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Dynamic Conditional Dependence for Turkey Earthquake Data: CD Vine Copula Approach

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ABSTRACT

The objective of this research was to use Turkey's major fault zones, which are located on fault lines, to describe the dependency structure. The current study also intended to show the dynamic structure of the conditional dependencies of the earthquake data in Turkey in terms of depth and magnitude, using the CD-vine copula method. Conditional dependence, also known as the CD-vine method, makes obtaining a complex dependency structure simpler. The current research uses 30 years of data from the Eastern Anatolian Fault line and the North Anatolian Fault line to describe the dynamic conditional dependency structure. As a consequence, this dependency structure is represented graphically and numerically in tables and graphs.

1. Introduction

[1] Investigated the dynamics co movement between capital markets in ASEAN Exchanges for during period of 2012-2013 and studied based on C-D Vine copula approach found the dynamics co-movement among of capital markets in ASEAN Exchanges. [2] stuided focuses on the pattern of relation among major world exchanges such as the United States stock market (S & P 500), European stock markets (the united kingdom and German), Asia stock markets (Japan and China) and ASEAN stock markets (Indonesia and Philippine) in the pre-crisis period (2000 ~ 2008) and post-crisis period (2000 ~ 2016). Using the C-D Vine copula approach and in their results, they found the dynamic change between stock markets in the world financial markets. [3] used the vine copula analysis to analyze the composite stock price index and some of the macroeconomics (inflation, IDR to USD exchange rate and interest rate). In addition, the dependent structure was applied to obtain the common density function and they calculated the expected value as a copula regression model. [4] presented, the canonical vine (C-vine) and D-vine copulas provide functions and tools for the extraction of stolen R-package presents CDVine. [5] examined the daily informational structure and common movements of the Thai Baht (THB) and Malaysian Ringgit (MYR) currencies in the period 2006-2013. [6] proposed vine copula mixes for the dependency structure hidden in multidimensional data. They tested a finite number of C- and D-vine model mixtures using the same copula family and finally studied the original CD-vine model, which enables the analysis of the dependency structure between many variables. [7] examined the relationship between bitcoin and other cryptocurrency indicators using the CD Vine Approach method.[8] To uncover and fully understand complex and latent dependence patterns in multivariate data, they proposed a D-vine copula mix that incorporates D-vine copula into a finite mix model. In addition, the model they propose can facilitate the scope of their multivariable complex and confidential content models.[9] presented the design of the R package copula, other implementation details.[10] have presented an intuitive systemic risk model for analyzing complex complex problems between different borrowers.

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Then, they used the state-of-the-art canonical (C-) and D-vine bridge to investigate the correlation structure between the divisions between the groups. [11] have used vine copula models to capture climate-efficiency dependency patterns, including the occurrence of extreme events (i.e. tail dependencies). Inc et al. [12] obtained approximate solutions of nonlinear time-dependent generalized Fitzhugh–Nagumo equation with time-dependent coefficients and Sharma–Tasso–Olver equation subjected to certain initial conditions and showed that this method is efficient and convenient; thus it can be applied to variety of problems. The approximate solutions are compared with the exact solutions. Then, Acay and Inc [13] proposed non-local singular fractional operators and examined this model, which has a very important place in everyday life. In 2020, Houwe et al. [14] studied analytical solutions of nonlinear differential equations (DED) with fractional derivatives and used the discrete tanh method for the calculations. In addition, Akinlar et al. [15] take an epidemic system with additional fractional white noise, build a new SIRS model and mix it into the fractional model, to show that epidemics can be modeled more competently in fractional-stochastic environments than those modeled by fractional-stochastic environments. deterministic differential equations. -stochastic systems and their chaotic behavior at unscathed and endemic equilibrium points have been studied. Later, Akınlar et al. [16], optimal control formulations, digital solutions, stability analysis for the fractional malchus model is considered a new contribution because it is studied for the first time in this article. Later in the same year, Korpınar et al. [17] analyzed the fractional-cubic-cubic-stochastic non-linear-stochastic schrödinger equation, which describes the propagation of the solitons through the optical fibers, and used it to obtain stochastic solutions in space White noise with hermit transform. In addition, Hashemi et al. [18] used the diagram of Adams-Bashforth-Moulton (ABMS) to determine the approximate solution of the fractional three-dimensional chaotic process at varying degrees, showing the results of the simulation. In this study, it is a question of explaining the dependency structure using the Copulule CD-Vigne approach for the main zones of faults located on the fault lines of Turkey. Conditional dependency, in other words CD-vine approach, provides convenience in obtaining the complex dependency structure. The present study explains the dynamic conditional dependency structure using 30 years of data of the regions on the Eastern Anatolian Fault line and the North Anatolian Fault line. As a result, this dependency structure is presented in graphics and tables.

2. Materials and Methods

Definition 2. 1. 1.Copula Function
A copula is a multivariate distribution whose marginals are all uniform over [0, 1]. For a *n*-dimensional vector *U* on the unit hyper cube, a copula *C* is defined as,

$$C(u_1,u_2,...,u_n)=\Pr\left(U_1\le u_1,U_2\le u_2,...,U_n\le u_n\right) \quad (1)$$

This definition is the main result of the Sklar's theorem [17], namely building block of the theory of copulas, given below.
Theorem 2.1.2. (Sklar's Theorem) Let *F* be a *n*-dimensional distribution function with univariate margins $F_1,F_2,...,F_n$.

Let A_i define the range of F_i and $A_i=[-\infty,\infty]$ where $i=1,2,...,n$.At that case, there exists a copula function C whole $(x_1,x_2,...,x_n)\in[-\infty,\infty]$

$$F(x_1,x_2,...,x_n)=C\left(F(x_1),F(x_2),...,F(x_n)\right) \quad (2)$$

where the random variables $(X_1,X_2,...,X_n)$ are assumed to be continuous.

Definition 2. 1. 2. Kendall Tau and Spearman Rho
rank correlations are defined as Spearman ρ and Kendall τ
For both rank based correlation measures, the general term in their formulations is the rank of the observation, indicated below

$$\widehat{\rho}_S(X,Y)=12\iint_U\left(C(u,v)-uv\right)dudv \quad (3)$$

$$\widehat{\rho}_\tau(X,Y)=4\iint_U C(u,v)\partial C(uv)-1 \quad (4)$$

where, $U=I^2$ is the unique square.

