

# Distributed Formation Control Algorithm for Improved String Stability in Heterogenic Vehicle Platoons

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**Abstract**— Achieving speed, safety and energy optimization in vehicle platoons is among the important topics of recent years. In this context, this work focuses on improving the string stability (SS) of heterogeneous platoons. Better SS allows for smaller gap between vehicles, which means shorter time headway for Cooperative Adaptive Cruise Control (CACC) or inter-vehicle following-distance for platooning. Shorter time headway or inter-vehicle distance results in better road use and less fuel consumption. Rather than compensating for dynamic differences by means of low level control schemes by implementing pre-compensators at vehicle level, this work takes a different approach, where a higher level of control is preferred to improve SS: The platoon formation. To achieve this, it is important to ensure that each vehicle is in its optimum position in the platoon based on its dynamics for highest string stability. This generally leads to the vehicle with highest inertia to lead the platoon and the lowest one as the last follower. In this study, a platoon formation algorithm is proposed to run on each vehicle, leading to an optimum overall string structure. The efficiency of the algorithm is demonstrated by simulations in Matlab(R) for a four-vehicle platoon.

**Index Terms**— Cruise control, distributed formation, heterogenic vehicle platoon, string stability.

## I. INTRODUCTION

IN RECENT years, traffic congestion and energy saving have been the utmost major issues in road transportation. Vehicle platooning has already proven to have potential to improve both. The basic idea in platooning is to have a string of vehicles with closer gap than normal driving by applying automation. In this sense, automation in its simplest form is a cooperative adaptive cruise control (CACC) in which a following-strategy is applied for a constant time gap. While

adaptive cruise control is based on measuring distance with the vehicle ahead using a forward-facing radar, CACC also requires vehicle to vehicle (V2V) communication, which is done via wireless radio frequency (RF) communication, the IEEE 802.11p protocol over 5.9GHz base frequency.


Benefits of platoons come up from its vehicle-following strategy. Platooning differs from CACC in that it has a constant-distance following strategy rather than a constant time gap. Less inter-vehicle gap means more vehicles per kilometer, which leads to better use of the road and less traffic jams. It also improves aerodynamics of the whole platoon reducing fuel cost and emission in parallel. It can also be said that the platoon aids safety since the reaction time of automation is much less than that of the human driver.

Besides the opportunities described above, there are some challenges with vehicle platoons. One of the major challenges is to ensure the string stability (SS). There are various definitions for SS in the literature [1]. In this study we define SS as the fact that the error for the distance between two consecutive vehicles remains positive as we move along the string. Here, the error is defined as the difference between the actual distance and the value calculated according to the predefined equation. SS ensures that there will be no collision between the members of the platoon or no split under acceleration. Thus, not only each vehicle needs to exhibit stability individually, but also the entire platoon needs to be stable as well.

Several studies are carried out in the literature regarding the string stability. In [2], a model reference adaptive control augmentation is proposed in which the control objective is to augment a baseline CACC, proven to be string stable in the homogeneous scenario, with an adaptive control term that compensates for each vehicle's unknown driveline dynamics. In [3] a constant time headway policy is employed for a platoon-stable controller design based on feedback linearization. In [4], a controller design method is developed that allows the string stability requirement to be explicitly included in the controller synthesis specifications. [5] presents a distributed finite-time adaptive integral-sliding-mode (ISM) control approach. In [6], novel criteria for string stability are proposed for mixed traffic platoons that consists of both automated and manual driving cars. A mixed traffic string stability definition is proposed to guarantee the boundedness of the motion states fluctuation upstream as well as the safety

