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A Numerical Solution of the Generalized Burgers-Huxley Equation

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Keywords
Crank Nicolson logarithmic finite difference method; Generalized Burgers-Huxley equation; von Neumann Stability analysis.

Abstract

In this study numerical solutions of the generalized Burgers-Huxley equation are obtained utilizing a new approach: The Crank Nicolson logarithmic finite difference method (CN-LFDM). The effectiveness of the suggested method is demonstrated by a numerical example for various parameter cases. Presented tables demonstrate that the obtained results are in excellent agreement with the exact solutions and better than numerical results acquired by other methods in the literature. The method was analyzed with the von-Neumann stability analysis method and it was shown that the method was unconditionally stable.

Genelleştirilmiş Burgers-Huxley Denkleminin Bir Sayısal Çözümü

Anahtar kelimeler
Crank Nicolson logaritmik sonlu fark yöntemi; Genelleştirilmiş Burgers-Huxley denklemi; von Neumann kararlılık analizi.

Öz

Bu çalışmada, genelleştirilmiş Burgers-Huxley denkleminin sayısal çözümleri yeni bir yaklaşım kullanılarak elde edilmiştir: Crank Nicolson logaritmik sonlu farklar yöntemi (CN-LSFY). Önerilen yöntemin etkinliği, çeşitli parametre durumları için sayısal bir örnekle gösterilmiştir. Sunulan tablolar, elde edilen sonuçların tam çözümlerle mükemmel bir uyum içinde olduğunu ve literatürdeki diğer yöntemlerle elde edilen sayısal sonuçlardan daha iyi olduğunu göstermektedir. Yöntem, von-Neumann kararlılık analizi yöntemi ile analiz edilmiş ve yöntemin koşulsuz kararlı olduğu gösterilmiştir.

1. Introduction

In subjects such as mathematics, chemistry, physics, biology, and engineering, nonlinear partial differential equations are often used to simulate many subjects. The generalized Burgers-Huxley equation is one of these nonlinear partial differential equations. The initial-boundary value problem for the generalized Burgers-Huxley equation is as follows:

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right), \quad a<x<b, \quad t>0 \quad (1)$$

$$u(a,t)=w_1(t), \quad u(b,t)=w_2(t), \quad t>0$$

$$u(x,0)=q(x), \qquad a<x<b$$

This problem illustrates a model that may be used to explain how convection effects, diffusion transports and reaction mechanisms interact (Satsuma 1987). Where $q(x)$, $w_1(t)$ and $w_2(t)$ are known functions, $\alpha>0$ is the advection coefficient, $\beta\geq 0$, $\delta>0$ and $\gamma\in(0,1)$ are model parameters modulating the interplay between non-standard nonlinear advection, diffusion and nonlinear reaction (or applied current) contributions (Khan *et al*. 2021).

Many scientists have used a variety of numerical approaches to numerically solve the generalized Burgers-Huxley equation. Wazwaz (2005) and Deng (2008) studied the ravelling wave solutions of equation. Hashim *et al*. (2006) used the Adomian decomposition approach to solve the equation numerically. Javidi (2006, 2009) employed the pseudospectral method and spectral collocation method to provide numerical solutions of the equation. Batiha *et al*. (2008) used the variational iteration technique to solve the equation. Darvishi *et al*. (2008) employed the spectral collocation technique and Darvishi's preconditionings to obtain numerical solutions to equations. Khattak (2009) employed a numerical strategy based on the collocation technique and radial basis functions to solve the problem. Sari and Gürarslan (2009) developed the differential quadrature method to solve the equation numerically. To solve the equation numerically, Javidi and Golbabai (2009) employed the spectral collocation technique with Chebyshev polynomials for spatial derivatives and the fourth order Runge-Kutta technique for integration. The differential transform method was used by Biazar and Mohammadi (2010) to solve the equation. To get numerical solutions to equation, Tomasiello (2010) employed the iterative differential quadrature method. Bratsos (2011) suggested a fourth order finite difference approach for numerical solutions of the equation in a two time level recurrence relation. The Galerkin approach was utilized by El-Kady *et al*. (2013) to obtain numerical solutions to the equation. Celik (2012, 2016) solved the equation using the haar wavelet method and the Chebyshev wavelet collocation method. For the equation, Duan *et al*. (2012) constructed a lattice Boltzman model. Mittal and Tripathi (2015) developed a numerical technique based on the collocation of modified cubic B-spline functions to solve the equation. Inan and Bahadır (2015) employed an implicit exponential finite difference approach to obtain the numerical solutions of the equation. In addition, Inan (2017) obtained the numerical solutions of the equation by using the explicit exponential finite difference approach. The numerical solutions of the equation were obtained by Singh *et al*. (2016) by using the modified cubic B-spline quadrature technique. Loyinmi and Akinfe (2020) used the Elzaki transform to solve the equation. Mohan and Khan (2021) established the existence and uniqueness of a global weak solution of the generalized Burgers-Huxley equation by using a Faedo-Galerkin approximation method.

