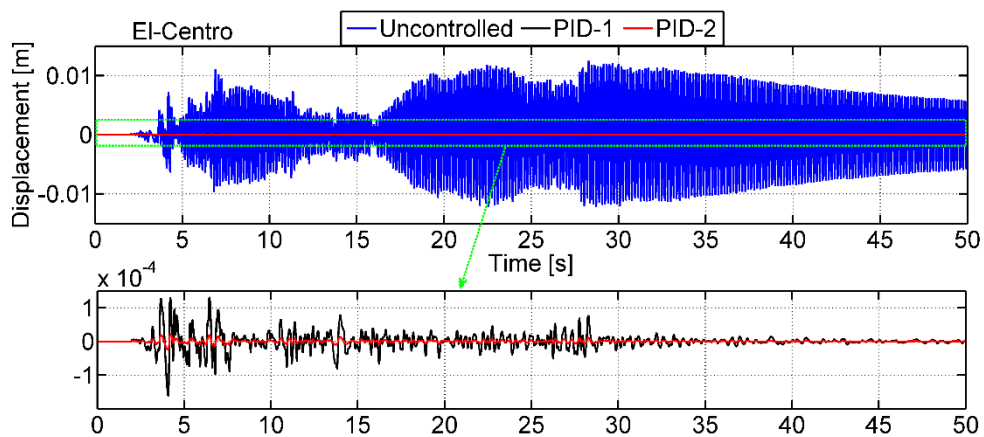
**Table 1.** Controllers parameters and values of the performance criteria for the system

PID controller gain				Values of the performance criteria			
	$k_p$	$k_i$	$k_d$	Settling time (s)	Overshoot (%)	Rise time (s)	Peak time (s)
PID-1	1750824.3	5818890.3	47747.115	0.4577	4.0177	0.0041	0.0110
PID-2	9999900	7.4216e+07	5.0861e+05	7.9226e-04	0.2762	4.5876e-04	0.0015

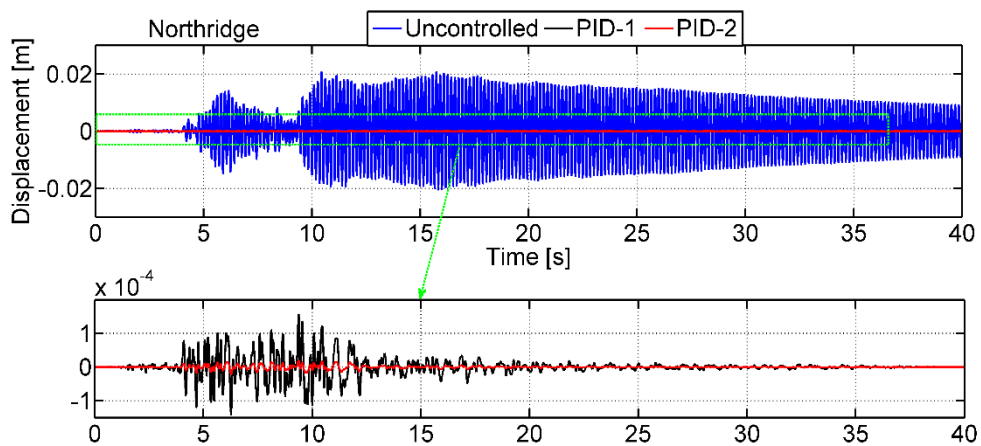
#### 4. SIMULATION STUDIES

The system performance in El-Centro and Northridge earthquake excitations has been investigated by applying a PID controller to the model in Figure 1. The efficiency of the determined PID parameters

has been compared with the parameters obtained using the Matlab-Simulink Toolbox. Among the applied control states, PID-1 represents the controllers, which are the parameters determined by the Matlab-Simulink toolbox, and PID-2, the parameters obtained with the proposed method. In addition, the displacement and acceleration responses of the SDOF system have been examined as evaluation criteria. Figure 6 and Figure 7 show the displacement responses of the system, and Figure 8 and Figure 9 show the acceleration responses of the system. In the displacement responses of the El-Centro and Northridge earthquake excitations in Figure 6 and Figure 7, respectively, both controllers successfully suppressed the system responses. But the best performance has been obtained in the PID-2 control case.

