Schwarzian Derivative of Third Chaotic Transition in Mercury Based Superconductors

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Abstract: The superconductivity displays some nonlinear quantum properties such as quantum chaotic transitions, effective mass of quasi-particles. The calculation of the effective mass equation of the mercury based superconductor has a crucial step in order to determine chaotic behavior of the sample. The sample has three quantum chaotic points; the critical transition (T_c), paramagnetic Meissner transition (T_{PME}) and the quantum gravitational transition temperatures (T_{QG}). T_c and T_{PME} were already investigated by means of Schwarzian derivative and it was found that the Schwarzian derivatives are both negative at two chaotic points. In the study, the third quantum chaotic point of the system, T_{QG} is investigated by the Schwarzian derivative of the effective mass of the sample. As is known, the object's mass varies in a gravitational field. Hence, it is determined that the Schwarzian derivative of the effective mass, which has the negative value, shows the quantum chaotic transition in the system. At the vicinity of T_{QG} where the first derivative of the net effective mass has a maximum value, the plasma frequency shifts from microwave to infrared. As a result, it is proposed that the Schwarzian derivative is a convenient mathematical method for precise prediction of chaotic points and transitions in superconducting systems.

Keywords: Mercury based superconductor, Nonlinear behavior of the superconducting system, Schwarzian Derivative, Quantum chaotic transition.

1. INTRODUCTION

The superconductivity possesses some nonlinear quantum properties such as magnetic flux quantization, Josephson Effect, solitonic behavior of the system, electron-phonon interaction (occurrence of Cooper pairs), quantum chaotic transitions, effective mass of quasi-particles etc [1-6]. The nonlinear quantum properties of the mercury based layered superconductors were investigated by Z.Güven Özdemir from our research group via Schwarzian derivative [1] in the context of two chaotic transitions; the critical transition temperature, T_c and paramagnetic Meissner transition temperature, T_{PME} which are determined from magnetic moment versus temperature data [2,7,8]. Moreover, it was proved by Z.Güven Özdemir for the first time that the Schwarzian derivative method, which was mathematically used in order to determine the chaotic points of the Hg-based cuprates, is a suitable mathematical model in order to predict the chaotic transition points in non-linear superconducting systems, precisely [1].

In this study, the third chaotic transition, which cannot be observed on the magnetic moment versus temperature graphic, was investigated. The net effective mass equation of the quasi-particles (Cooper pairs or electron pairs) of the mercury based superconducting sample calculated by Ongüas Equation has a crucial step in order to determine third chaotic transition point of the sample. At the third chaotic transition called T_{QG}, a special quantum gravitational transition appears where the plasma frequency, f_p of the Hg-based cuprate shifts from microwave to infrared region at superconducting state [9-11]. As is known, when an object enters a gravitational field, its mass decreases [12]. From this point of view, at the vicinity of T_{QG}, the first derivation of the net effective mass of the electron pairs, which has a maximum value in negative region, is important for the superconducting system where some symmetry breakings accompany the quantum chaotic transitions [11]. Hence, the study is devoted to the mathematical treatment of the third quantum chaotic transition by means of negative values of the Schwarzian derivative in order to show chaotic behavior of the mercury based nonlinear superconducting system.

2. $HgBa_2Ca_2Cu_3O_{8+x}$ SUPERCONDUCTOR and EFFECTIVE MASS of QUASI-PARTICLES

Mercury cuprates exhibit the highest critical parameters such as the critical transition temperature, high critical magnetic fields and critical current density values among the other high temperature superconducting materials. Due to these special features of the bulk superconducting $HgBa_2Ca_2Cu_3O_{8+x}$ (Hg-1223) samples, the preparation methods such as oxygen annealing procedures and the determination of magnetic and electrodynamics properties the superconducting sample such as the plasma frequency, anisotropy factor and Josephson penetration depth have importance for not only theoretical investigations, but also technological applications. In order to obtain electrodynamics parameters mentioned, the grain size of the superconductor, *t* and the average spacing of copper oxide bilayers, *d*, which are in nano or micrometer scale, are required to be measured. The average spacing of CuO_2 layers, *d* is shown in the primitive cell of the Hg-cuprates shown in Figure 1a [13-17].