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of the entire platoon. [7] presents a periodic switching control method for an automated vehicle platoon to minimize the overall fuel consumption. Considering the nonlinearity of switching operation, the concept of bounded stability for a platoon of vehicles is defined to replace conventional internal stability and string stability. The distributed servo loop controller based on dual-pulse-and-glide operation is designed for each vehicle, wherein a sectionalized switching map is adopted for proper mode selection. [8] investigates the nonlinear vehicle platoon control problems with external disturbances. The authors apply the quadratic spacing policy (QSP) into the platoon control, in which the desired inter-vehicle distance is a quadratic function in terms of the vehicle's velocities. They propose a platoon control scheme based on the distributed integrated sliding mode (DISM) and they also perform a string stability analysis accordingly. In [9] the authors propose a centralized collision-free solution on the basis of model predictive control that guarantees asymptotic platoon tracking of speed changes and satisfaction of system constraints during the transient process. In [10] the authors investigate the impact of applying a constant distance gap (CDG) policy for starting platoons at traffic lights, where they claim that the applicability of CDG in real traffic is limited, due to its demand on complex communication topologies in order to achieve string stability. However, they explain that they were able to show its capability to increase the capacity of traffic light controlled intersections.

This work focuses on improving SS of a heterogeneous platoon where each member of the platoon has different dynamics in contrast to a homogenous platoon where each vehicle is considered to present the same dynamic behavior. Improving SS allows for smaller gap between the vehicles which means shorter time headway for CACC. Shorter time headway or inter-vehicle distance results in better road use and less fuel consumption. Ideally, this leads to road trains (touch condition) which is practically not possible due to marginal SS behavior under physical constraints.

Rather than compensating for dynamic differences by means of low level control schemes by implementing pre-compensators, this study takes a different approach to improve SS at a higher level control: The platoon formation. We try to ensure that each vehicle in the platoon is in its optimum position based on its dynamics for highest string stability. Considering the vehicle dynamics to be proportional to engine dynamics, it results that the vehicle with the highest inertia takes the leadership of the platoon and the one with the lowest is the last follower. A platoon formation algorithm is developed and run on each vehicle leading to an optimum overall string structure.

In line with the explanations above, the rest of this paper is arranged as follows: Section 2 focuses on vehicle model with CACC in a platoon and drive cycle following a brief information about SS. Section 3 covers the proposed algorithm and Section 4 provides the simulation results. In Section 5 we draw conclusions along with future steps of the study.

## II. VEHICLE MODEL WITH CACC

### II.1 String stability

If we consider the changes in vehicle dynamics (acceleration, speed or position) as noise, the string stability can be described as the condition that this noise attenuates as it propagates along the vehicle string [11]. String stability is concerned with the interactions in a platoon such that vehicles must not collide with each other [12].

### II.2 Mathematical background

Figure 1 illustrates a heterogeneous vehicle string, where inter-vehicle communication is considered to be established over wireless network. Each vehicle receives data from the vehicle in front and sends its own to the one behind over the wireless communication channel.

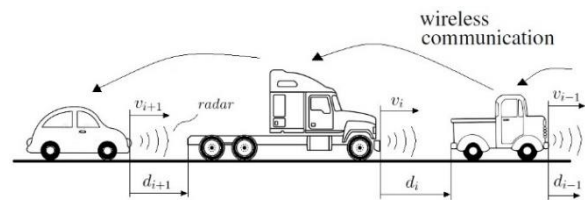


Fig. 1. CACC-equipped heterogeneous vehicle platoon [1]

In a vehicle string, the desired following-distance is given as:

$$d_{r,i}(t) = r_i + h v_i(t), \quad i \in S_m \quad (1)$$

where  $h$  is the following-time gap in seconds and  $r_i$  is the desired distance in meters between the vehicles at full stop condition;  $v_i$  is the speed of the  $i$ th vehicle in m/s.  $S_m = \{i \in \mathbb{N} \mid 1 \leq i \leq m\}$  is the set of all vehicles in a platoon of length (number of vehicles)  $m \in \mathbb{N}$ .  $i = 0$  is reserved for the platoon leader. The inter-vehicle distance decreases as the platoon speed decreases and it increases with an increase in the platoon speed [13]