Definition 2. 1. 3.Tail Dependence
The case of queue dependency is directly related to the extreme value relation defined mainly as a function of the queues. Suppose *X* and *Y* are two random variables having distributions F_X and F_Y respectively. Therefore, two important asymptotic measures for tail dependence, called upper and lower tail dependency coefficients, are mentioned below.

$$\lambda_l=\lim_{u\to 0}P(\mathrm{F}_X(x)\le u\mid F_y(x)\le u)=\lim_{u\to 0}C(u,u)/u \quad (5)$$

$$\lambda_u=\lim_{u\to 1}P(\mathrm{F}_X(x)>\mathrm{u}\mid F_y(x)>u)=\lim_{u\to 1}1-2u-C(u,u)/1-u \quad (6)$$

where λ_l and $\lambda_u\in[0,1]$.

Definition 2. 1. 4. Elliptical Copulas
Let *F* be the multivariate cumulative distribution function (cdf) of an elliptical distribution. Let F_i be CDF of the *i*'th margin and F_i^{-1} be its inverse function for $i=1,2,...,n$, the elliptical copula determined by F is;

$$C(u_1,u_2,...,u_n)=F\left[F_1^{-1}(u_1)+...+F_n^{-1}(u_n)\right] \quad (7)$$

For example; normal copulas (derived from the bivariate normal with zero mean, unitary and correlated variance) and Student's t pairs (derived from the bivariate t distribution with zero mean, ν degrees of freedom and association) are two types of elliptic families.

Definition 2. 1. 5. Archimedean Copulas

An Archimedean copula is built using a generator ϕ as;

$$C(u_1,u_2,...,u_n)=\phi^{[-1]}\left[\phi_1(u_1)+...+\phi_n(u_n)\right] \tag{8}$$

where, $\phi^{[-1]}$ is the pseudo-inverse of the generator, ϕ defined by,

$$\phi^{[-1]}=\begin{cases}\phi^{-1}(t) & 0\le t\le\phi(0)\\ 0 & \phi(0)\le t\le\phi(\infty)\end{cases} \tag{9}$$

In bivariate case,

$$C(u_1,u_2)=\phi^{[-1]}\left(\phi(u_1)+\phi(u_2)\right) \tag{10}$$

defines the so-called Archimedean bivariate copula function.

Definition 2.1.6. Vine Copulas

Let $T=U_1,U_2,...,U_{n-1}$ denote the regular vine for *n* variables, where U_i is a connected tree with nodes $M_i=1,2,...,n$ and edges E_i for $i=2,...,n-1$. In this tree structure, U_i is a connected tree with nodes $M_i=E_{i-1}$ A regular vine with p variables is a vine where two edges in tree *i* are connected by an edge in tree *i + 1*, only if these edges share a common node. In general, there are totally $n(n-2)/2$ possible edges in a regular vine for *n* variables [19].

Definition 2.1.7. Conditional Copula

Given [19]

$$f(x_1,x_2,...,x_n)=\left(\sum_{k=2}^{n}f(x_k\mid x_1,x_2,...,x_{k-1})\right)f_1(x_1) \tag{11}$$

and for distinct values of $i,j,i_1,...,i_m$ with i < j and $i_1<...<i_m$ and describe

$$c_{i,j|i_1,i_2,...i_m}=c_{i,j|i_1,i_2,...i_m}\left(F(x_i\mid x_{i1},...,x_{im}),F(x_j\mid x_{i1},...,x_{im})\right) \tag{12}$$

where *f* and *c* respectively define the marginal probability density function (pdf.) and the copula density function. Then the conditional pdf can be written.;

$$f(x_k\mid x_1,x_2,...,x_{k-1})=c_{1,k|2,...,k-1}f(x_k\mid x_2,...,x_{k-1})=\sum_{q=1}^{k-2}\left(c_{q,k|q+1,...,k-1}\right)c_{k-1,k}f_k(x_k) \quad ...(13)$$

by using equation (12) and (13) writing,

$$f(x_1,x_2,...,x_n)=\left(\sum_{j=1}^{n-1}\sum_{i=1}^{n-j}c_{i,i+j|i+1,...,i+j-1}\right)\sum_{m=1}^{n}f_m(x_m). \tag{14}$$

CD Vine Copula

A vine copula structure is simply defined as a nested set of trees that define binary copula functions unconditionally in the first tree and conditionally for the rest of the related trees. This structure is as follows;

Definition 2.1.8.C Vine Copula

It is a sort of regular vine distribution where each tree has a unique node connected to all the other nodes in the tree. It only uses star shaped trees and is useful for sorting by importance. The corresponding probability density function (pdf) can be written as:

$$f(x_1,x_2,...,x_n)=\prod_{k=1}^{n}f(x_k)\prod_{j=1}^{n-1}\prod_{i=1}^{n-j}c_{j,j+i|1,\dots,j-1}\left\{F\left(x_j|x_1,\dots,x_{j-1}\right),F\left(x_{j+i}|x_1,\dots,x_{j-1}\right)\right\}. \tag{15}$$

Definition 2.1.8.D Vine Copula

D-vine is another special case for the normal vine tree structure which has no nodes in any tree but connects to more than two edges. It uses paths like trees and is useful for the temporal ordering of variables. The density function (pdf) can be written:

$$f(x_1,x_2,...,x_n)=\prod_{k=1}^{n}f(x_k)\prod_{j=1}^{n-1}\prod_{i=1}^{n-j}c_{i,i+j|i+1,\dots,i+j-1}\left\{F\left(x_i|x_{i+1},\dots,x_{i+j-1}\right)F\left(x_{i+j}|x_{i+1},\dots,x_{i+j-1}\right)\right\}.$$

3. Results

We use magnitude and depth of Eastern Anatolian fault line and North Anatolian fault line data of stations the period 1999-2019. Table 1 and Table 2 summarizes statistics of magnitude and depth of Eastern Anatolian fault line and North Anatolian fault line data. In Table 1, Table 2 and Figure 1, Figure 2 shows different mean values for the for data set, and the corresponding standard deviations are different. Skewness of data set is positive, indicating that this data is skewed right. The high kurtosis of data set reveals that extreme value changes often occur when the tail of return distributions shows fatness. The Jarque-Bera (JB) test shows that the normality of each return series distribution is strongly rejected at 0.05 level, which means all price index distributions are non-normal. The empirical distribution functions used in modelling the dependence of Eastern Anatolian fault line and North Anatolian fault line depth and magnitude data pairs are as shown in Figure 3,4,5,6,7,8,9,10 and Table 3,4. For this, we used C Vine and D Vine copula. In table 3,4,5 and Figure 1, 2, 3, 4 it is shown that for the relationship between North Anatolian fault line depth and North Anatolian fault line magnitude, in table 3,4,5 Figure5,6,7,8 it is shown that for the relationship between Eastern Anatolian fault line depth and Eastern Anatolian fault line magnitude, we use C Vine and D Vine copula modelling. In Table 3,4, the preferred copula families for branching and the parameter summaries of these families are given. In Figure figure 3,4,5,6,7,8,9,10 for C Vine, D Vine matrix summaries and appropriate branch graphs are shown. From table 3,4,5 Figure 3,4,5,6,7,8,9,10 for dependency structure of North Anatolian fault line depth, C vine branching was found to be appropriate according to the information criteria. Similarly, for North Anatolian fault line magnitude data. From table 3,4,5 Figure 3,4,5,6,7,8,9,10, for dependency structure of North Anatolian fault line magnitude, C vine branching was found to be appropriate according to the information criteria, for Eastern Anatolian fault line depth data, in table 3,4,5 Figure 3,4,5,6,7,8,9,10, for dependency structure of Eastern Anatolian fault line depth, D vine branching was found to be appropriate according to the information criteria and for Eastern Anatolian fault line magnitude data, in table 3,4,5 Figure 3,4,5,6,7,8,9,10 for dependency structure of Eastern Anatolian fault line magnitude, D vine branching was found to be appropriate according to the information criteria.