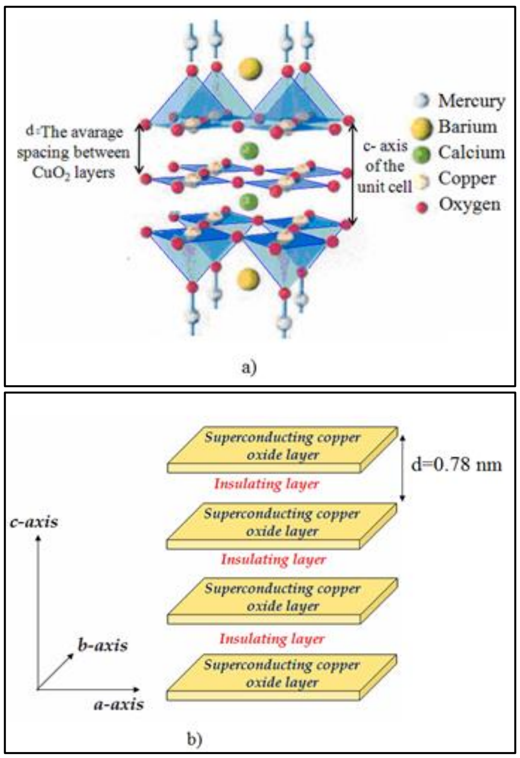


Figure 1. (a)The primitive cell of Hg-based cuprate. (b)The schematic presentation of the intrinsic Josephson structure (IJJ).

The primitive cell of the Hg-based CuO_2 layered superconducting sample contains three superconducting copper oxide planes that are separated by insulating layers of the distance of 0.78 nm and the primitive cell of the sample is considered as an intrinsic Josephson junction (IJJ) array (Figure 1b) [13-17]. In other words, the primitive cell of the superconducting sample, which has the lattice parameters within *a*=*b*=3.8684 Å and *c*=15.7152 Å, displays superconducting properties in nano structure. Moreover, the intrinsic Josephson junction including superconducting-insulating-superconducting layers is considered as nano-capacitor. Furthermore, it was shown that the micro-whiskers in the mercury based samples was spontaneously grown for the over oxygen annealed sample [15]. Hence, the sample investigated has a promising potential for nano-electronic devices, the high frequency applications and new electronic inventions. As a result, the superconducting sample investigated in the study, which displays nonlinear quantum properties in nano and micro dimensions, has some advanced technological application properties.

The high temperature superconducting structure is totally characterized by the same superconducting order parameter ψ (i.e.wave function). As is known, the only variable of the order parameter is the phase difference, φ. The calculation procedure of the net effective mass, m^* of electron pairs of the Hg-1223 sample is established by invoking an advanced analogy between the supercurrent density J_s and the third derivative of the phase of the quantum wave function of the superconducting relativistic system. The derivation method of the net effective mass equation is given in details in references [9-11].

The net effective mass equation of the electron pairs of Hg-cuprates called as Ongüas Equation, is given in Equation (1) [9,11,18],

$$\frac{1}{\varphi_0 m^*} = \frac{dJ_s}{dx} = \frac{c\phi_o}{8\pi^2 d}\left[-\frac{1}{\lambda_j}\right]^3 exp\left[-\frac{x}{\lambda_j}\right] \qquad (1)$$

where λ_j, c, ϕ_o, d and φ_o are the Josephson penetration depth, the speed of light, the magnetic flux quantum, the average distance between the CuO_2 layers and the phase value at $x=0$ in the Josephson junction, respectively. In order to investigate the temperature dependence of the net effective mass, the distance parameter, *x* in Equation (1) has been chosen as 0.3 μm which is smaller than the lowest λ_J values for both the optimally and over oxygen doped samples.

In the study, the parameter *x* in Equation (1), which is selected within the Josephson penetration depth, is taken as the constant value for every temperature range for both samples. Hence, the only variation of the superconducting systems is temperature. The net effective mass versus temperature graphics for the optimally and over doped samples obtained by the Ongüas equation are illustrated in Figure 2a and b, respectively.