The following-distance error between two vehicles travelling consecutively is given as:

$$\begin{aligned} e_i(t) &= d_i(t) - d_{r,i}(t) \\ &= (q_{i-1}(t) - q_i(t) - L_i) - (r_i + h v_i(t)) \end{aligned} \quad (2)$$

Here,  $q_i$  and  $q_{i-1}$  are the rear-bumper positions of vehicles in the rear and front, respectively;  $L_i$  stands for the vehicle length. Nonlinear model for the  $i$ th vehicle in a platoon is given as [13]:

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= f_i(v_i, a_i) + g_i(v_i) \eta_i \end{aligned} \quad (3)$$

where  $v_i$  and  $a_i$  stand for the velocity and acceleration of the  $i$ th vehicle, respectively and  $\eta_i$  represents the engine input. The functions  $f_i$  and  $g_i$  are given by the following equations:

$$f_i(v_i, a_i) = -\frac{2K_{di}}{m_i} v_i a_i - \frac{1}{\tau_i(v_i)} \left[ a_i + \frac{K_{di}}{m_i} v_i^2 + \frac{d_{mi}}{m_i} \right] \quad (4)$$

$$g_i(v_i) = \frac{1}{m_i \tau_i(v_i)}$$

where  $m_i$  is the vehicle mass,  $\tau_i$  is the time constant of its engine,  $K_{di}$  is the aerodynamic drag coefficient and  $d_{mi}$  is the mechanical drag force [13]. In order to obtain a control system structure, we define

$$\eta_i = m_i u_i + K_{di} v_i^2 + d_{mi} + 2\tau_i K_{di} v_i a_i \quad (5)$$

where  $u_i$  represents the external input signal of the  $i$ th vehicle. Substituting (5) and (4) in (3) and assuming that  $\tau_i$  is constant, the resulting linearized vehicle model will be

$$\begin{pmatrix} \dot{a}_i \\ \dot{v}_i \\ \dot{u}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ a_i \\ -\frac{1}{\tau_i} a_i + \frac{1}{\tau_i} u_i \end{pmatrix}, i \in S_m \quad (6)$$

This model is actually derived by applying pre-compensator to a more complex model by input-output linearization with state feedback [11]. To reach a state space model of the followers in the platoon, a new control input  $\xi_i$  is defined as follows [2], [14]:

$$h\dot{u}_i = -u_i + \xi_i \quad (7)$$

$$\xi_i = \begin{cases} K_p e_i + K_d \dot{e}_i + u_{i-1}, & \forall i \in S_m \\ u_r, & i = 0 \end{cases} \quad (8)$$

Here,  $u_r$  stands for the input corresponding to the desired acceleration of the leader; the input  $u_{i-1}$  is received over the wireless communication between the vehicles  $i$  and  $i-1$ . Finally, string stable reference dynamics for the heterogeneous platoon is obtained as [2], [14]

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \\ \dot{u}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/\tau_i & 1/\tau_i \\ K_p/h & -K_d/h & -K_d & -1/h \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \\ u_i \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_d/h & 1/h \end{pmatrix} \begin{pmatrix} v_{i-1} \\ u_{i-1} \end{pmatrix} \quad (9)$$

Here,  $\tau_i$  is the time constant representing the driveline dynamics of the  $i$ th vehicle in seconds. Equation (9) can be generalized as;

$$\dot{x}_i = A_i x_i + B w_i, \forall i \in S_m \quad (10)$$

where  $x_i$  is the  $i$ th vehicle's state vector and  $w_i$  represents the external input vector. Since the leading vehicle follows the reference signal only, its state space model will be:

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \\ \dot{u}_0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/\tau_0 & 1/\tau_0 \\ 0 & 0 & 0 & -1/h \end{pmatrix} \begin{pmatrix} e_0 \\ v_0 \\ a_0 \\ u_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/h \end{pmatrix} u_r \quad (11)$$