Table 1. Summary Statistics for Eastern Anatolian fault line

	Adıyaman	Elazığ	K.Maraş	Malatya	Şırnak
Depth					
Mean	8,243636	16,15455	18,00909	17,65455	15,19273
Maksimum	15,35000	45,40000	64,20000	45,40000	35,00000
Minumum	2,600000	2,000000	6,900000	9,500000	5,000000
Std.Dev	3,393636	15,43421	16,86265	11,13008	10,19934
Skewness	0,344330	0,929670	2,087951	1,594602	1,348772
Kurtosis	3,165211	2,248423	6,270340	4,557203	3,287966
Jarqure Bera	0,229876	1,843424	12,89442	5,773121	3,373180
Probability	0,891421	0,397837	0,001585	0,055768	0,185150
Magnitude					
Mean	4,500000	4,309091	4,145455	4,318182	4,300000
Maksimum	5,500000	5,100000	4,500000	5,100000	5,100000
Minumum	4,000000	4,000000	4,000000	4,000000	4,000000
Std.Dev	0,473286	0,311302	0,180907	0,345885	0,354965
Skewness	0,908170	1,601171	0,874018	1,192958	1,097454
Kurtosis	2,771923	4,845286	2,344167	3,250882	3,163643
Jarqure Bera	1,535924	6,260866	1,597635	2,637954	2,220351
Probability	0,463958	0,043699	0,449861	0,267409	0,329501

Table 2. Summary Statistics for North Anatolian fault line

	Çankırı	Düzce	Erzincan	İzmit	Sakarya
Depth					
Mean	11,04286	20,00000	27,87143	13,51429	17,64286
Maksimum	22,30000	65,60000	66,10000	22,60000	56,00000
Minumum	7,000000	10,00000	7,000000	7,000000	3,700000
Std.Dev	5,110074	20,70394	23,37404	5,863852	17,79671
Skewness	1,791256	1,828218	0,512661	0,553095	1,668841
Kurtosis	4,681279	4,624956	1,852352	1,783947	4,292637
Jarqure Bera	4,567817	4,669584	0,690778	0,788212	3,736549
Probability	0,101885	0,096831	0,707945	0,674283	0,154390
Magnitude					
Mean	4,042857	4,685714	5,200000	4,928517	4,300000
Maksimum	4,200000	7,100000	6,600000	7,600000	4,500000
Minumum	4,000000	4,000000	4,600000	4,000000	4,100000
Std.Dev	0,078680	1,080785	0,797914	1,237894	0,163299
Skewness	1,357727	1,916392	0,822842	1,652398	0,248039
Kurtosis	3,233728	4,893136	2,175256	4,331803	1,421875
Jarqure Bera	2,166593	5,329972	0,988305	3,702817	0,798167
Probability	0,338478	0,069600	0,610088	0,157016	0,670935

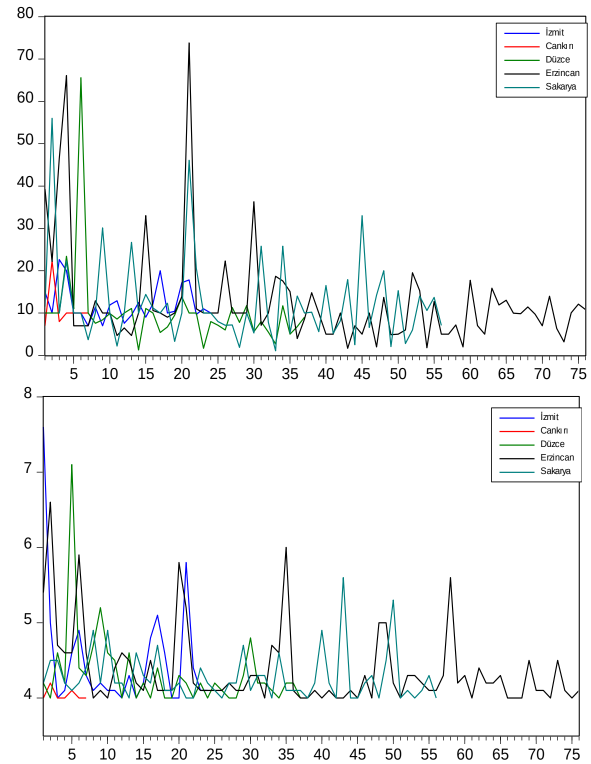

Figure 1. Respectively, Depth and Magnitude of North Anatolian fault line.

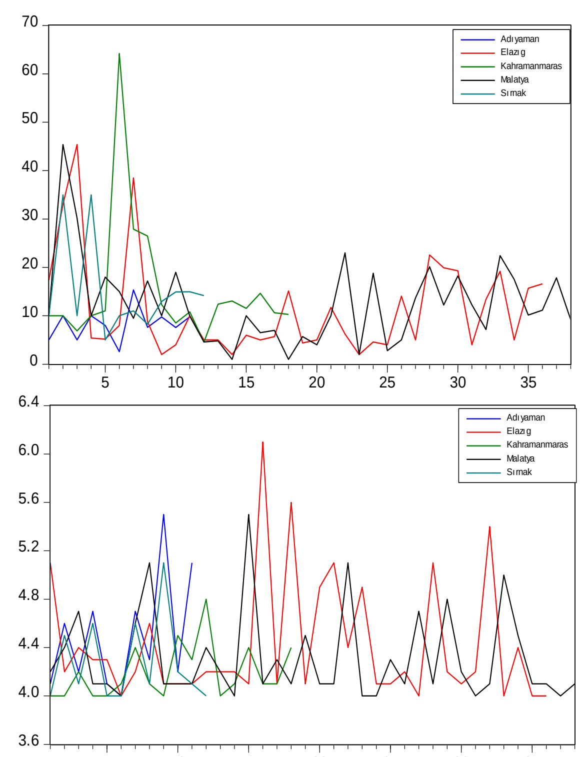

Figure 2. Respectively, Depth and Magnitude of Eastern Anatolian fault

Table 3. C-vine copula estimation results.