In this study, numerical solutions of the generalized Burgers-Huxley equation were obtained by using Crank Nicolson logarithmic finite difference method which is an accurate, reliable, easily understandable, unconditionally stable and suitable alternative method.

CN-LFDM

2. Material And Methods

2.1 Crank Nicolson logarithmic finite difference method

We demonstrate the finite difference approximation of $u(x,t)$ at the node point (x_i, t_n) by u_i^n in which $x_i = ih(i = 0,1,\ldots,N)$, $t_n = t_0 + nk(n = 0,1,2,\ldots)$, $h = \dfrac{b-a}{N}$ is the node size in x direction and k is the time step.

We reorganize equation (1) to acquire

$$\frac{\partial u}{\partial t} = \beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right) - \alpha u^{\delta}\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}. \quad (2)$$

Multiplying equation (2) by e^u, we acquire the following equation:

$$\frac{\partial e^u}{\partial t} = e^u\left(\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right) - \alpha u^{\delta}\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}\right) \quad (3)$$

In equation (3) utilizing finite difference approximations for derivatives the following Crank Nicolson logarithmic finite difference scheme is acquired

$$u_i^{n+1} = u_i^n + \ln\left\{1 + k\left[\beta u_i^n\left(1-\left(u_i^n\right)^\delta\right)\left(\left(u_i^n\right)^\delta - \gamma\right) - \alpha\left(u_i^n\right)^\delta \frac{u_{i+1}^{n+1} - u_{i-1}^{n+1} + u_{i+1}^n - u_{i-1}^n}{4h} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1} + u_{i+1}^n - 2u_i^n + u_{i-1}^n}{2h^2}\right]\right\} \quad (4)$$

where $1 \le \mathrm{i} \le \mathrm{N}\text{-}1$. A system of nonlinear difference equations is equation (4). This nonlinear system of equations is supposed to be in the form:

$$G(W) = 0 \quad (5)$$

where $G = \left[g_1, g_2, \ldots, g_{N-1}\right]^T$ and $W = \left[u_1^{n+1}, u_2^{n+1}, \ldots, u_{N-1}^{n+1}\right]^T$. The nonlinear equation (5) is linearized using Newton's iterative approach, which yields the following iteration:

1) Determine $W^{(0)}$, a first guess.
2) For $m = 0,1,2,3\ldots$ up to convergency do:

Resolve $J\left(W^{(m)}\right)\delta^{(m)} = -G\left(W^{(m)}\right)$;

Adjust $W^{(m+1)} = W^{(m)} + \delta^{(m)}$ where $J(W^{(m)})$ the Jacobian matrix which is appraised analytically. The initial estimate is based on the solution from the previous time step. The Newton iteration is halted at every time step when $\left\|G(W^{(m)})\right\| \le 10^{-5}$.

2.2 Stability Analysis

We will utilize the von Neumann stability analysis to analyze the scheme's stability, where the growth factor of a characteristic Fourier mode is specified as follows:

$$u_i^n = \varepsilon^n e^{I\phi ih}, \ I = \sqrt{-1}. \quad (6)$$

The stability of finite difference approaches implemented to linear partial differential equations is investigated using von Neumann stability analysis. So we'll look into the scheme's linear form's stability. The nonlinear term of the scheme (4) have been linearized by replacing the quantity $\left(u_i^n\right)^\delta$ by local constant $\tilde{U}$. Hence the numerical scheme (4), convert into

$$u_i^{n+1} = u_i^n + \ln\left\{1 + k\left[\beta u_i^n\left(1-\tilde{U}\right)\left(\tilde{U} - \gamma\right) - \alpha\tilde{U} \frac{u_{i+1}^{n+1} - u_{i-1}^{n+1} + u_{i+1}^n - u_{i-1}^n}{4h} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1} + u_{i+1}^n - 2u_i^n + u_{i-1}^n}{2h^2}\right]\right\} \quad (7)$$