### II.3 Simulation example

Based on Eq. (10), different definitions are proposed in the literature for SS and stability analyses are performed accordingly [2, 14]. However, the common attitude is that the stability is studied by observing the change in distance error, velocity and/or acceleration. At this point we recall that we define SS as the condition that the absolute change in the abovementioned dynamics attenuates as it propagates along the vehicle string and relate the heterogeneity of the string to different time constants of vehicle engines in the platoon. As a novelty of this study, we do not search for controllers to compensate for dynamic differences between the vehicles for SS; instead, we arrange the platoon formation for optimum stability. That is, the vehicle with the highest inertia leads the platoon, whereas the one with the lowest inertia is the last follower.

We formed a sample heterogeneous 4-vehicle platoon where we determined the time constants of the members as given in Table I. The number of the vehicles in the string was limited to 4 for the sake of simplicity and the fact that longer strings would not be practical [12].

TABLE I  
TIME CONSTANTS ( $\tau_i$  [s]) CHOSEN FOR A SAMPLE 4-VEHICLE  
HETEROGENEOUS PLATOON

$i$	0	1	2	3
$\tau_i$	0,1	0,2	0,4	0,5

A 60-seconds drive cycle was created from stand-still, acceleration, deceleration to full-stop as shown in Figure 2.

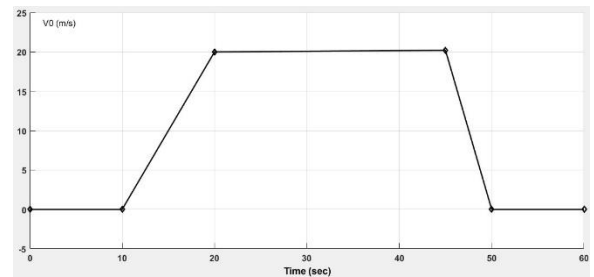


Fig. 2. Sample drive cycle

### III. THE PLATOON FORMATION ALGORITHM

We developed a formation algorithm which takes the dynamics of all vehicles in the string into consideration. For this purpose, the state-flow method was used in Matlab(R) environment. The algorithm includes the operation modes, interfaces, transition rules and actions described below. The corresponding flowchart for a single vehicle is presented in Fig. 3.

III.1 Operation Modes

In a 4-vehicle platoon, a vehicle can only be in one of the following operation modes at a time:

1. Manual Drive (MD)
2. Free Agent (FA)
3. Platoon Leader (PL)
4. Follower-1 (F1)
5. Follower-2 (F2)
6. Follower-3 (F3)

Possible transitions between modes of operation are listed in Table II, where rows and columns indicate the modes before and after the transition, respectively.

TABLE II  
OPERATION MODES AND TRANSITIONS

	MD	FA	PL	F1	F2	F3
MD		X				
FA	X		X	X		X
PL		X		X		
F1		X	X		X	
F2		X		X		X
F3		X			X	

III.2 Interfaces

Event triggers are:

1. Brake pedal
2. Accelerator pedal
3. Manual steering
4. Vehicle failure

System inputs are:

1. Enable switch
2. Join acknowledge
3. Split acknowledge

System outputs are:

1. Join request
2. Split request

III.3 Transition rules

The transition rules are determined as follows:

From manual drive to free agent mode:

1. No failure
2. Vehicle speed above 30km/h
3. Driver should press enable switch

From free agent mode to leader position:

1. Join request should be accepted (either sent or received over V2V)
2. Should have the smallest time constant among the platoon members

From free agent to the *i*th follower:

1. Join request should be accepted (either sent or received over V2V)
2. Must have a time constant greater than that of the (i-1)th vehicle and smaller than that of the (i+1)th vehicle.

III.4 Actions

Manual drive:

1. Stop V2V broadcasting,
2. Switch to free agent when conditions are fulfilled.

Free agent:

1. Start V2V broadcasting,
2. Switch to manual drive when conditions are fulfilled,
3. Switch to leader when conditions are fulfilled,
4. Switch to follower when conditions are fulfilled.