Depth of North Anatolian fault line	**Copula**	**Par1**	**Par2**	**Tau**	**Lower tail dependency**	**Upper tail dependency**
$C_{\text{Çankırı}-\text{İzmit}}$	I	-	-	0.00	-	-
$C_{\text{Düzce}-\text{İzmit}}$	C	2.88	0.00	0.59	-	0.79
$C_{\text{Sakarya}-\text{İzmit}}$	C	1.05	0.00	0.34	-	0.52
$C_{\text{Erzincan}-\text{İzmit}}$	C	0.56	0.00	0.22	-	0.29
$C_{\text{Erzincan,Çankırı/İzmit}}$	I	-	-	0.00	-	-
$C_{\text{Erzincan,Düzce/İzmit}}$	Tawn	18.00	0.90	0.85	0.89	-
$C_{\text{Erzincan,Sakarya/İzmit}}$	Tawn2_180	16.10	0.90	0.85	-	0.89
$C_{\text{Sakarya,Çankırı,Erzincan/İzmit}}$	I	-	-	0.00	-	-
$C_{\text{Sakarya,Düzce,Erzincan/İzmit}}$	SBB6	6.00	6.00	0.95	-	0.98
$C_{\text{Düzce,ÇankırıSakarya,Erzincan/İzmit}}$	I	-	-	0.00	-	-
Magnitude of North Anatolian fault line	**Copula**	**Par1**	**Par2**	**Tau**	**Lower tail dependency**	**Upper tail dependency**
$C_{\text{Çankırı}-\text{İzmit}}$	I	-	-	0.00	-	-
$C_{\text{Düzce}-\text{İzmit}}$	C	2.88	0.00	0.59	-	0.79
$C_{\text{Sakarya}-\text{İzmit}}$	N	0.03	0.00	0.02	-	-
$C_{\text{Erzincan}-\text{İzmit}}$	C	0.56	0.00	0.22	-	0.29
$C_{\text{Erzincan,Çankırı/İzmit}}$	I	-	-	0.00	-	-
$C_{\text{Erzincan,Düzce/İzmit}}$	Tawn	18.00	0.90	0.85	0.89	-
$C_{\text{Erzincan,Sakarya/İzmit}}$	J	8.48	0.00	0.79	0.91	-
$C_{\text{Sakarya,Çankırı/Erzincan İzmit}}$	I	-	-	0.00	-	-
$C_{\text{Sakarya,Düzce/Erzincan İzmit}}$	C90	-0.96	0.00	-0.32	-	-
$C_{\text{Düzce,Çankırı/Sakarya,Erzincan İzmit}}$	I	-	-	0.00	-	-
Depth of Eastern Anatolian fault line	**Copula**	**Par1**	**Par2**	**Tau**	**Lower tail dependency**	**Upper tail dependency**
$C_{\text{Elazığ}-\text{Şırnak}}$	Tawn2_90	-5.32	0.06	-0.06	-	-
$C_{\text{Malatya}-\text{Şırnak}}$	Tawn90	-20.00	0.30	-0.29	-	-
$C_{\text{Adıyaman}-\text{Şırnak}}$	Tawn2	5.45	0.47	0.42	0.46	-
$C_{\text{Kahramanmaraş}-\text{Şırnak}}$	Tawn2_270	-18.59	0.10	-0.10	-	-
$C_{\text{Kahramanmaraş,Elazığ/Şırnak}}$	Tawn2_270	-20.00	0.06	-0.06	-	-
$C_{\text{Kahramanmaraş,Malatya/Şırnak}}$	J90	-1.24	0.00	-0.12	-	-
$C_{\text{Kahramanmaraş,Adıyaman/Şırnak}}$	Tawn	4.17	0.43	0.38	0.43	-
$C_{\text{Adıyaman,Elazığ,Kahramanmaraş/Şırnak}}$	SBB1	0.93	1.00	0.32	0.48	0.00
$C_{\text{Adıyaman,Malatya,Kahramanmaraş/Şırnak}}$	Tawn90	-20.00	0.20	-0.20	-	-
$C_{\text{Malatya,Elazığ,Adıyaman,Kahramanmaraş/Şırnak}}$	t	0.07	2.00	0.04	0.20	0.20

Magnitude of Eastern Anatolian fault line	Copula	Par1	Par2	Tau	Lower tail dependency	Upper tail dependency
$C_{\text{Elazığ}-\text{Şırnak}}$	t	0.03	30.00	0.02	0.00	0.00
$C_{\text{Malatya}-\text{Şırnak}}$	J270	-1.07	0.00	-0.04	-	-
$C_{\text{Adıyaman}-\text{Şırnak}}$	SJ	15.94	0.00	0.88	-	0.96
$C_{\text{Kahramanmaraş}-\text{Şırnak}}$	SJ	15.94	0.00	0.88	-	0.96
$C_{\text{Kahramanmaraş,Elazığ/Şırnak}}$	C270	-0.00	0.00	-0.00	-	-
$C_{\text{Kahramanmaraş,}\textit{Malatya}\text{/Şırnak}}$	F	-0.88	0.00	-0.10	-	-
$C_{\text{Kahramanmaraş,}\textit{Adıyaman}\text{/Şırnak}}$	BB7_270	-1.02	-0.00	-0.01	-	-
$C_{\text{Adıyaman,Elazığ,Kahramanmaraş/Şırnak}}$	Tawn	2.76	0.32	0.26	0.30	-
$C_{\text{Adıyaman,}\textit{Malatya}\text{,Kahramanmaraş/Şırnak}}$	Tawn2_180	1.85	0.00	0.00	-	0.00
$C_{\textit{Malatya}\text{,Elazığ,Adıyaman,Kahramanmaraş/Şırnak}}$	G	1.09	0.00	0.08	0.11	-

Table 4. D-vine copula estimation results.

