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Research Article

ASYMPTOTICALLY $\mathcal{I}$-STATISTICAL EQUIVALENT FUNCTIONS DEFINED ON AMENABLE SEMIGROUPS

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ABSTRACT

In this study, we introduce the notions of asymptotically $\mathcal{I}$-equivalence, asymptotically $\mathcal{I}^*$-equivalence, asymptotically strongly $\mathcal{I}$-equivalence and asymptotically $\mathcal{I}$-statistical equivalence for functions defined on discrete countable amenable semigroups. Also, we examine some properties of these notions and relationships between them.
Keywords: Statistical convergence, ideal convergence, asymptotically equivalence, folner sequence, amenable semigroups.

1. INTRODUCTION

In [1], Fast introduced the notion of statistical convergence for real sequences. Also this notion was studied in [2], [3] and [4], too. The idea of $\mathcal{I}$-convergence was introduced by Kostyrko et al. [5] which is based on the structure of the ideal $\mathcal{I}$ of subset of the set $\mathbb{N}$ (natural numbers). Then, by using ideal, Das et al. [6] introduced a new notion, namely $\mathcal{I}$-statistical convergence.

In [7], Day studied on amenable semigroups. Then, the notions of summability in amenable semigroups were examined in [8], [9], [10] and [11]. Recently, Nuray and Rhoades [12] introduced the notions of convergence, strongly summability and statistical convergence for functions defined on amenable semigroups. Also, the notions of $\mathcal{I}$-summable and $\mathcal{I}$-statistical convergence for functions defined on amenable semigroups were studied by Ulusu et al. [13].

In [14], Marouf presented definitions for asymptotically equivalent sequences and asymptotic regular matrices. Then, the notion of asymptotically equivalence has been developed by many researchers (see, [15, 16, 17]). Recently, the notions of asymptotically equivalence, strongly asymptotically equivalence and asymptotically statistical equivalence for function defined on amenable semigroups were introduced by Nuray and Rhoades [18].

Now, we recall the basic definitions and concepts that need for a good understanding of our study (see, [5, 6, 12, 13, 14, 18]).

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Let G be a discrete countable amenab le semigroup with identity in which both right and left cancelation laws hold, and

$$w(G) = \{f \mid f: G \to \mathbb{R}\} \text{ and } m(G) = \{f \in w(G): f \text{ is bounded}\}.$$

$m(G)$ is a Banach space with the supremum norm $\| f \|_\infty = \sup\{|f(g)|: g \in G\}$.

Namioka [19] showed that, if G is a countable amenable group, there exists a sequence $\{S_n\}$ of finite subsets of G such that

i. $G = \bigcup_{n=1}^{\infty} S_n$,
ii. $S_n \subset S_{n+1}$ $(n = 1,2,\dots)$,
iii. $\lim\limits_{n\to\infty} \frac{|S_n g \cap S_n|}{|S_n|} = 1$, $\lim\limits_{n\to\infty} \frac{|g S_n \cap S_n|}{|S_n|} = 1$,

for all $g \in G$, where $|A|$ denotes the number of elements inside set A.

Any sequence of finite subsets of G satisfying (i), (ii) and (iii) is called a Folner sequence for G.

The sequence $S_n = \{0,1,2,\dots,n-1\}$ is a familiar Folner sequence giving rise to the classical Cesàro method of summability.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. A function $f \in w(G)$ is said to be convergent to s for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$, there exists a $n_0 \in \mathbb{N}$ such that $|f(g) - s| < \varepsilon$ holds, for all $n > n_0$ and $g \in G \backslash S_n$.

A function $f \in w(G)$ is said to be strongly Cesàro summable to s for any Folner sequence $\{S_n\}$ of G if

$$\lim_{n\to\infty} \frac{1}{|S_n|} \sum_{g\in S_n} |f(g) - s| = 0$$

holds.