Since the scheme (7) is logarithmic, the examination will be improved by expanding the logarithmic term of the scheme into a Taylor's series. Hilal *et al.* (2020) applied the same procedure to calculate the local truncation error of exponential finite difference schemes and examine their stability. If the scheme's logarithmic term is expanded to a Taylor series and the first term is used, the scheme can be expressed as:

$$u_i^{n+1} = u_i^n + k\beta u_i^n\left(1-\tilde{U}\right)\left(\tilde{U} - \gamma\right) - \alpha k\tilde{U}\left[\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1} + u_{i+1}^n - u_{i-1}^n}{4h}\right] + k\left[\frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1} + u_{i+1}^n - 2u_i^n + u_{i-1}^n}{2h^2}\right] \quad (8)$$

By substituting the (6) equality into the (8) linear form of the scheme, we get the growth factor as follows:

$$\varepsilon = \frac{1 + k\beta\left(1-\tilde{U}\right)\left(\tilde{U}-\gamma\right) - \frac{\alpha k\tilde{U}}{2h} i\sin\left(\phi h\right) - \frac{k}{h^2}\sin^2\frac{\phi h}{2}}{1 + \frac{\alpha k\tilde{U}}{2h} i\sin\left(\phi h\right) + \frac{k}{h^2}\sin^2\frac{\phi h}{2}}$$

Stability condition in von-Neumann method is $|\varepsilon| \leq 1$.

$|\varepsilon| \leq 1$ since $\beta \geq 0$ and $\gamma \in (0.1)$.Therefore CN-LFDM generalized Burgers-Huxley equation is unconditionally stable.

3. Results and Discussion

Crank Nicolson logarithmic finite difference method is used to acquire the numerical solutions of the generalized Burgers-Huxley equation. To demonstrate that the results are correct the error norms L_2, L_∞ and absolute error:

$$A.E. = \left|U(x_i,t_n) - u(x_i,t_n)\right|,$$

$$L_\infty = \left\|U - u_N\right\|_\infty = \max_j \left|U_j - (u_N)_j\right|,$$

$$L_2 = \left\|U - u_N\right\|_2 = \sqrt{h\sum_{j=0}^{N}\left|U_j - (u_N)_j\right|^2}$$

are used, where u and U indicate computed numerical solutions and exact solutions, respectively.

3.1 Numerical example of generalized Burgers-Huxley equation

Consider the generalized Burgers-Huxley equation in the form of equation (1) for $0 \leq x \leq 1,\ t > 0$ with initial condition

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1 x)\right]^{\frac{1}{\delta}}$$

and boundary conditions

$$u(0,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(-A_1 A_2 t)\right]^{\frac{1}{\delta}},$$

$$u(1,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1(1 - A_2 t))\right]^{\frac{1}{\delta}}.$$

This problem's exact solution is

$$u(x,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1(x - A_2 t))\right]^{\frac{1}{\delta}}$$

where

$$A_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4\beta(1+\delta)}}{4(1+\delta)}\gamma,$$

$$A_2 = \frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)\left(-\alpha + \sqrt{\alpha^2 + 4\beta(1+\delta)}\right)}{2(1+\delta)}.$$