Depth of North Anatolian fault line	Copula	Par1	Par2	Tau	Lower tail dependency	Upper tail dependency
$C_{\text{Erzincan}-\text{Sakarya}}$	C	4.32	0.00	0.68	-	0.85
$C_{\text{Sakarya}-\text{İzmir}}$	C	1.05	0.00	0.34	-	0.52
$C_{\text{İzmir}-\text{Düzce}}$	C	2.88	0.00	0.59	-	0.79
$C_{\text{Düzce}-\text{Çankırı}}$	I	-	-	0.00	-	-
$C_{\text{İzmit},\text{Erzincan}/\text{Sakarya}}$	N	-0.78	0.00	-0.57	-	-
$C_{\text{Düzce},\text{İzmit}/\text{Sakarya}}$	Tawn2	20.00	0.93	0.89	0.92	-
$C_{\textit{Çankırı}\text{,İzmit/Düzce}}$	I	-	-	0.00	-	-
$C_{\textit{Düzce,Erzincan}\text{/Sakarya,}\textit{İzmir}}$	BB7_270	-1.00	-1.65	-0.45	-	-
$C_{\text{Çankırı,}\textit{Sakarya/Düzce,İzmir}}$	I	-	-	0.00	-	-
$C_{\textit{Çankırı}\text{,Erzincan/Düzce,}\textit{İzmir,Sakarya}}$	I	-	-	0.00	-	-

Magnitude of North Anatolian fault line	Copula	Par1	Par2	Tau	Lower tail dependency	Upper tail dependency
$C_{\text{Erzincan}-\text{İzmir}}$	C	0.56	0.00	0.22	-	0.29
$C_{\text{İzmir}-\text{Sakarya}}$	N	0.03	0.00	0.02	-	-
$C_{\text{Sakarya}-\text{Düzce}}$	C	0.02	0.00	0.01	-	0.00
$C_{\text{Düzce}-\text{Çankırı}}$	I	-	-	0.00	-	-
$C_{\text{Sakarya,İzmit/Erzincan}}$	J	8.48	0.00	0.79	0.91	-
$C_{\text{Düzce},\text{İzmit}/\text{Sakarya}}$	C	3.57	0.00	0.64	-	0.82
$C_{\text{Çankırı,Sakarya/Düzce}}$	I	-	-	0.00	-	-
$C_{\textit{Düzce,Erzincan}\text{/Sakarya,}\textit{İzmit}}$	Tawn90	-10.77	0.31	-0.30	-	-
$C_{\text{Çankırı,}\textit{İzmit/Düzce}\text{,Sakarya}}$	I	-	-	0.00	-	-
$C_{\textit{Çankırı}\text{,Erzincan/Düzce,}\textit{Sakarya}\text{,İzmit}}$	I	-	-	0.00	-	-

Depth of Eastern Anatolian fault line	**Copula**	**Par1**	**Par2**	**Tau**	**Lower tail dependency**	**Upper tail dependency**
$C_{Şırnak-Kahramanmaraş}$	Tawn90	-18.59	0.10	-0.10	-	-
$C_{Kahramanmaraş-Adıyaman}$	t	0.22	2.00	0.14	0.26	0.26
$C_{Adıyaman-Malatya}$	C270	-0.83	0.00	-0.29	-	-
$C_{Malatya-Elazığ}$	t	0.48	2.00	0.32	0.38	0.38
$C_{Adıyaman,Şırnak/\text{Kahramanmaraş}}$	J	2.88	0.00	0.50	0.73	-
$C_{Malatya,\text{Kahramanmaraş},/Adıyaman}$	C90	-0.70	0.00	-0.26	-	-
$C_{\text{Elazığ},Adıyaman/Malatya}$	J	1.20	0.00	0.10	0.21	-
$C_{Malatya,\text{Şırnak/Adıyaman,Kahramanmaraş}}$	Tawn2_270	-19.27	0.43	-0.42	-	-
$C_{\text{Elazığ,Kahramanmaraş}/Malatya,Adıyaman}$	Tawn	12.92	0.09	0.09	0.09	-
$C_{\text{Elazığ,Şırnak/Malatya,Adıyaman,Kahramanmaraş}}$	BB1_90	-1.13	-1.00	-0.36	-	-
Magnitude of Eastern Anatolian fault line	**Copula**	**Par1**	**Par2**	**Tau**	**Lower tail dependency**	**Upper tail dependency**
$C_{Adıyaman-Şırnak}$	SJ	15.94	0.00	0.88	-	0.96
$C_{Şırnak-Kahramanmaraş}$	SJ	15.94	0.00	0.88	-	0.96
$C_{Kahramanmaraş-Malatya}$	SC	0.13	0.00	0.06	0.00	-
$C_{Malatya-Elazığ}$	Tawn270	-5.29	0.12	-0.11	-	-
$C_{\text{Kahramanmaraş,Adıyaman/Şırnak}}$	BB7_270	-1.02	-0.00	-0.01	-	-
$C_{\text{Malatya,Şırnak/Kahramanmaraş}}$	C	0.00	0.00	0.00	-	0.00
$C_{\text{Elazığ,Kahramanmaraş/Malatya}}$	SJ	1.01	0.00	0.00	-	0.01
$C_{\text{Malatya,Adıyaman/Şırnak,Kahramanmaraş}}$	N	0.06	0.00	0.04	-	-
$C_{\text{Elazığ,Şırnak/Kahramanmaraş,Malatya}}$	Tawn2_180	20.00	0.01	0.00	-	0.01
$C_{\text{Elazığ,Adıyaman/Malatya,Kahramanmaraş,Şırnak}}$	N	0.16	0.00	0.10	-	-

Table 5. Comparison of the C-vine and D-vine.

Tree	**C vine**	**D vine.**
Depth of North Anatolian fault line		
Loglike	62.7	55.79
AIC	-107.39576	-95.57715
BIC	-86.42	-76.93
Magnitude of North Anatolian fault line		
Loglike	38.83	39.39
AIC	-63.66569	-60.78
BIC	-47.35	-44.47
Depth of Anatolian fault line		
Loglike	21.47	22.52
AIC	-10.93	-13.034703
BIC	20.18	13.17
Magnitude of Anatolian fault line		
Loglike	30267.22	30285.76
AIC	-60506.43	-60545.53
BIC	-60483.51	-60524.24

