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THE log(ft) VALUES IN SPHERICAL AND DEFORMED NUCLEI FOR SOME ODD-A GERMANIUM ISOTOPES

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ABSTRACT

The log(ft) values of the allowed $\beta\pm$ decay between odd-A spherical and deformed nuclei are studied for germanium isotopes in this paper. The Pyatov Method (PM) and the Schematic Model (SM)) are used to the GT strength distributions, including the schematic residual spin-isospin interaction between nucleons in the particle-hole and particle-particle channels. Particle-hole and particle-particle interaction parameters are calculated respectively with $\chi_{ph}^{GT} = 5.2\ A^{0.7}$ MeV and $\chi_{pp}^{GT} = 0.58\ A^{0.7}$ MeV. Deformed Woods-Saxon potential is used in calculations of single-particle energies and wave functions. The results are also compared with previous experiment values wherever available.

Keywords: Pyatov Method, Schematic Model, Log(ft), Beta decay, Gamow-Teller Transitions

1. INTRODUCTION

The field devoted to the study of exotic nuclei is one of the most productive fields in Nuclear Physics today. Weak proportions in germanium play an integral role in supernova dynamics. Electron capture and beta decay of germanium isotopes dictated by Gamow Teller transitions significantly alter the lepton fraction of stellar matter.

Gamow-Teller (GT) transitions are known to be effective in stellar collapse events, and GT transitions strengths (B(GT)) are needed to explain this core collapse. Therefore, Taddeucci et al., in their study in 1987 [1], stated that there is a linear relationship between the measured cross-section and B(GT). It is seen that these intensity distributions are possible over experimental cross-sections. Considering the experimental studies on this study, we see that experimental B(GT) values were obtained from experiments with (p,n) reactions done by Rapaport et al. in 1994. In addition, Anderson et al. also conducted a study using the same reaction [2]. Experimental B(GT) values are obtained not only by (p,n) reactions but also by other charge-exchange reactions such as (n,p), (^{3}He,t), (t,^{3}He), and (d,^{2}He). Regarding this, the studies of (n,p) reactions used by El-Kateb et al. [3] in 1994, (^{3}He,t) reactions used by Fujita et al. in 1999 [4], (d,^{2}He) reactions used by Baumer et al. in 2003 [5], and (t,^{3}He) reactions used by Cole et al. [6] in 2006 are seen in the literature. As is known, GT_- strength distributions are obtained through (p,n) and (^{3}He,t) reactions, while GT_+ strength distributions are obtained through (n,p) and (d,^{2}He) reactions. These reactions convert a proton (neutron) to a neutron (proton). The isospin change in this transformation is $\Delta T = 1$ (either $\Delta S = 0$ or $\Delta S = 1$). The quantum number selection rules for GT transitions are $\Delta L = 0$, $\Delta Tz = \pm 1$, $\Delta S = 1$. Here L and S are orbital and spin quantum numbers. These transitions are important studies as they are the best tool for testing nuclear structural models. These transitions are denoted by $\sigma\tau_\pm$ (σ denotes Pauli spin matrix and $\tau_\pm$ denotes isospin raising/lowering operators) and depend on similar initial and final states as $\beta^\pm$ decays. In astrophysics, GT transitions are important for the formation of elements in pre-supernova and supernova events for model calculations of supernovae. Therefore, charge exchange reactions are an important tool for searching nuclei structures, nuclei with multibody systems, and matching GT responses [7].

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In this study, the models used within the QRPA formalism are the Schematic Model (SM) and the Pyatov Method (PM). Particle-hole and particle-particle interaction parameters are calculated respectively with $\mathcal{X}_{ph}^{GT} = 5.2\, A^{0.7}$ MeV and $\mathcal{X}_{pp}^{GT} = 0.58\, A^{0.7}$ MeV. These models have also been categorized into two categories for this study:

1- Schematic model for deformed nuclei with the particle-hole (ph) interaction (SM (C))
2- Pyatov method for spherical nuclei with the particle-particle (pp)+particle-hole (ph) interaction (PM (B))

The theoretical formalism used to calculate log (ft) values using SM and PM theories is given in the next section. In Chapter 3, the calculated log (ft) of odd Germanium isotopes are presented and our results are compared with the experimental results. The main results of this study are given in Chapter 4.