The results obtained by using the presented method are presented in Table 1-8 and Figure 1-2. In Table 1-6 we took as $h = 0.01$ and $k = 0.000001$.The absolute errors acquired by CN-LFDM and by some other methods (Batiha *et al*. 2008, Hashim *et al*. 2006) in literature are compared in Table 1-3. The comparisons for the parameters $\delta = 1$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$ are shown in Table 1 while the comparisons for the parameters $\delta = 2$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.01$ are given in Table 2 and for the parameters $\delta = 4$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.01$ are shown in Table 3. As evidenced by the tables, the absolute errors acquired by the CN-LFDM are less than the absolute errors acquired by some other methods in the literature. The error norms L_2 and L_∞ for the parameters $\delta = 1$, $\alpha = 1$, $\gamma = 0.01$ and various values of β are presented in Table 4. The error norms L_2 and L_∞ for the parameters $\delta = 1$, $\alpha = 1, \beta = 1$ and various values of γ are presented in Table 5. Table 6 presents L_2 and L_∞ error norms for the parameters $\alpha = 1$, $\beta = 1$, $\gamma = 0.001$ and varied values of δ . As evidenced by the tables, the L_2 and L_∞ error norms acquired by the CN-LFDM are quite small in all cases. Table 7 presents the error norms L_2 and L_∞ for the parameters $\alpha = 0.1$, $\beta = 0.1$, $\gamma = 0.001$ and different values of k and $h = 0.05$ at $t = 1$. As evidenced by the table, as the value of k decreases, the error norms L_2 and L_∞ are also decrease. Table 8 presents the error norms L_2 and L_∞ for the parameters $\delta = 2$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$ for different values of h and $k = 0.00001$ at $t = 2$. As evidenced by the table, as the value of h decreases, the error norms L_2 and L_∞ are increase. This increase in error norms is not very significant, but using large values of h provides great convenience in calculations. Figure 1 presents exact and numerical solutions for the parameters $\delta = 2$, $\beta = 10$, $\alpha = 5$ and $\gamma = 0.001$ at different times. As evidenced by the figure, exact solutions and numerical solutions are very close to each other. Figure 2 presents absolute errors for the parameters $\delta = 1$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$ at different times. As evidenced by the figure, the absolute errors are very small and become very close to each other as t increases. In Figure 1-2 we took as $h = 0.05$ and $k = 0.00001$.

Table 1. Absolute errors for the parameters $\delta = 1$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$.

x	t	CN-LFDM	ADM (Hashim et al. 2006)	VIM (Batiha et al. 2008)
0.1	0.05	7.97215 e-9	1.87406 e-8	1.87405 e-8
	0.1	1.31873 e-8	3.74812 e-8	3.74813 e-8
	1	2.31122 e-8	3.74812 e-7	3.74812 e-7
0.5	0.05	2.44677 e-8	1.87406 e-8	1.87405 e-8
	0.1	4.52485 e-8	3.74812 e-8	1.37481 e-8
	1	7.80880 e-8	3.74812 e-7	3.74813 e-7
0.9	0.05	5.47148 e-8	1.87406 e-8	1.87405 e-8
	0.1	6.27359 e-8	3.74812 e-8	3.74813 e-8
	1	7.31144 e-8	3.74812 e-7	3.74813 e-7

Table 2. Absolute errors for the parameters $\delta = 2$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.01$.

x	t	CN-LFDM	ADM (Hashim et al. 2006)	VIM (Batiha et al. 2008)
0.1	0.1	1.89263 e-5	5.51554 e-5	5.51580 e-5
	0.2	2.73074 e-5	1.10342 e-4	1.10310 e-4
	0.3	3.05222 e-5	1.65529 e-4	1.65457 e-4
	0.4	3.17171 e-5	2.20708 e-4	2.20598 e-4
	0.5	3.21570 e-5	2.75950 e-4	2.75734 e-4
0.3	0.1	4.51489 e-5	5.51381 e-5	5.51340 e-5
	0.2	6.73694 e-5	1.10293 e-4	1.10262 e-4
	0.3	7.57965 e-5	1.65458 e-4	1.65385 e-4
	0.4	7.89279 e-5	2.20635 e-4	2.20502 e-4
	0.5	8.00813 e-5	2.75832 e-4	2.75614 e-4
0.5	0.1	6.25195 e-5	5.51134 e-5	5.51099 e-5
	0.2	9.05364 e-5	1.10243 e-4	1.10214 e-4
	0.3	1.00966 e-4	1.65402 e-4	1.65313 e-4

	0.4	1.04837 e-4	2.20543 e-4	2.20406 e-4
	0.5	1.06261 e-4	2.75716 e-4	2.75493 e-4

Table 3. Absolute errors for the parameters $\delta = 4$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.01$.