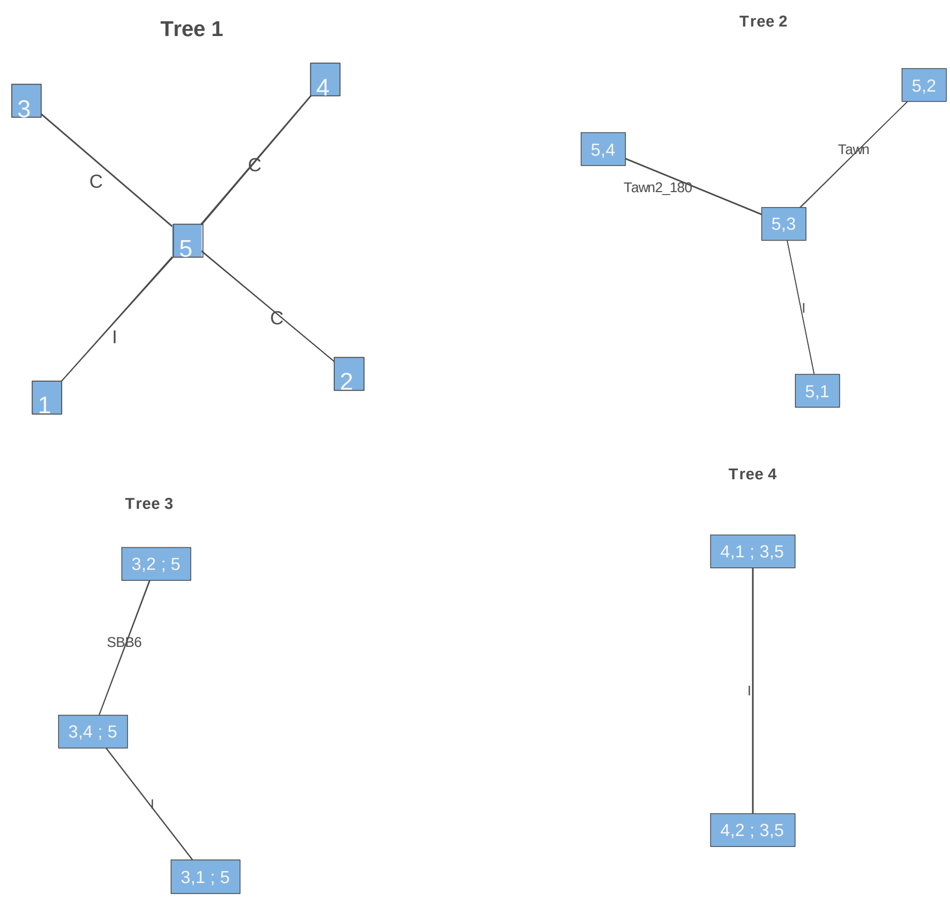

Figure 3. C Vine Copula Summary for Depth of North Anatolian fault line (1. Cankırı, 2. Düzce, 3. Erzincan, 4. Sakarya, 5. İzmit)

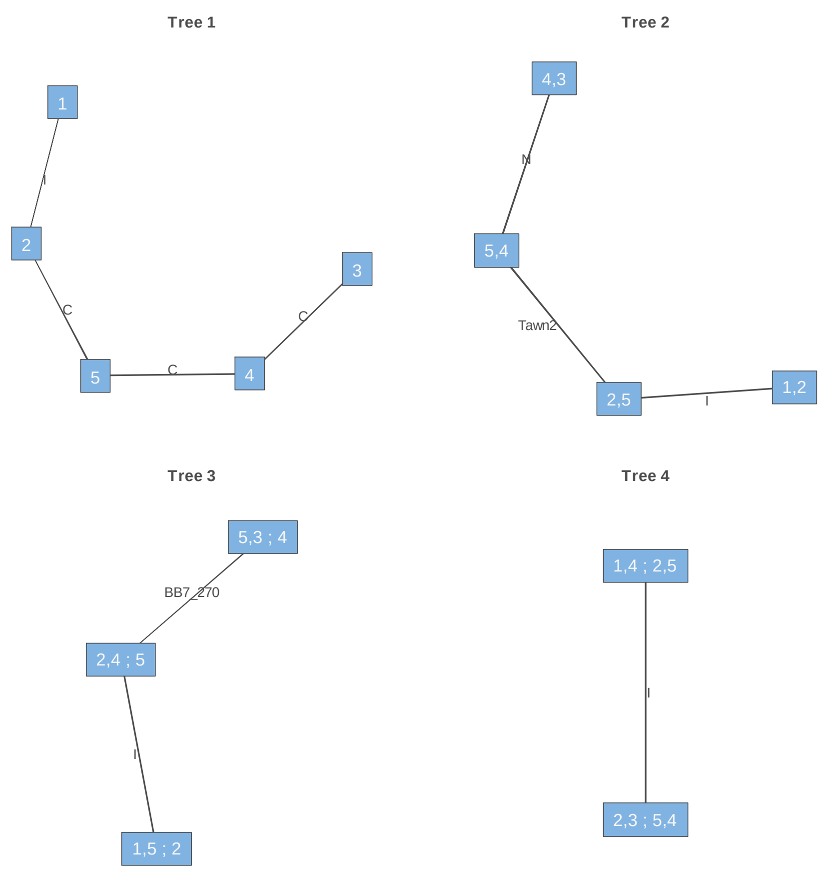

Figure 4. D Vine Copula Summary for Depth of North Anatolian fault line (1. Cankırı, 2. Düzce, 3. Erzincan, 4. Sakarya, 5. İzmit).