A function $f \in w(G)$ is said to be statistically convergent to s for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{|S_n|} |\{g \in S_n: |f(g) - s| \geq \varepsilon\}| = 0.$$

Let X is a non-empty set. A family of sets $\mathcal{I} \subset 2^X$ is called an ideal on X if

i. $\emptyset \in \mathcal{I}$,
ii. For each $A, B \in \mathcal{I}$, $A \cup B \in \mathcal{I}$,
iii. For each $A \in \mathcal{I}$ and each $B \subset A$, $B \in \mathcal{I}$.

An ideal $\mathcal{I} \subset 2^X$ is called non-trivial if $X \notin \mathcal{I}$ and a non-trivial ideal $\mathcal{I} \subset 2^X$ is called admissible if $\{x\} \in \mathcal{I}$ for each $x \in X$.

An admissible ideal $\mathcal{I} \subset 2^X$ is said to satisfy the condition (AP) if for every countable family of mutually disjoint sets $\{A_1, A_2, \dots\}$ belonging to $\mathcal{I}$ there exists a countable family of sets $\{B_1, B_2, \dots\}$ such that $A_j \Delta B_j$ is a finite set for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{I}$.

A non-empty family of sets $\mathcal{F} \subset 2^X$ is called a filter on X if

i. $\emptyset \notin \mathcal{F}$,
ii. For each $A, B \in \mathcal{F}$, $A \cap B \in \mathcal{F}$,
iii. For each $A \in \mathcal{F}$ and each $B \supset A$, $B \in \mathcal{F}$.

$\mathcal{I} \subset 2^X$ is a non-trivial ideal if and only if $\mathcal{F}(\mathcal{I}) = \{M \subset X: (\exists A \in \mathcal{I})(M = X \backslash A)\}$ is a filter on X, called the filter associated with $\mathcal{I}$.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{I} \subset 2^G$ be an admissible ideal. A function $f \in w(G)$ is said to be $\mathcal{I}$-convergent to s for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$,

$$\{g \in G: |f(g) - s| \geq \varepsilon\} \in \mathcal{I}$$

holds.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. $f \in w(G)$ is said to be $\mathcal{I}$-statistical convergent to s, for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon, \delta > 0$

$$\left\{n \in \mathbb{N}: \frac{1}{|S_n|}|\{g \in S_n: |f(g) - s| \geq \varepsilon\}| \geq \delta\right\} \in \mathcal{I}$$

holds.

Two nonnegative sequences (x_k) and (y_k) are said to be asymptotically equivalent if

$$\lim_k \frac{x_k}{y_k} = 1$$

and it is denoted by $x \sim y$.

Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically equivalent for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that

$$\left|\frac{f(g)}{h(g)} - 1\right| < \varepsilon$$

holds, for all $n > n_0$ and $g \in G \backslash S_n$. It is denoted by $f \sim h$.

Two nonnegative functions $f, h \in w(G)$ are said to be strongly Cesàro asymptotically equivalent for any Folner sequence $\{S_n\}$ of G if

$$\lim_{n\to\infty} \frac{1}{|S_n|} \sum_{g \in S_n} \left|\frac{f(g)}{h(g)} - 1\right| = 0$$

and it is denoted by $f \overset{w}{\sim} h$.

Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically statistical equivalent for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{|S_n|} \left|\left\{g \in S_n: \left|\frac{f(g)}{h(g)} - 1\right| \geq \varepsilon\right\}\right| = 0,$$

and it is denoted by $f \overset{S}{\sim} h$.

2. MAIN RESULTS

In this section, we introduce the notions of asymptotically $\mathcal{I}$-equivalence, asymptotically $\mathcal{I}^*$-equivalence, asymptotically strongly $\mathcal{I}$-equivalence and asymptotically $\mathcal{I}$-statistical equivalence for functions defined on discrete countable amenable semigroups. Also, we examine some properties of these notions and relationships between of them. For the particular case; when the amenable semigroup is the additive positive integers, our definitions and theorems yield the results of [15, 17].