x	t	CN-LFDM	ADM (Hashim et al. 2006)	VIM (Batiha et al. 2008)
0.1	0.1	7.26930 e-5	2.17787 e-4	2.17687 e-4
	0.2	1.02969 e-4	4.35690 e-4	4.35293 e-4
	0.3	1.14492 e-4	6.53711 e-4	6.52817 e-4
	0.4	1.18726 e-4	8.71847 e-4	8.70258 e-4
	0.5	1.20232 e-4	1.09010 e-3	1.08762 e-3
0.3	0.1	1.70679 e-4	2.17552 e-4	2.17453 e-4
	0.2	2.50825 e-4	4.35222 e-4	4.34824 e-4
	0.3	2.81042 e-4	6.53008 e-4	6.52113 e-4
	0.4	2.92150 e-4	8.70910 e-4	8.69320 e-4
	0.5	2.96116 e-4	1.08893 e-3	1.08644 e-3
0.5	0.1	2.29482 e-4	2.17318 e-4	2.17218 e-4
	0.2	3.30248 e-4	4.34753 e-4	4.34354 e-4
	0.3	3.67631 e-4	6.52304 e-4	6.51408 e-4
	0.4	3.81349 e-4	8.69972 e-4	8.68380 e-4
	0.5	3.86233 e-4	1.08776 e-3	1.08527 e-3

Table 4. The error norms L_2 and L_∞ for the parameters $\delta = 1$, $\alpha = 1$ and $\gamma = 0.01$.

	L_2		
t	$\beta = 1$	$\beta = 10$	$\beta = 100$
0.01	1.66976 e-6	8.23320 e-6	5.24735 e-5
0.1	4.50870 e-6	3.40692e-5	3.01463 e-4
1	6.61984 e-6	5.29130 e-5	4.13360 e-4
10	6.61590 e-6	4.55478 e-5	2.16492 e-7
	L_∞		
t	$\beta = 1$	$\beta = 10$	$\beta = 100$
0.01	5.93770 e-6	2.40722 e-5	8.56440 e-5
0.1	6.29866 e-6	4.26142 e-5	3.93010 e-4
1	8.30963 e-6	6.90598 e-5	5.58153 e-4
10	8.30472 e-6	5.94715 e-5	2.94023 e-7

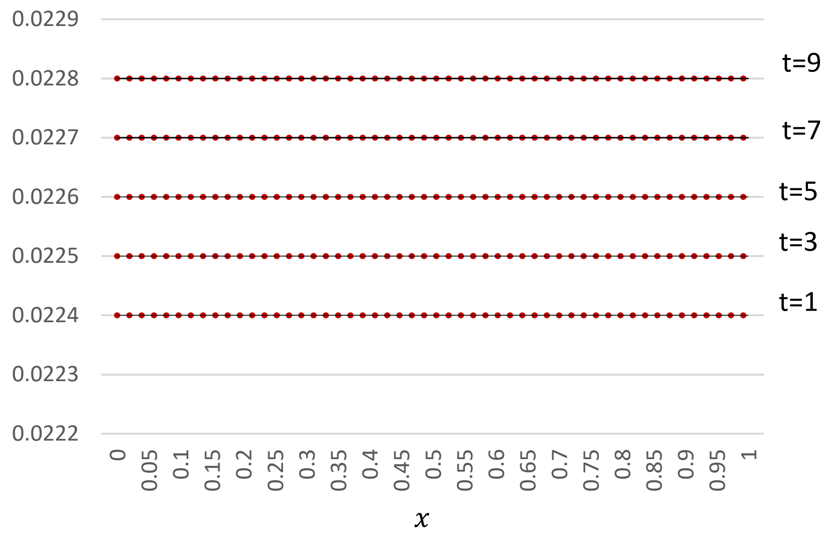

Figure 1. Exact and numerical solutions for the parameters $\delta = 2$, $\beta = 10$, $\alpha = 5$ and $\gamma = 0.001$ at different times.