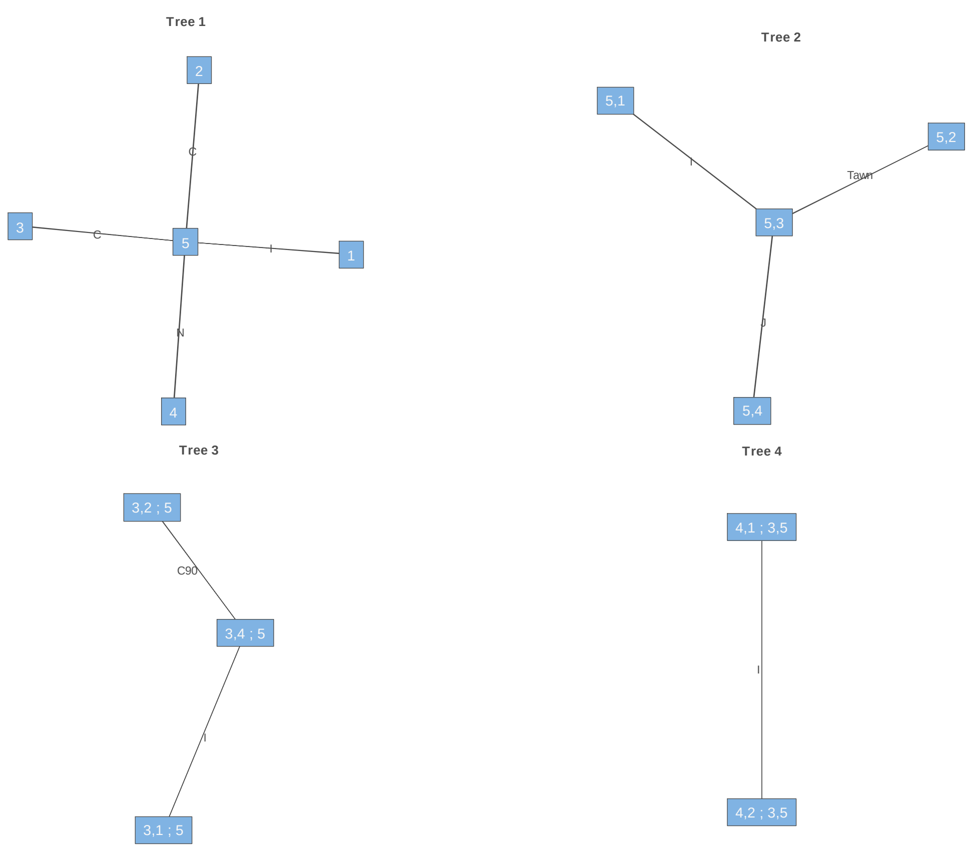

Figure 5. C Vine Copula Summary for magnitude of North Anatolian fault line (1. Cankırı, 2. Düzce, 3. Erzincan, 4. Sakarya, 5. İzmit).

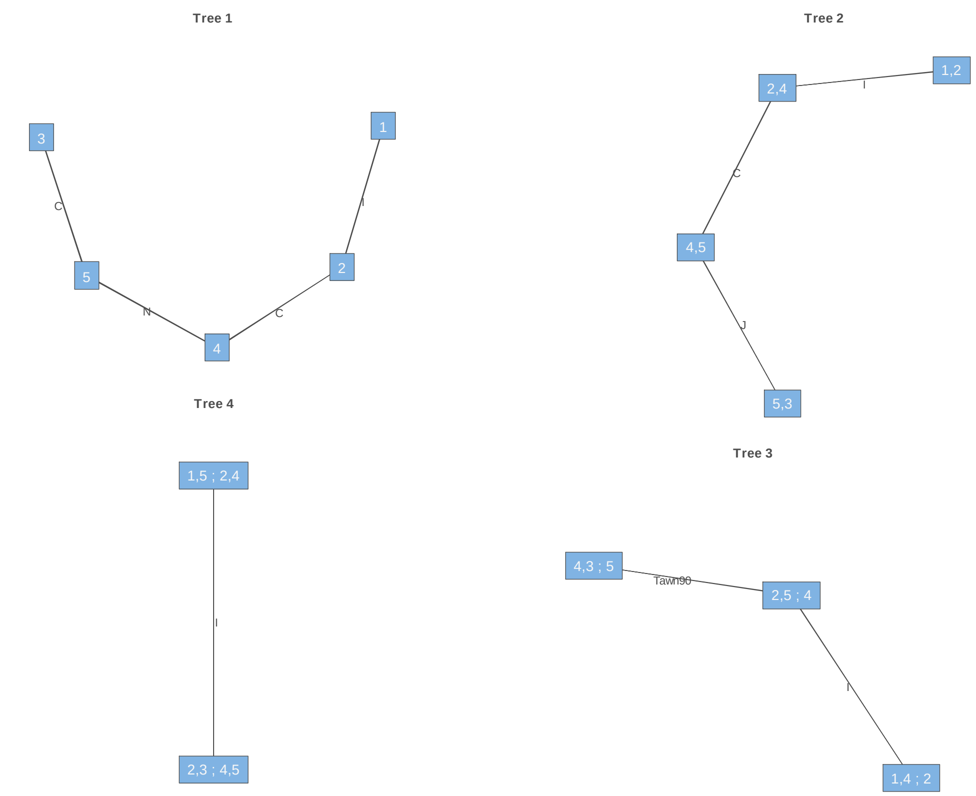

Figure 6. D Vine Copula Summary for Magnitude of North Anatolian fault line (1. Cankırı, 2. Düzce, 3 .Erzincan, 4. Sakarya, 5. İzmit).

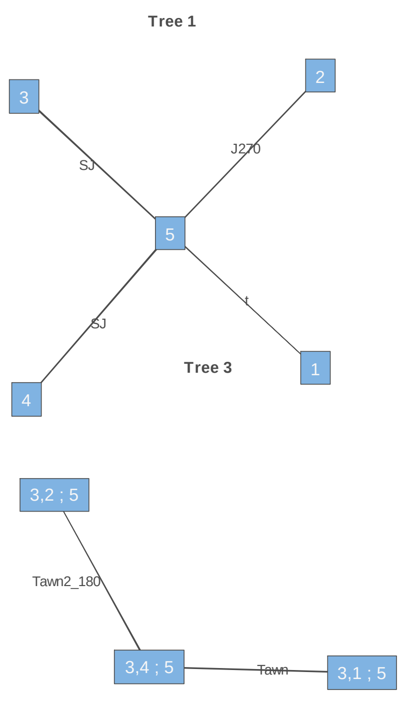

Figure 7. C Vine Copula Summary for depth of Eastern Anatolian fault line (1. Elazıg, 2. Malatya, 3. Kahramanmaras, 4. Adıyaman, 5. Sırnak)

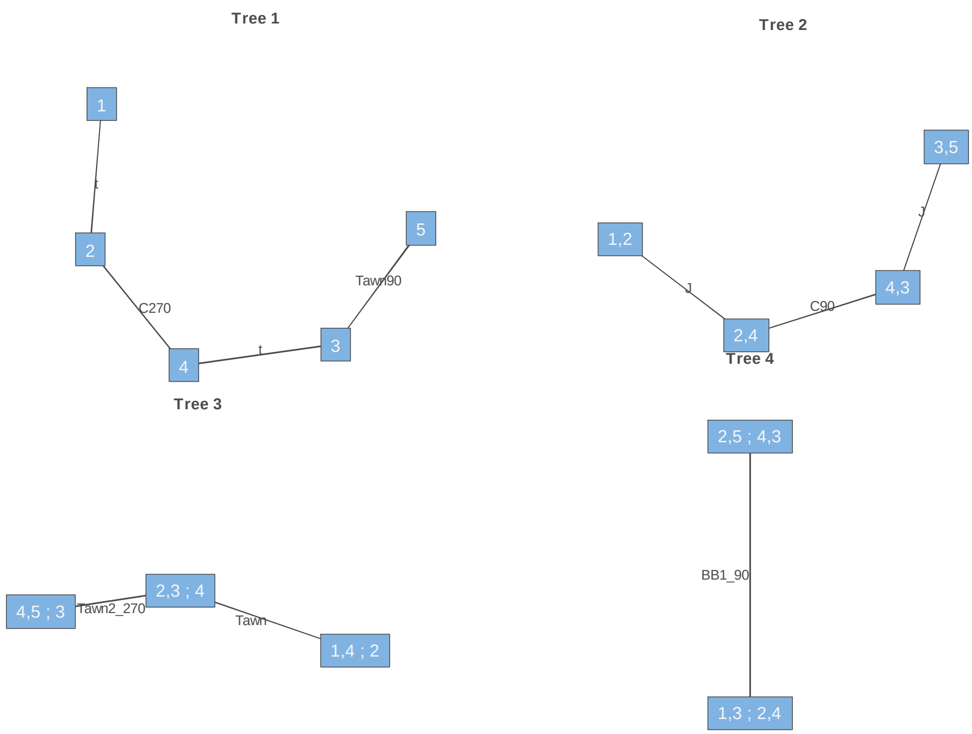

Figure 8. D Vine Copula Summary for depth of Eastern Anatolian fault line (1. Elazıg, 2. Malatya, 3. Kahramanmaras, 4. Adıyaman, 5. Sırnak).