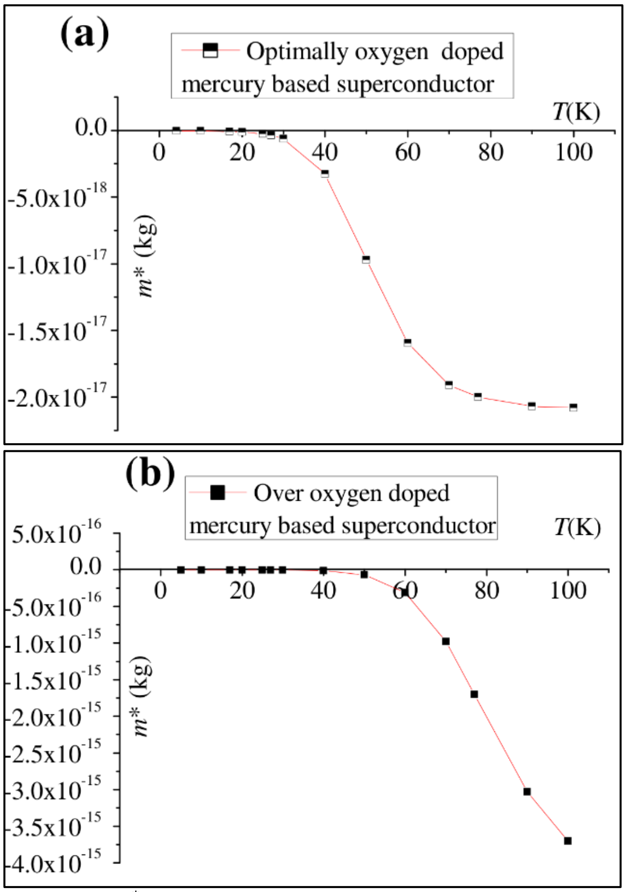


Figure 2. m^* versus temperature graphics for (a) optimally and (b) over oxygen doped samples.

Hg-based cuprates have hole-type of conductivity. Hence, the negative sign of m^* verify the hole-type conductivity in the Hg-based samples. The other explanation is that the negative net effective mass values can be interpreted as the formation of an anti-gravitational force which has the reverse sign to the gravitational field of the Earth [9,11]. The mass of an object is variable in a gravitational field [12]. Therefore the third quantum critical transition phenomenon of the superconducting system is identified from the first derivative m^* to temperature.

In the study, the third quantum critical transition temperature region for the superconducting samples, where f_p shifts from microwave to infrared in the electromagnetic spectrum [13,14,16,19] (Table 1) that corresponds to quantum gravitational phenomenon is studied. The superconducting system has constant entropy hence the temperature is the only variable. In superconducting system, temperature variation can be considered as time variation. Hence the quantum gravitational effect, which is observed by the variation of temperature, is consistent with the El Naschie's quantum gravity. According to El Naschie's quantum gravity, the gravitational field slows down time flowing. It is also true that the changing the speed of the passing time (or temperature) can create a

Table 1. f_p values for (a) optimally and (b) over oxygen-doped Hg-1223 superconductors.

(a)

T(K)	$f_p(Hz)$
4.2	8.303×10^{13}
27	3.363×10^{13}
77	8.303×10^{12}

(b)

T(K)	$f_p(Hz)$
5	3.295×10^{13}
17	2.175×10^{13}
25	1.981×10^{13}
77	1.866×10^{12}
90	1.537×10^{12}

gravitational effect [20]. Due to this reason, the derivatives with respect to temperature can be considered as the derivatives with respect to time. At the vicinity of T_{QG}, the first derivation of the net effective mass to the temperature, $(dm^*)/dt$ for the various oxygen doped samples, which has also the maximum value in negative region, is considered as an indicator of the intrinsic gravitational field of the superconducting sample investigated. In the study, the maximum value of the $(dm^*)/dt$ is called as the super critical temperature, T_{sc}. The phenomenon of the quantum gravitational effect appears at the temperature interval between 25-77 K for optimally and over doped samples [9,11,13,16].