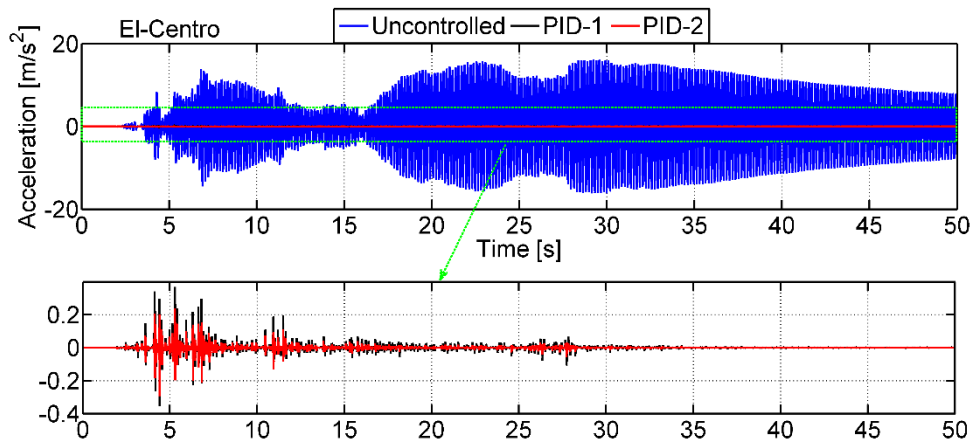


**Figure 6.** Displacement responses of the system under the El-Centro Earthquake

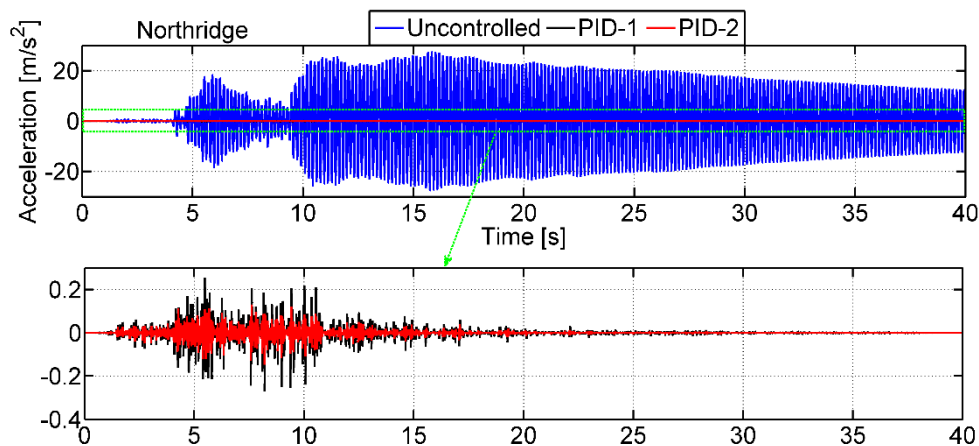


**Figure 7.** Displacement responses of the system under the Northridge Earthquake

In Figure 8 and Figure 9, acceleration responses in El-Centro and Northridge earthquake excitations, similar to displacements, both controllers successfully suppressed the system responses, and the best performance has been obtained in the PID-2 control condition.



**Figure 8.** Acceleration responses of the system under the El-Centro Earthquake



**Figure 9.** Acceleration responses of the system under the Northridge Earthquake

## 5. CONCLUSION

In this study, a design method is presented for optimum tuning of PID controller parameters. In the proposed method, other PID controller parameters are determined in response to the optimum proportional gain ( $k_p$ ) setting in a stable loop so that the error rate of  $t_s$  and  $M_p$  is minimized. The effectiveness of this method, which has the advantages of not destroying parameters and not involving the complex mathematical formulation, is tested in an SDOF structural system under the influence of El-Centro and Northridge excitations. In addition, the performance of the PID controller (PID-2) obtained by the proposed method is compared with the controller (PID-1) obtained with the Matlab-Simulink PID toolbox.

The results reveal that PID-2 is not only effective at suppressing system responses but also outperforms PID-1.



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