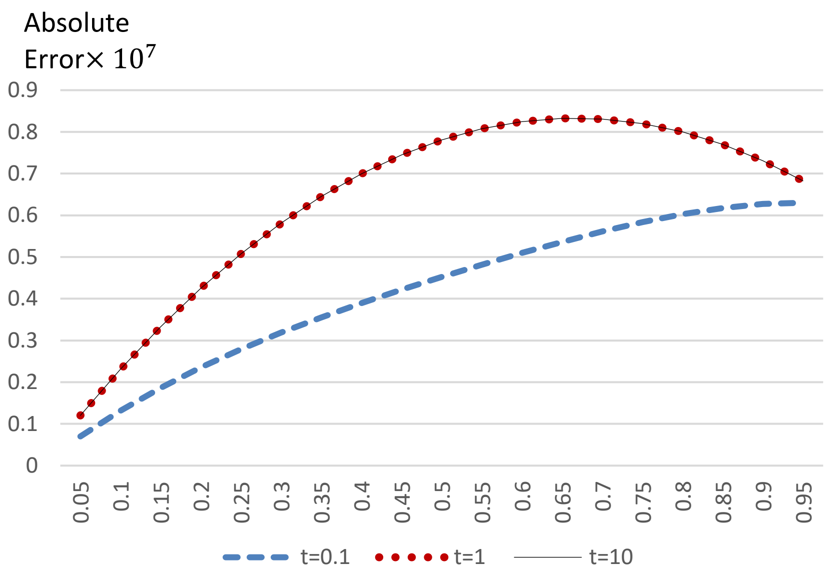

Figure 2. Absolute errors for the parameters $\delta = 1$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$ at different times.

Table 5. The error norms L_2 and L_∞ for the parameters $\delta = 1$, $\alpha = 1$ and $\beta = 1$.

	L_2		
t	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.0001$
0.01	1.669765 e-6	1.670664 e-8	1.670936 e-10
0.1	4.508696 e-6	4.518389 e-8	4.515046 e-10
1	6.619839 e-6	6.636429 e-8	6.614811 e-10
10	6.615903 e-6	6.637146 e-8	6.614709 e-10
	L_∞		

t	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.0001$
0.01	5.937700 e-6	5.938021 e-8	0.837297 e-10
0.1	6.298663 e-6	6.302047 e-8	0.888415 e-10
1	8.309630 e-6	8.330121 e-8	1.170686 e-10
10	8.304716 e-6	8.331040 e-8	1.170670 e-10

Table 6. The error norms L_2 and L_∞ for the parameters $\alpha = 1$, $\beta = 1$, $\gamma = 0.001$

	L_2		
t	$\delta = 1$	$\delta = 2$	$\delta = 4$
0.01	1.670664 e-8	0.661227 e-6	3.766536 e-6
0.1	4.518389 e-8	1.904844 e-6	1.187424 e-5
1	6.636429 e-8	2.832022 e-6	1.790726 e-5
10	6.637146 e-8	2.824892 e-6	1.775187 e-5
	L_∞		
t	$\delta = 1$	$\delta = 2$	$\delta = 4$
0.01	5.938021 e-8	2.309319 e-6	1.276667 e-5
0.1	6.302047 e-8	2.527796 e-6	1.511204 e-5
1	8.330121 e-8	3.568203 e-6	2.274354 e-5
10	8.331040 e-8	3.559230 e-6	2.254628 e-5

Table 7. The error norms L_2 and L_∞ for the parameters $\alpha = 0.1$, $\beta = 0.1$, $\gamma = 0.001$ and different values of k at $t = 1$.

k	$L_2 \times 10^8$	$L_\infty \times 10^8$
0.001	1.742699	2.482851
0.0001	1.741400	2.481706
0.00001	1.741416	2.481782
0.000001	1.740698	2.481631
0.0000001	1.732642	2.479340

Table 8. The error norms L_2 and L_∞ for the parameters $\delta = 2$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$ for different values of h at $t = 2$.

h	$L_2 \times 10^6$	$L_\infty \times 10^6$
0.05	2.810044	3.567088
0.025	2.823963	3.567090
0.0125	2.830683	3.568187
0.01	2.832009	3.568186

4. Conclusion

The numerical solutions of the generalized Burgers-Huxley equation are achieved via the Crank Nicolson logarithmic finite difference method in this study. Tables compare the absolute errors obtained by the provided method to those obtained by earlier studies in the literature. The tables show that the results obtained by CN-LFDM are better than those obtained by other methods in the literature. In addition, L_2 and L_∞ error norms have been calculated. The obtained error norms are quite small. The results clearly show that the present method is accurate, reliable and convenient alternative method. The method was analyzed with the von-Neumann stability analysis method and it was shown that the method was unconditionally stable. Consequently, the present method can be used to find numerical solutions to a wide variety of nonlinear problems.

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