3. THE SCHWARZIAN DERIVATIVE of THE THIRD CHAOTIC TRANSITION

In order to investigate the chaotic behaviors of the systems, the most convenient mathematical method is the derivative process. Schwarzian derivative method is utilized to find a sufficient condition for the chaotic transitions of nonlinear dynamical systems. The Schwarzian derivative $Sf(x)$ of a locally univalent analytic function f at point x, is defined by

$$Sf(x)=\left(\frac{f''(x)}{f'(x)}\right)'-\frac{1}{2}\left(\frac{f''(x)}{f'(x)}\right)^2 \qquad (2)$$

where $f(x)$ is a function with one variable, $f'(x)$ and $f''(x)$ are first and second continuous derivatives of the equation, respectively. The Schwarzian derivative of a function, which is used for the limiting the behavior of dynamical systems, has a negative when the system behaves chaotically [21-24]. In order to investigate the chaotic behavior in Hg-cuprate layered superconductors, the Schwarzian derivative of the m^* of Hg-cuprate superconductors are calculated the equation given below,

$$S[m^*(T)] = \frac{(m^*)'''(T)}{(m^*)'(T)} - \frac{3}{2}\left(\frac{(m^*)''(T)}{(m^*)'(T)}\right)^2 \qquad (3)$$

where $(m^*)'(T)$, $(m^*)''(T)$ and $(m^*)'''(T)$ represent the first, second and third order derivatives of m^* with respect to temperature, respectively. The variations of the related derivatives with temperature for the optimally and over doped samples are given in Figure 3a and b.

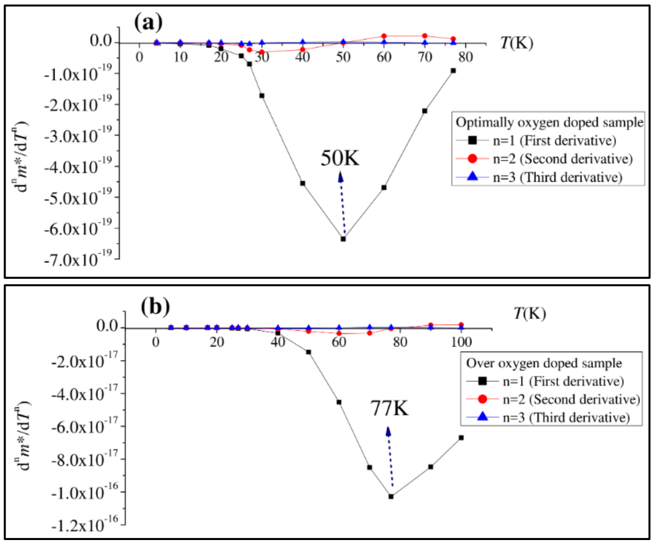
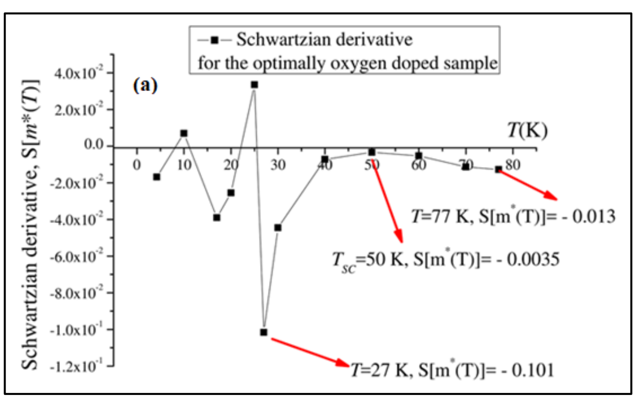


Figure 3. The variation of the first, second and third-order derivatives of m^* to temperature for (a) the optimally and (b) over oxygen doped samples.

The first, second and third order derivatives components of the m^* are taken in order to calculate $S[m^*(T)]$. The Schwarzian derivatives of the net effective mass of the optimally and over oxygen doped samples are shown Figure 4a and b, respectively.