2. THEORETICAL FORMALISM

A summary of the formalism required for Schematic and Pyatov methods used in this study is presented in this section. A nucleon system in the axial symmetry means the domain interacts with a charge exchange via pairing and spin-spin interactions. The Hamiltonian of the schematic model (SM) in quasi-particle representation is given by

$$H_{SM} = H_{SQP} + h_{ph} + h_{pp} \tag{1}$$

where H_{SQP} is the single quasi-particle Hamiltonian and described by:

$$H_{SQP} = \sum_{s,\tau,\rho} E_s(\tau)\alpha_{s\rho}^{\dagger}\,\alpha_{s\rho}, \quad \tau = n, p \tag{2}$$

where $E_s(\tau\ )$ is the single quasi-particle energy of nucleons, $\alpha_{s\rho}^{\dagger}$ ($\alpha_{s\rho}$) is the quasi-particle creation (annihilation) operator. h_{GT}^{ph} and $h_{GT}^{pp}h$ are the GT effective interactions in the particle-hole and particle-particle channels, respectively, and given as

$$h_{ph}^{GT} = 2\mathcal{X}_{GT}^{ph}\sum_{\mu}\beta_{\mu}^{+}\beta_{\mu}^{-}$$

$$h_{pp}^{GT} = -2\mathcal{X}_{GT}^{pp}\sum_{\mu}P_{\mu}^{+}P_{\mu}^{-}\ , \quad \mu = 0, \pm 1; \tag{3}$$

with

$$\beta_{\mu}^{+} = \sum_{n,p,\rho,\rho'}\langle n\rho|\sigma_{\mu} + (-1)^{\mu}\sigma_{-\mu}|p\rho'\rangle a_{np}^{\dagger}\,a_{p\rho'}, \quad \beta_{\mu}^{-} = (\beta_{\mu}^{+})^{\dagger}$$

$$P_{\mu}^{+} = \sum_{n,p,\rho,\rho'}\langle n\rho|\sigma_{\mu} + (-1)^{\mu}\sigma_{-\mu}|p\rho'\rangle a_{np}^{\dagger}\,a_{pp'}^{\dagger}, \quad P_{\mu}^{-} = (P_{\mu}^{+})^{\dagger} \tag{4}$$

where $a_{np}^{\dagger}$ ($a_{p\rho'}$) is the nucleon creation (annihilation) operator, σ_{μ} is the spherical component of the Pauli operator. In the quasi-particle representation, the $\beta_{\mu}^{\pm}$ and $P_{\mu}^{\pm}$ operators are introduced as:

$$\beta_{\mu}^{+} = \sum_{n,p}\left[\frac{1}{\sqrt{2}}\left(\overline{d_{np}}D_{np}^{\dagger} + d_{np}D_{np}\right) + \left(\overline{b_{np}}C_{np}^{\dagger} - b_{np}C_{np}\right)\right]$$

$$P_{\mu}^{+} = \sum_{n,p}\left[\frac{1}{\sqrt{2}}\left(b_{np}D_{np}^{\dagger} - \overline{b_{np}}D_{np}\right) + \left(d_{np}C_{np}^{\dagger} + \overline{d_{np}}C_{np}\right)\right] \tag{5}$$

D_{np} corresponds to quasi-particle scattering operator, $C_{np}^{\dagger}(C_{np})$ is a two quasi-particle creation (annihilation) operator for neutron-proton pair (for details see [8]). It satisfies the following bosonic commutation rules in the quasi-boson approximation.

$$[C_{np}, C_{np}^{\dagger}] \approx \delta_{nn'}\delta_{pp'}, \qquad [C_{np}, C_{n'p'}] = 0 \tag{6}$$

From here, the effective Gamow-Teller (GT) interactions in quasi-particle space may be expressed as follows (see [8] for details):

$$h_{ph}^{GT} = h_{ph}^{CC} + h_{ph}^{DD} + h_{ph}^{CD}$$
$$h_{pp}^{GT} = h_{pp}^{CC} + h_{pp}^{DD} + h_{pp}^{CD} \tag{7}$$

The constants effective interacting with the two channels were fixed according to the measured value of the GT resonance energy. Terms not commuting with GT operators were removed from the total Hamiltonian. The mean-field approximation was rewritten by adding an effective interaction term h_0 [9], which is given as:

$$h_0 = \sum_{\rho=\pm} \frac{1}{2\gamma_\rho} \sum_{\mu=0,\pm1} [H_{sqp} - V_c - V_{ls} - V_{1,}G_{1\mu}^{\rho}]^{\dagger} [H_{sqp} - V_c - V_{ls} - V_{1,}G_{1\mu}^{\rho}]. \tag{8}$$