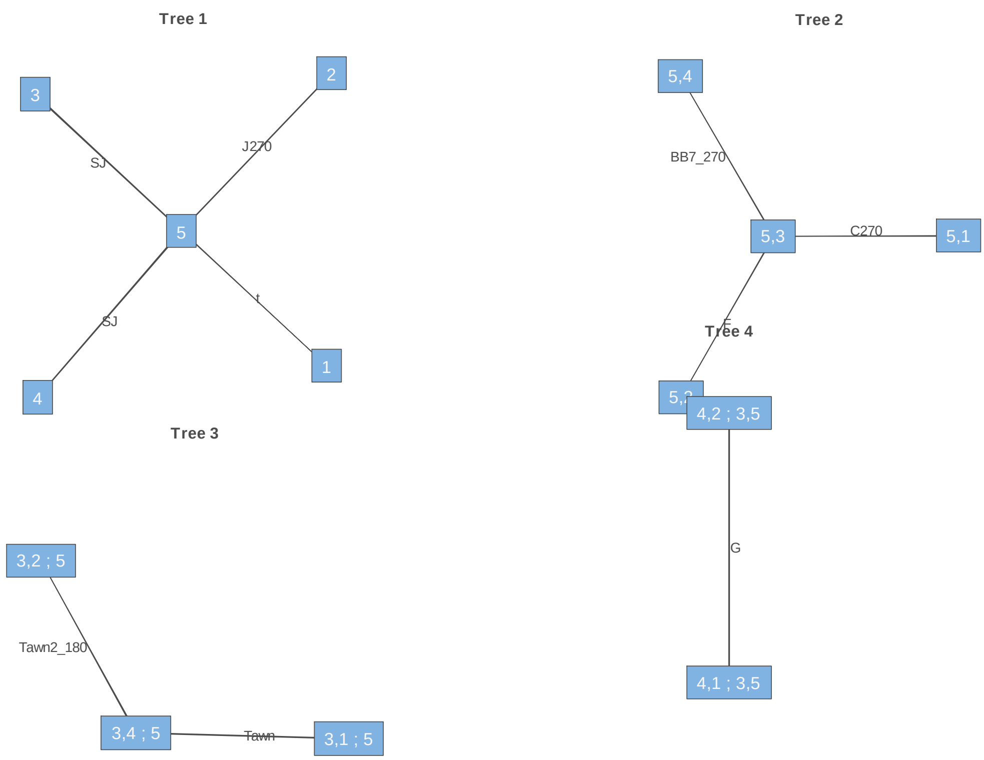

Figure 9. C Vine Copula Summary for Magnitude of Eastern Anatolian fault line (1. Elazıg, 2. Malatya, 3. Kahramanmaras, 4. Adıyaman, 5. Sırnak)

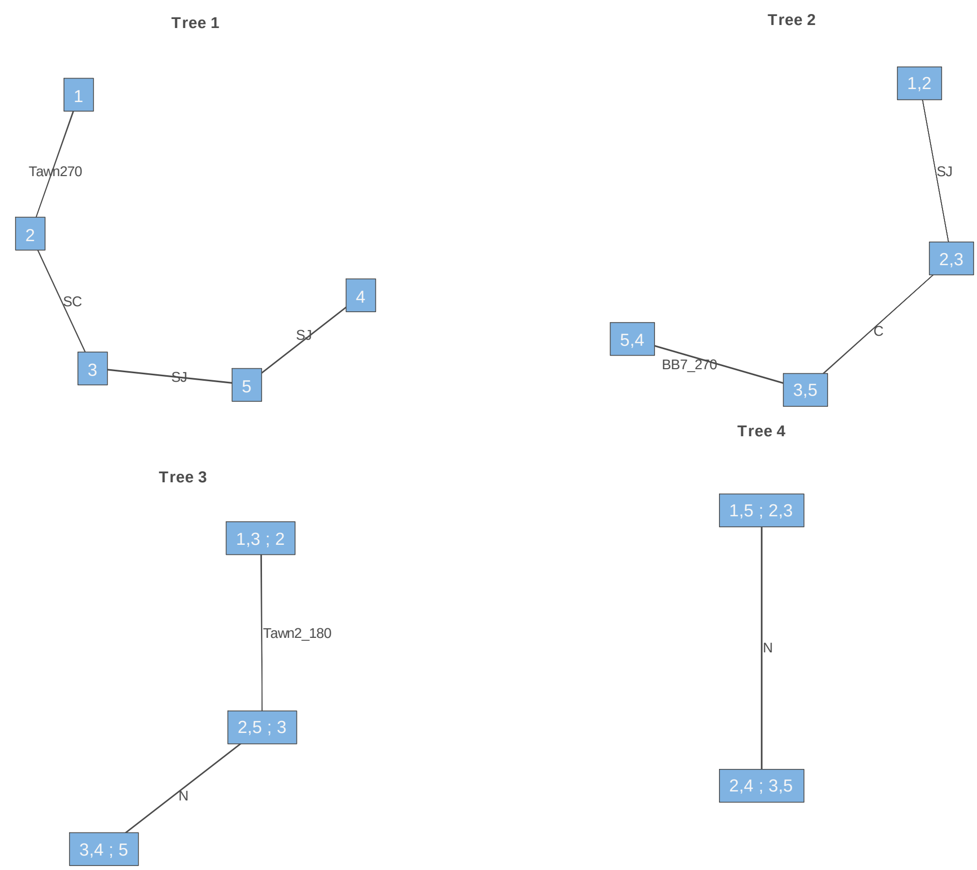

Figure 10. D Vine Copula Summary for Magnitude of Eastern Anatolian fault line (1. Elazıg, 2. Malatya, 3. Kahramanmaras, 4. Adıyaman, 5. Sırnak)

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