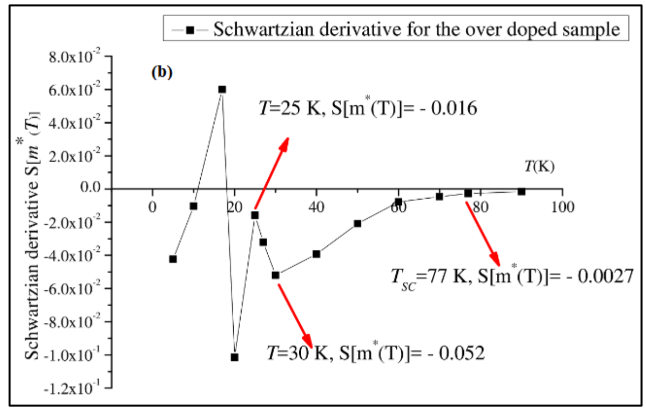


Figure 4. The Schwarzian derivative of m^* for (a) the optimally and (b) over oxygen doped samples

4. DISCUSSION

In the work, the Schwarzian derivative method is used for predicting the chaotic behavior mathematically in the mercury based high temperature nonlinear superconducting system for determining the third chaotic point. The third chaotic point called the quantum gravitational transition temperature, T_{QG}, where the quantum gravitational field appears, manifest itself as the maximum of negativity in the Schwarzian derivatives of the net effective mass-temperature data for both the optimally and over doped samples between 25-77 K temperature range. As shown in Figures 4a and b, the negative tendency of Schwarzian derivative at T_{SC}, at which the first derivatives of m^* for various oxygen doped samples have also maximum negative value, goes on the temperature interval of the third chaotic point, T_{QG} that locates at the temperature interval of 25-77 K. The phenomenon of the third chaotic transition temperature, T_{QG} is coincided to the fact that when a particle enters a gravitational field, its mass decreases [12]. Moreover, it was determined that Schwarzian derivative values at T_{SC} for both optimum and over doped samples have negative values, which corresponds to -0.0035 and -0.0027, respectively (Figure 4a and b). As is seen in Figure 5, the Schwarzian derivative has same values for both samples at some temperatures.

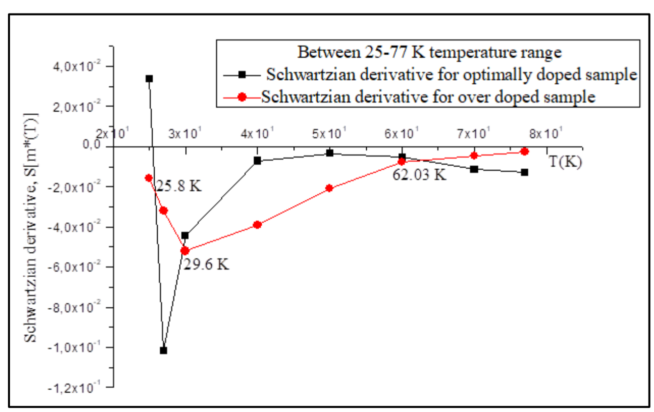


Figure 5. Schwarzian derivative for optimally and over doped samples between 25-77 K temperature interval.

According to Table 1, the various doped samples behave as microwave cavity. By lowering the temperature from 77 K to 25 K, it starts to emit infrared region. After the shifting to infrared region, the third chaotic transition is completed for both samples (Figure 4a and 4b). Moreover, it is understood that the oxygen doping does not the effect of the existence of the quantum gravitational field and also the chaotic transition of the system, that is proved by negative Schwarzian derivative peaks but it lowers the super critical temperature, T_{sc}.

5. CONCLUSION

The superconducting order parameter, of the superconducting system that totally represents the system, is utilized to derive m^* of the superconducting system. By recalling the phase difference of the wave function which is the only variable of the system, m^* of the mercury based superconductor is established by using an advanced analogy between the supercurrent density and third derivative of the phase of the quantum wave function of the superconducting system to temperature. Ultimately, Onguas Equation (Equation 1) gives the relationship between m^* and the phase of the superconducting state. As is known, the mass of object decreases in a gravitational field. Hence, the existence of the gravitational field, i.e. the third transition temperature T_{QG} is determined by means of the variation of the effective mass of the quasi-particles. As a result of the mathematical study, it is proposed that the negative Schwarzian derivative of m^* is a convenient mathematical method for precise prediction of chaotic transitions in nonlinear superconducting condensed matter systems as well. Hence, the Schwarzian method used in the study, which explains chaotic properties of the nonlinear superconducting system, is consistent with the fundamentals of superconductivity.

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