The strength parameter γ_ρ of the effective interaction was found as follows since it requires commutation conditions (see [9,10] for details).

$$\gamma_\rho = \frac{\rho}{2} \langle 0 | [[H_{sqp} - V_c - V_{ls} - V_{1,}G_{1\mu}^{\rho}], G_{1\mu}^{\rho}] | 0 \rangle.$$

The GT operator $G_{1\mu}^{\pm}$, which commutes with the Hamiltonian, is given as

$$G_{1\mu}^{\pm} = \frac{1}{2} \sum_{k=1}^{A} [\sigma_{1\mu}(k) t_{+}(k) + \rho(-1)^{\mu} \sigma_{1-\mu}(k) t_{-}(k)] \qquad (\rho = \pm 1), \tag{9}$$

where $\sigma_{1\mu}(k) = 2s_{1\mu}(k)$ are the spherical components of the Pauli operators, $t_{\pm} = t_x(k) \pm i t_y(k)$ are the isospin raising/lowering operators.
The total Hamiltonian of Pyatov Method is

$$H_{PM} = H_{SQP} + h_0 + h_{ph} + h_{pp} \tag{10}$$

The GT transition strengths were calculated by summing the nuclear matrix elements

$$B_{GT}^{(\pm)}(\omega_i) = \sum_{\mu} \left| \mu_{\beta^{\pm}}^{i} (0^{+} \to 1^{+}) \right|^2, \tag{11}$$

where ω_i are the excitation energies in the daughter nucleus. The $\beta^{\pm}$ transition strengths were finally calculated using

$$B(GT)_{\pm} = \sum_i B_{GT}^{(\pm)}(\omega_i). \tag{12}$$

The calculated GT strengths should fulfill the Ikeda sum rule (ISR) [11]

$$ISR = B(GT)_{-} - B(GT)_{+} \cong 3(N - Z) \tag{13}$$

The GT strength values calculated from PM and SM may be different because of the effective interaction term (h_0) (for more, see [12, 13, 14, 15]). Spherical calculations are made within the framework of the Pyatov method and deformed calculations are done within the frame of the Schematic Model. The Hamiltonian of the Gamow-Teller interaction in odd-odd nuclei and the necessary formalism for deformed nuclei are detailed in [8].

3. RESULTS AND DISCUSSIONS

Numerical calculations have been made for deformed and spherical nuclei for odd nuclei Ge**.** Nilsson's single-particle energies and wave functions have been calculated with a deformed Woods-Saxon potential [40]. The log (ft) values for odd germanium isotopes are calculated using Eq. 12 for β^- and β^+ decays as in the formula below:

$$(ft)_{\beta^+} = \mathrm{D}\ \frac{g_v^2}{4\pi B_{GT}^{\pm}} = \frac{D}{(\frac{g_A}{g_V})^2\ (I_i K_i 1 K_f - K_i/I_f K_f)^2\ |M_{\beta^{\pm}}|^2} \tag{14}$$

We used the constants D≡ $\frac{2\pi^3\ \hbar^7 ln2}{g_v^2 m_e^5 c^4}$ = 6295 s and $\frac{g_A}{g_v}$ = -1.254 in our calculations. The log(ft) values found in the literature were compared with the experimental values found in the literature.

In Figure 1, the log(ft) value for the [67]Ge isotope in the β^- direction, the transitions of $\frac{1}{2}^- \rightarrow \frac{1}{2}^-$ $1^+ \xrightarrow{\beta^-} 0^+$ were compared with experimental values corresponding to different energies. This experiment is done by Junde et al. [16]. Spherical PM (B) and deformed SM (C) models were used for this comparison. These values were found to be quite close to the experimental values.