Definition 2.1 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{I} \subset 2^G$ be an admissible ideal. Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically $\mathcal{I}$-equivalent of multiple L, for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$*

$$\left\{g \in G: \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\} \in \mathcal{I}.$$

In this case, we write $f \overset{\mathcal{I}_L}{\sim} h$ and simply asymptotically $\mathcal{I}$-equivalent, if $L = 1$.

Example 2.1

- If we take $\mathcal{I} = \mathcal{I}_f$ be an ideal of all finite subsets of G, then we get asymptotically equivalent in [18] with respect to Folner sequence.

- Let $\mathcal{I}_d = \{H \subset G: \delta(H) = 0\}$. Then, $\mathcal{I}_d$ is an admissible ideal and asymptotically $\mathcal{I}_d$-equivalence coincides with asymptotically statistical equivalent in [18] with respect to the Folner sequence.

Remark 2.1 *The asymptotical $\mathcal{I}$-equivalence of $f, h \in w(G)$ depends on the particular choice of the Folner sequence.*

By assuming $\mathcal{I} = \mathcal{I}_d$, let us show this by an example.

Example 2.2 *Let $G = \mathbb{Z}^2$ and take two Folner sequences as follows:*

$\{S_n^1\} = \{(i,j) \in \mathbb{Z}^2: |i| \le n, |j| \le n\}$ and $\{S_n^2\} = \{(i,j) \in \mathbb{Z}^2: |i| \le n, |j| \le n^2\}$, and define $f, h \in w(G)$ by

$$f(g) := \begin{cases} \frac{|ij|+3}{|ij|+2} & , \text{ if } (i,j) \in A, \\ 1 & , \text{ if } (i,j) \notin A \end{cases} \quad \text{and} \quad h(g) := \begin{cases} \frac{|ij|+1}{|ij|+2} & , \text{ if } (i,j) \in A, \\ 1 & , \text{ if } (i,j) \notin A \end{cases}$$

where $A = \{(i,j) \in \mathbb{Z}^2: i \le j \le n, \ i = 0,1,2,\dots,n; \ n = 1,2,\dots\}$.
Since for the Folner sequence $\{S_n^2\}$

$$\lim_{n\to\infty} \frac{1}{|S_n^2|} \left| \left\{ g \in S_n^2: \left| \frac{f(g)}{h(g)} - 1 \right| \ge \varepsilon \right\} \right| = \lim_{n\to\infty} \frac{\frac{(n+1)(n+2)}{2}}{(2n+1)(2n^2+1)} = 0,$$

then $f(g), h(g)$ are asymptotically $\mathcal{I}_d$-equivalent.
But, since for the Folner sequence $\{S_n^1\}$

$$\lim_{n\to\infty} \frac{1}{|S_n^1|} \left| \left\{ g \in S_n^1: \left| \frac{f(g)}{h(g)} - 1 \right| \ge \varepsilon \right\} \right| = \lim_{n\to\infty} \frac{\frac{(n+1)(n+2)}{2}}{(2n+1)^2} = \frac{1}{4} \ne 0,$$

then $f(g), h(g)$ are not asymptotically $\mathcal{I}_d$-equivalent.

Definition 2.2 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{I} \subset 2^G$ be an admissible ideal. Two nonnegative functions $f, g \in w(G)$ are said to be asymptotically $\mathcal{I}^*$-equivalent of multiple L, for any Folner sequence $\{S_n\}$ for G if there exists $M \subset G$ such that $M \in \mathcal{F}(\mathcal{I})$ (i.e., $G \backslash M \in \mathcal{I}$) and an $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $\varepsilon > 0$*

$$\left| \frac{f(g)}{h(g)} - L \right| < \varepsilon,$$

for all $n > n_0$ and all $g \in M \backslash S_n$. In this case, we write $f \overset{\mathcal{I}_L^*}{\sim} h$ and simply asymptotically $\mathcal{I}^*$-equivalent, if $L = 1$.