Follower vehicle:

1. Keep the following distance,
  2. Split when needed,
  3. Switch to leader when conditions are fulfilled,
- Switch to manual drive when conditions are fulfilled.

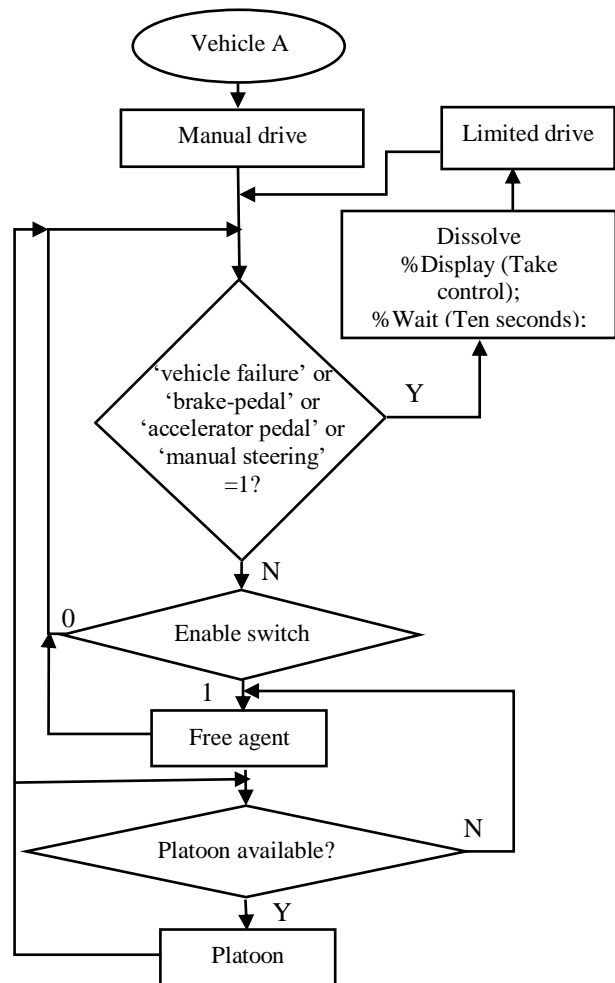


Fig. 3. Flow chart for a single vehicle (e.g. vehicle A)

Leader:

1. Broadcast platoon ID,
  2. Broadcast target gap,
  3. Switch to follower when conditions are fulfilled,
- Dissolve the platoon when conditions are fulfilled.

Fig. 4 illustrates the join to platoon based on vehicle dynamics.

#### IV. SIMULATION RESULTS

In this section, we demonstrate how the proposed algorithm improves the string stability by considering the vehicle dynamics in contrast to a platoon disregarding this fact. Four vehicles with different dynamics listed in Table 1 are run in the simulation environment, each time arranging them in a different order to observe the difference in the string stability behavior of the platoon. The error signal is considered as the difference between the reference following-distance (determined as 5 meters in this study) and the actual one.