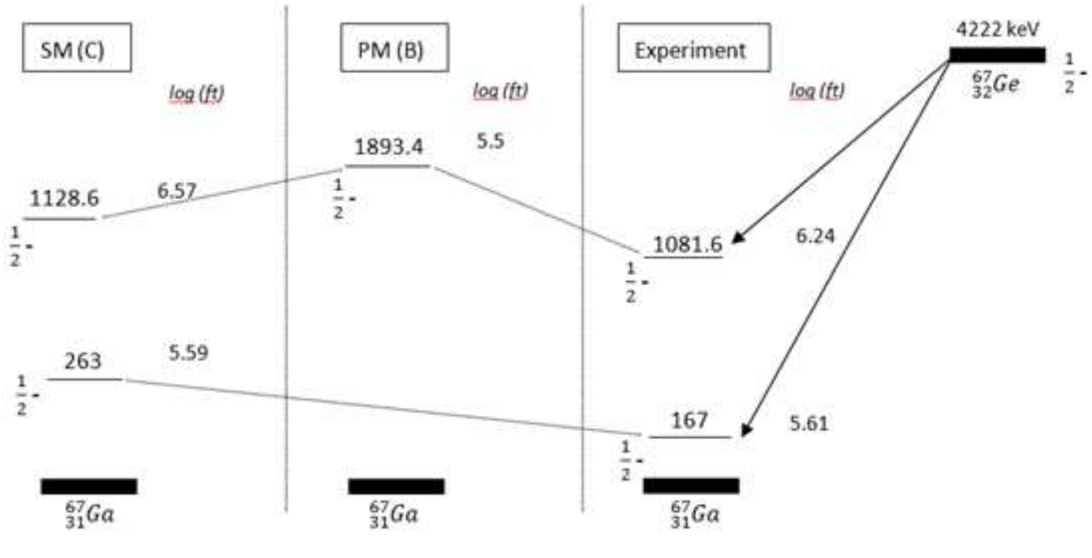

Figure 1. Comparison of log (ft) values for [67]Ge with the experimental values for transitions of $1^+ \xrightarrow{\beta^-} 0^+$

Comparison of the calculated log(ft) value for $1^+ \xrightarrow{\beta^-} 0^+$ transitions in the β^- direction with the measured log(ft) values found by Nesaraja [17] corresponding to the different energies was given in Fig. 2 for the [69]Ge isotope. The log (ft) value of 5.33, which corresponds to an energy of 1531.7 KeV in the transitions from the 1+ states to the ground state, is the same as the experiment. These results show that we found good agreement with the experiment.

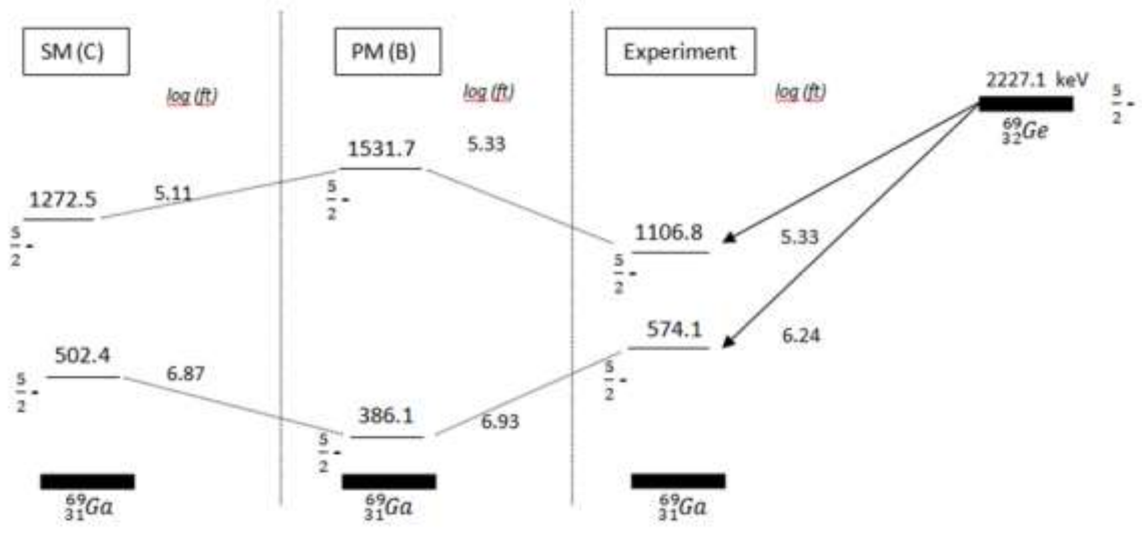

Figure 2. Comparison of log (ft) values for ^{69}Ge with the experimental values for transitions of $1^+ \xrightarrow{\beta^-} 0^+$

For ^{71}Ge isotope, the log(ft) values of the ground state $1^+ \xrightarrow{\beta^-} 0^+$ transition calculated from the schematic and the Pyatov method were compared with the experimental values found by Abusaleem and Singh [18] in Figure 3. As can be seen in Figure 3, the log(ft) values calculated by the Pyatov method for these transitions are much larger than the schematic model values, and the energy value is closer to the experimental one.