Theorem 2.1 *Let $\mathcal{I} \subset 2^G$ be an admissible ideal. If two nonnegative functions $f, h \in w(G)$ are asymptotically $\mathcal{I}^*$-equivalent of multiple L, for Folner sequence $\{S_n\}$ for G, then f, h are asymptotically $\mathcal{I}$-equivalent of multiple L for same sequence.*

Proof. Suppose that $f, g \in w(G)$ are asymptotically $\mathcal{I}^*$-equivalent of multiple L for Folner sequence $\{S_n\}$ for G. Then, there exists $M \subset G$, $M \in \mathcal{F}(\mathcal{I})$ (i.e., $H = G \backslash M \in \mathcal{I}$) and an $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $\varepsilon > 0$

$$\left| \frac{f(g)}{h(g)} - L \right| < \varepsilon,$$

for all $n > n_0$ and all $g \in M \backslash S_n$. Therefore, obviously

$$A_\varepsilon^\sim = \left\{ g \in G: \left| \frac{f(g)}{h(g)} - L \right| \ge \varepsilon \right\} \subset H \cup S_{n_0}.$$

Since $\mathcal{I}$ is admissible, $H \cup S_{n_0} \in \mathcal{I}$ and so $A_\varepsilon^\sim \in \mathcal{I}$. Hence, $f, g \in w(G)$ are asymptotically $\mathcal{I}$-equivalent of multiple L.

Theorem 2.2 *Let $\mathcal{I} \subset 2^G$ be an admissible ideal that satisfy the condition (AP). If two nonnegative functions $f, h \in w(G)$ are asymptotically $\mathcal{I}$-equivalent of multiple L, for Folner sequence $\{S_n\}$ for G, then f, g are asymptotically $\mathcal{I}^*$-equivalent of multiple L for same sequence.*

Proof. Let $\mathcal{I}$ satisfies the condition (AP) and suppose that $f(g), h(g) \in w(G)$ are asymptotically $\mathcal{I}$-equivalent of multiple L for Folner sequence $\{S_n\}$ for G. Then, for every $\varepsilon > 0$ we have

$$\left\{g \in G: \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\} \in \mathcal{I}.$$

Denote by

$$A_1 = \left\{g \in G: \left|\frac{f(g)}{h(g)} - L\right| \geq 1\right\} \text{ and } A_n = \left\{g \in G: \frac{1}{n} \leq \left|\frac{f(g)}{h(g)} - L\right| < \frac{1}{n+1}\right\}$$

for $n \geq 2$, $n \in \mathbb{N}$. Obviously, $A_i \cap A_j = \emptyset$ for $i \neq j$. By the condition (AP), there exists a sequence of sets $(B_n)_{n\in\mathbb{N}}$ such that $A_j \bigtriangleup B_j$ are infinite sets for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{I}$. It is sufficient to prove that there exist $M \subset G$, $M \in \mathcal{F}(\mathcal{I})$ (i.e., $M = G \backslash B$) and an $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $\varepsilon > 0$

$$\left|\frac{f(g)}{h(g)} - L\right| < \varepsilon,$$

for all $n > n_0$ and all $g \in M \backslash S_n$.

Let $\eta > 0$. Choose $k \in \mathbb{N}$ such that $\frac{1}{k+1} < \eta$. Then, for every $\eta > 0$ we have

$$\left\{g \in G: \left|\frac{f(g)}{h(g)} - L\right| \geq \eta\right\} \subset \bigcup_{j=1}^{k+1} A_j.$$

Since $A_j \bigtriangleup B_j$ $(j = 1,2,\ldots,k+1)$ are finite sets, there exists n_0 such that

$$\bigcup_{j=1}^{k+1} B_j \cap \left(M \backslash S_{n_0}\right) = \bigcup_{j=1}^{k+1} A_j \cap \left(M \backslash S_{n_0}\right). \tag{1}$$

If $g \in M \backslash S_{n_0}$ and $g \notin \bigcup_{j=1}^{k+1} B_j$, then $g \notin \bigcup_{j=1}^{k+1} A_j$ by (1). But we have

$$\left|\frac{f(g)}{h(g)} - L\right| < \frac{1}{k+1} < \eta.$$

Hence, $f, g \in w(G)$ are asymptotically $\mathcal{I}^*$-equivalent of multiple L.