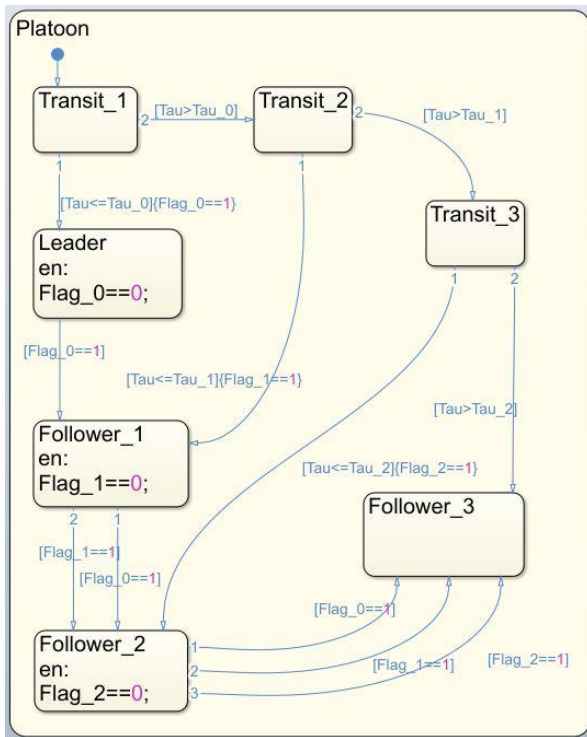


Fig. 4. Join to platoon based on vehicle dynamics (Tau:  $\tau$ )

According to the definition of the string stability, the error signal should attenuate as it propagates along the vehicle string. We note that only the positive values of the error signal have to be taken into consideration, since this corresponds to the condition that vehicles are getting closer than allowed. Thus, these are the only values that have potential to reduce the driving safety of the vehicles in the platoon.

The simulation results are summarized in Figure 5. The stability of the string is analyzed based on the error propagation results. Here, the inspection is focused on how the following-distance changes as we move through the platoon. As an example, for scenario A, error signals remain positive as

Scenario	Vi	Min. Following Distance between V1-V2 (m)		Min. Following Distance between V2-V3 (m)		Min. Following Distance between V3-V4 (m)		String Stable?
		V1	V2	V2	V3	V3	V4	
A	1/τi	10	5	5	2,5	2,5	2	String Stable? YES
B	1/τi	10	2	2	2,5	2,5	5	String Stable? No
C	1/τi	2	2,5	2,5	5	5	10	String Stable? No
D	1/τi	2	10	5	5	2,5	2,5	String Stable? Yes

Fig. 5. Minimum following-distances and string stability for different scenarios

we move through the string. The minimum following-distances have approximately the same value (5.1m and 5.2m) being greater than the reference value set at 5m. Thus, we can conclude that the platoon in scenario A is string-stable. The vehicle following-distances for this scenario are depicted in Figure 6.

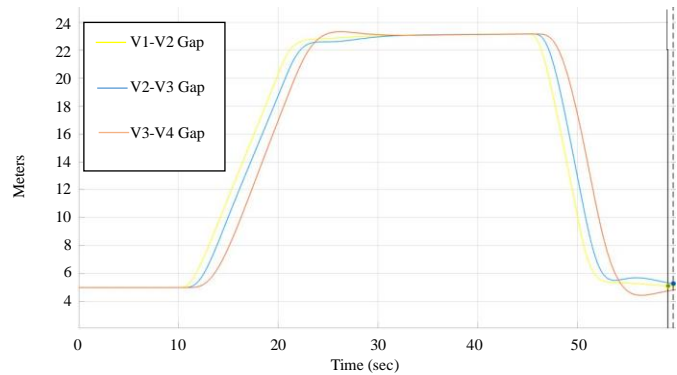


Fig. 6. Following-distances of the vehicles in scenario A

In Figure 7, vehicle following-distances are shown for the platoon in scenario B. The error signal goes to negative as we move through the string ( $e_i$ : 5.2-5=0.2m, 4.5-5=-0.5m, 4.4-5=-0.6m, respectively); that is, the following-distance decreases when we move towards the back of the platoon. Thus, we can say that the platoon in scenario B is not string-stable.