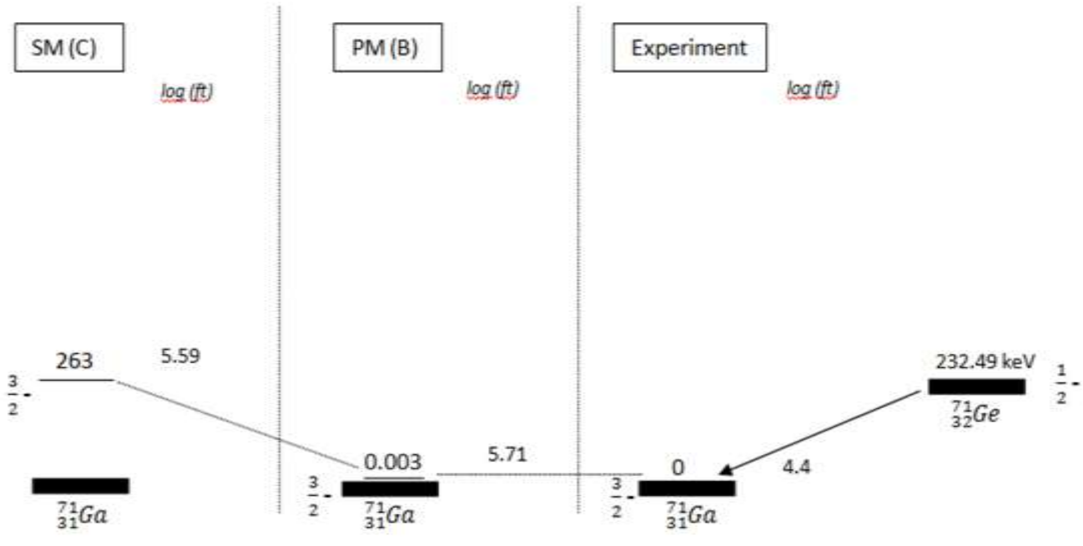

Figure 3. Comparison of log (ft) values for ^{71}Ge with the experimental values for transitions of $1^+ \xrightarrow{\beta^-} 0^+$

A comparison of log (ft) values for ^{75}Ge with the experimental values found by Negret and Singh [19] was given in Figure 4 for the transitions of $1^+ \xrightarrow{\beta^+} 0^+$. Here, it is seen that in the β^+ direction for the deformed SM model, both the energy and the log (ft) value are very close to the experimental value.

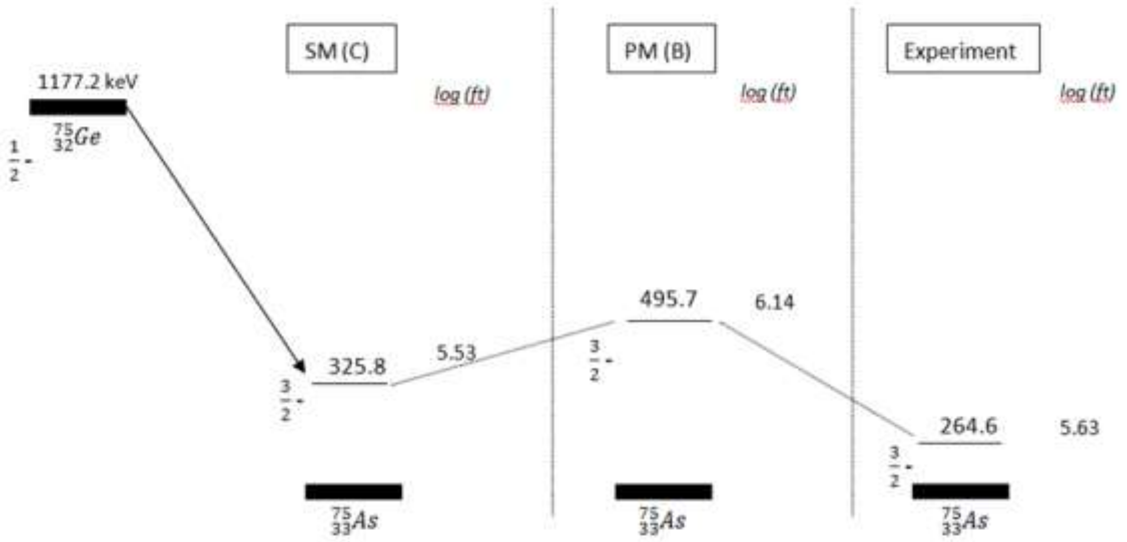

Figure 4. Comparison of log (ft) values for ^{75}Ge with the experimental values for transitions of $1^+ \xrightarrow{\beta^+} 0^+$