Definition 2.3 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically strongly $\mathcal{I}$-equivalent of multiple L, for any Folner sequence $\{S_n\}$ for G if for every $\varepsilon > 0$*

$$\left\{n \in \mathbb{N}: \frac{1}{|S_n|} \sum_{g \in S_n} \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\} \in \mathcal{I}.$$

In this case, we write $f \overset{[\mathcal{I}_L]}{\sim} h$.

Definition 2.4 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically $\mathcal{I}$-statistical equivalent of multiple L, for any Folner sequence $\{S_n\}$ for G if for every $\varepsilon, \delta > 0$*

$$\left\{n \in \mathbb{N}: \frac{1}{|S_n|} \left|\left\{g \in S_n: \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\}\right| \geq \delta\right\} \in \mathcal{I}.$$

In this case, we write $f \overset{S(\mathcal{I}_L)}{\sim} h$.

Remark 2.2 *The asymptotically $\mathcal{I}$-statistical equivalence of $f, h \in w(G)$ depend on the particular choice of the Folner sequence.*

Theorem 2.3 *Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. If two nonnegative function $f, h \in w(G)$ are asymptotically strongly $\mathcal{I}$-equivalent of multiple L, for Folner sequence $\{S_n\}$ of G, then f and h are asymptotically $\mathcal{I}$-statistical equivalent to multiple L for same sequence.*

Proof. Suppose that $f, h \in w(G)$ are asymptotically strongly $\mathcal{I}$-equivalent of multiple L, for Folner sequence $\{S_n\}$ for G. For any fixed $\varepsilon > 0$, we have

$$\sum_{g\in S_n} \left|\frac{f(g)}{h(g)} - L\right| = \sum_{\substack{g\in S_n \\ \left|\frac{f(g)}{h(g)}-L\right|\geq\varepsilon}} \left|\frac{f(g)}{h(g)} - L\right| + \sum_{\substack{g\in S_n \\ \left|\frac{f(g)}{h(g)}-L\right|<\varepsilon}} \left|\frac{f(g)}{h(g)} - L\right| \geq \left|\left\{g \in S_n : \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\}\right| \cdot \varepsilon$$

and this inequality gives that

$$\frac{1}{\varepsilon\cdot|S_n|}\sum_{g\in S_n} \left|\frac{f(g)}{h(g)} - L\right| \geq \frac{1}{|S_n|}\left|\left\{g \in S_n : \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\}\right|.$$

Hence, for any $\delta > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|}\left|\left\{g \in S_n : \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\}\right| \geq \delta\right\} \subseteq \left\{n \in \mathbb{N} : \frac{1}{|S_n|}\sum_{g\in S_n} \left|\frac{f(g)}{h(g)} - L\right| \geq \delta\cdot\varepsilon\right\}$$

holds. Therefore, due to our acceptance, the set in the right of above inclusion belongs to $\mathcal{I}$, so we get

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|}\left|\left\{g \in S_n : \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\}\right| \geq \delta\right\} \in \mathcal{I}.$$

This completes the proof.

Theorem 2.4 *Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. If $f, h \in m(G)$ are asymptotically $\mathcal{I}$-statistical equivalent of multiple L for Folner sequence $\{S_n\}$ for G, then f, h are asymptotically strongly $\mathcal{I}$-equivalent of multiple L for same sequence.*