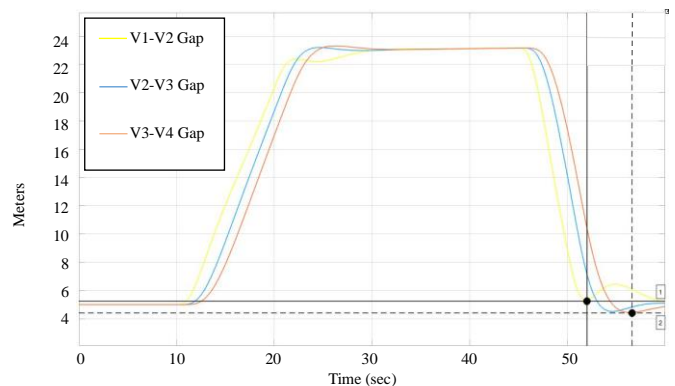


Fig. 7. Following-distances of the vehicles in scenario B

Figure 8 presents the graphics for the vehicle following-distances for the platoon in scenario C. Though the following-distance between the third couple of vehicles (V3&V4) is greater than that between the second couple (V2&V3), the error signal is negative along the whole string. Thus, we can disclose that the platoon in scenario C is not string stable.

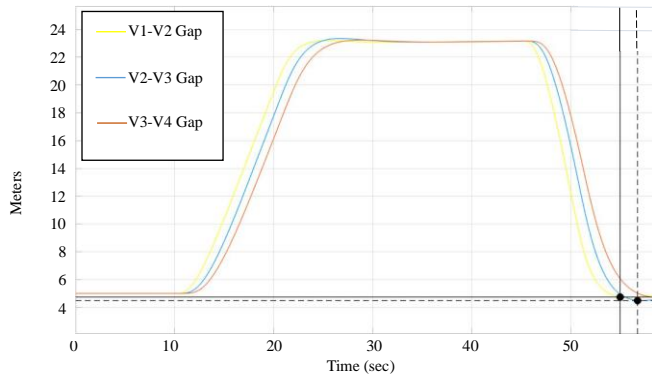


Fig. 8. Following-distances of the vehicles in scenario C

Finally, Figure 9 illustrates the vehicle following-distances for the platoon in scenario D. The error signal being negative between the leader and its follower (3.9-5=-1.1m) turns to positive as we move through the platoon. Thus, we can conclude that the platoon in scenario D is string stable.

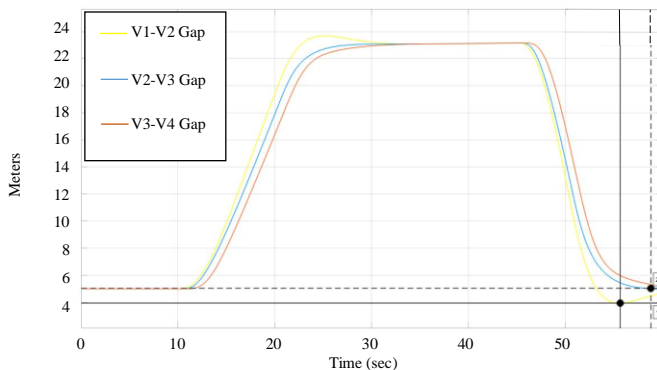


Fig. 9. Following-distances of the vehicles in scenario D

## V. CONCLUSIONS

This work has focused on improving string stability of heterogeneous platoons by implementing a new algorithm: The platoon formation. This algorithm takes the dynamics of the platoon members into consideration and decides on the position of each member regarding its inertia in contrast to algorithms in the literature, which apply complex control schemes by implementing pre-compensators at vehicle level. The proposed algorithm is studied under different platoon scenarios. Based on the simulation results, we can conclude that when constructing a platoon, taking each member's dynamics into consideration helps to improve the string stability. Obviously, it is possible to compensate the differences of each vehicle's longitudinal dynamics by implementing pre-filters to longitudinal controllers of the

vehicles, but that would limit the overall performance of the platoon. This method is considered to be an alternative to low level adaptive controllers. Even for the string stable platoons, the proposed formation algorithm has potential to further decrease the following-distances which results in improvement of overall fuel economy and road usage. Exchanging vehicle information (load, power, brake performance etc.) between the vehicle members is a part of draft platoon standards, which makes the algorithm possible for real world applications.

The next step as the future work is that the algorithm will be extended to cover the determination of the correct time that the free agent will join the platoon.

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## BIOGRAPHIES



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