Comparison of log (ft) values for ^{77}Ge with the experimental values found by Singh and Nica [20] for $1^+ \longrightarrow 0^+$ transitions in the β^+ direction was given in Figure 5. Here, there are transitions from $\frac{1}{2}^-$ states to $\frac{3}{2}^-$ states. The log (ft) value calculated with the PM (B) model is closer to the experimental value

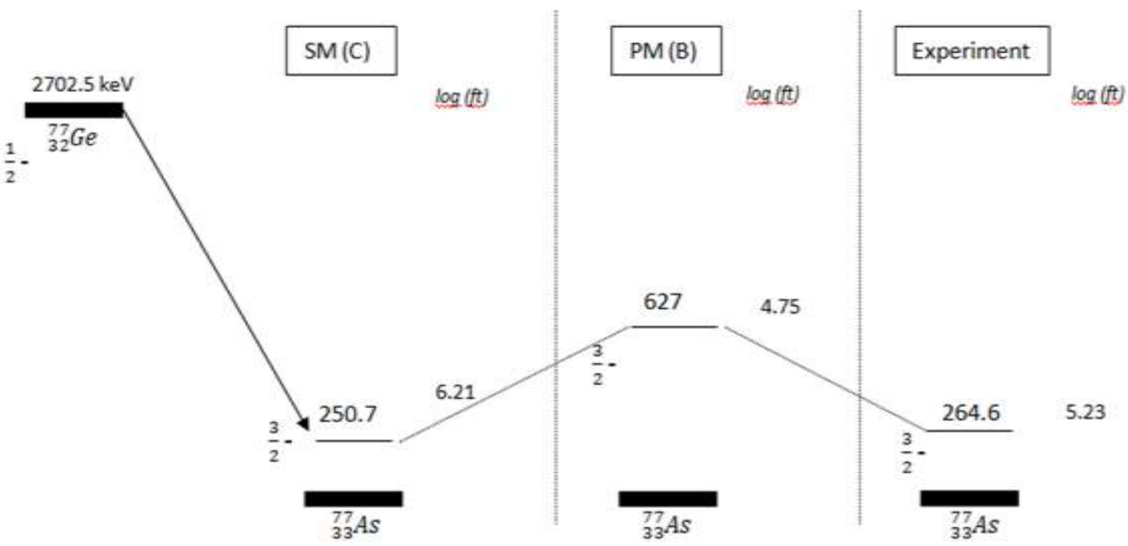

Figure 5. Comparison of log (ft) values for ^{77}Ge with the experimental values for transitions of $1^+ \xrightarrow{\beta^+} 0^+$

4. SUMMARY AND CONCLUSIONS

The Gamow-Teller transition is known to have an important place in astrophysical events (eg electron capture, β-decay, supernova explosion, etc.) [21 ,22, 23, 24]. Therefore, it is aimed to investigate the property of the Gamow-Teller transition in astrophysical conditions and to motivate the processing of odd germanium isotopes with fp shell nuclei in this study. In this paper, Gamow-Teller transitions were studied by using Schematic Model and Pyatov Method in ^{67,69,71,75,77}Ge isotope for the first time. Within this framework, the effects of particle-hole, particle-particle force, and deformation were taken into account. The calculated log(ft) values by using our models in the *β*-decay directions were compared with the corresponding measured data wherever available.

Log (ft) values for $1^+ \xrightarrow{\beta^+} 0^+$. transitions for odd-A-mass germanium nuclei were compared with experimental values. Results of PM model with spherical ph+pp channel and SM model with deformed ph+pp channel were found to be close to experimental values.

For astrophysical applications, microscopic and confidential calculations of the GT strength distributions for hundreds of iron regimen cores are needed. A significant advance in our understanding of supernova explosions and heavy element nucleosynthesis, which will be developed from next-generation radioactive ion beam facilities, is hypothesized when the measured GT strength distribution of many more nuclei (including unstable isotopes) is obtained. Computed GT strength functions for other key fp-shell nuclei (including many neutron-rich unstable nuclei) are being studied and it is hoped that our findings will be presented soon.

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CONFLICT OF INTEREST

The author stated that there are no conflicts of interest regarding the publication of this article.

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