Proof. Suppose that $f, h \in m(G)$ are asymptotically $\mathcal{I}$-statistical equivalent of multiple L for Folner sequence $\{S_n\}$ of G. Since $f, g \in m(G)$, set $\| \frac{f}{g} \|_\infty + L = M$. Then, for given $\varepsilon > 0$ we have

$$\frac{1}{|S_n|}\sum_{g\in S_n} \left|\frac{f(g)}{h(g)} - L\right| = \frac{1}{|S_n|}\sum_{\substack{g\in S_n \\ \left|\frac{f(g)}{h(g)}-L\right|\geq\frac{\varepsilon}{2}}} \left|\frac{f(g)}{h(g)} - L\right| + \frac{1}{|S_n|}\sum_{\substack{g\in S_n \\ \left|\frac{f(g)}{h(g)}-L\right|<\frac{\varepsilon}{2}}} \left|\frac{f(g)}{h(g)} - L\right| \leq \frac{M}{|S_n|}\left|\left\{g \in S_n : \left|\frac{f(g)}{h(g)} - L\right| \geq \frac{\varepsilon}{2}\right\}\right| + \frac{\varepsilon}{2},$$

and so

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|}\sum_{g\in S_n} \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\} \subseteq \left\{n \in \mathbb{N} : \frac{1}{|S_n|}\left|\left\{g \in S_n : \left|\frac{f(g)}{h(g)} - L\right| \geq \frac{\varepsilon}{2}\right\}\right| \geq \frac{\varepsilon}{2M}\right\}.$$

Therefore, due to our acceptance, the right set belongs to $\mathcal{I}$, so we get

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|}\sum_{g\in S_n} \left|\frac{f(g)}{h(g)} - L\right| \geq \varepsilon\right\} \in \mathcal{I}.$$

This completes the proof.

REFERENCES

[1] Fast H., (1951) Sur la convergence statistique, *Colloq. Math.* 2, 241–244.
[2] Connor J.S., (1988) The statistical and strong p-Cesaro convergence of sequences, *Analysis* 8, 46–63.
[3] Fridy J.A., (1985) On statistical convergence, *Analysis* 5, 301–313.
[4] Šalát T., (1980) On statistically convergent sequences of real numbers, *Math. Slovaca* 30(2), 139–150.
[5] Kostyrko P., Šalát T. and Wilczyński W., (2000) $\mathcal{I}$-Convergence, *Real Anal. Exchange* 26(2), 669–686.

[6] Das P., Savaş E. and Ghosal S.Kr., (2011) On generalizations of certain summability methods using ideals, *Appl. Math. Letters* 24, 1509–1514.
[7] Day M., (1957) Amenable semigroups, *Illinois J. Math.* 1, 509–544.
[8] Douglass S.A., (1968) On a concept of summability in amenable semigroups, *Math. Scand.* 28, 96–102.
[9] Douglass S.A., (1973) Summing sequences for amenable semigroups, *Michigan Math. J.* 20, 169–179.
[10] Mah P.F., (1971) Summability in amenable semigroups, *Trans. Amer. Math. Soc.* 156, 391–403.
[11] Mah P.F., (1972) Matrix summability in amenable semigroups, *Proc. Amer. Math. Soc.* 36, 414–420.
[12] Nuray F. and Rhoades B.E., (2011) Some kinds of convergence defined by Folner sequences, *Analysis* 31(4), 381–390.
[13] Ulusu U., Dündar E. and Nuray F., (2019) Some generalized convergence types using ideals in amenable semigroups, *Bull. Math. Anal. Appl.* 11(1), 28-35.
[14] Marouf M., (1993) Asymptotic equivalence and summability, *Int. J. Math. Math. Sci.* 16(4), 755–762.
[15] Hazarika B., (2015) On asymptotically ideal equivalent sequences, *Journal of the Egyptian Mathematical Society* 23(1), 67-72.
[16] Patterson R.F., (2003) On asymptotically statistically equivalent sequences, *Demostratio Mathematica* 36(1), 149-153.
[17] Savaş E., (2013) On ℐ-asymptotically lacunary statistical equivalent sequences, *Adv. Difference Equ.* 111, 7 pages. doi:10.1186/1687-1847-2013-111.
[18] Nuray F. and Rhoades B.E., (2013) Asymptotically and statistically equivalent functions defined on amenable semigroups, *Thai J. Math.* 11(2), 303–311.
[19] Namioka I., (1964) Følner's conditions for amenable semigroups, *Math. Scand.* 15, 18–28.

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