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Research Article

SOLITON SOLUTIONS FOR KUDRYASHOV-SINELSHCHIKOV EQUATION

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ABSTRACT

This paper acquires the closed form solutions for the Kudryashov-Sinelshchikov (KS) equation. The Riccati-Bernoulli (RB) sub-ODE method is used to acquire such solitons whose structure include trigonmetric, hyperbolic and algebraic structures. Some interesting figures for the obtained solutions are presented in order to shed light on the characteristics of the solutions.
Keywords: KS equation, RB sub-ODE method, soliton.

1. INTRODUCTION

Soliton solutions to NPDEs play an important role in mathematical physics. Recently, many powerful methods for acquiring soliton and other solutions to NPDEs have been proposed [1-19]. In this study, we analyze and investigate the soliton solutions for the KS equation given by

$$u_t + \gamma u u_x + u_{xxx} - \varepsilon (u u_{xx})_x - \beta u_x u_{xx} - \upsilon u_{xx} - \delta (u u_x)_x = 0, \tag{1}$$

where γ, ε, β, υ, δ are real parameters. Eq. (1) characterizes the pressure waves in the liquid with gas bubbles taking into account the heat transfer and viscosity [12-14] and it is called KS equation. If $\varepsilon = k = 0$, Eq. (1) becomes Burgers-KdV [15-17].

2. DESCRIPTION OF RB SUB-ODE METHOD

Let there be given a NLPDE, say, in two variables,

$$F(r, r_t, r_x, r_{xx}, r_{tx}, \dots) = 0, \tag{2}$$

where F is a polynomial function in $r(x,t)$. The main step of this method is as follows:

Step 1. We consider its traveling wave solution

$$r(x,t) = r(\xi), \tag{3}$$

$$\xi = x + Vt, \tag{4}$$

where $r(\xi)$ travel s with speed V. Then equation (2) is reduced to an ODE:

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$$H(r, r', r'', r''', \dots) = 0, \tag{5}$$

where H is a polyno mial in $r(\xi)$ and its total derivatives.

Step 2. Let Eq. (5) has the following solution

$$r' = ar^{2-s} + br + cr^{s}, \tag{6}$$

where a, b, c, and s are constants to be determined later. From Eq.(6), we acquire

$$r'' = ab(3-s)r^{2-s} + a^2(2-s)r^{3-2s} + sc^2r^{2s-1} + bc(s+1)r^s + (2ac+b^2)r, \tag{7}$$

$$r''' = ab(2-s)(3-s)r^{1-s} + a^2(2-s)(3-2s)r^{2-2s} + s(2s-1)c^2r^{2s-2} + bcs(s+1)r^{s-1} + (2ac+b^2)r'. \tag{8}$$

Remark *Eq.(6) is a Riccati equation for $ac \neq 0$ and $s = 0$. Eq.(6) is a Bernoulli equation $a \neq 0$, $c = 0$, and $s \neq 1$. Therefore, this equation is called Riccati-Bernoulli equation.*

The types of the solutions for Eq. (6):

1. For $s = 1$, we acquire

$$r(\xi) = Ce^{(a+b+c)\xi}. \tag{9}$$

2. For $s \neq 1$, $b = 0$, and $c = 0$, we acquire

$$r(\xi) = (a(s-1)(\xi + C))^{\frac{1}{s-1}}. \tag{10}$$

3. For $s \neq 1$, $b \neq 0$, and $c = 0$, we acquire

$$r(\xi) = (-\frac{a}{b} + Ce^{b(s-1)\xi})^{\frac{1}{s-1}}. \tag{11}$$

4. For $s \neq 1$, $a \neq 0$, and $b^2 - 4ac < 0$, we acquire

$$r(\xi) = \left(-\frac{b}{2a} + \frac{\sqrt{4ac-b^2}}{2a}\tan(\frac{(1-s)\sqrt{4ac-b^2}}{2}(\xi + C))\right)^{\frac{1}{1-s}}, \tag{12}$$

and

$$r(\xi) = \left(-\frac{b}{2a} - \frac{\sqrt{4ac-b^2}}{2a}\cot(\frac{(1-s)\sqrt{4ac-b^2}}{2}(\xi + C))\right)^{\frac{1}{1-s}}. \tag{13}$$

5. For $s \neq 1$, $a \neq 0$, and $b^2 - 4ac > 0$, we acquire

$$r(\xi) = \left(-\frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\coth(\frac{(1-s)\sqrt{b^2-4ac}}{2}(\xi + C))\right)^{\frac{1}{1-s}}, \tag{14}$$

and

$$r(\xi) = (-\frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\tanh(\frac{(1-s)\sqrt{b^2-4ac}}{2}(\xi + C)))^{\frac{1}{1-s}}. \tag{15}$$

6. For $s \neq 1$, $a \neq 0$, and $b^2 - 4ac = 0$, we acquire

$$r(\xi) = (\frac{1}{a(s-1)(\xi+C)} - \frac{b}{2a})^{\frac{1}{1-s}}, \tag{16}$$

here C is an arbitrary constant.

2.1. Bäcklund Transformation of the RB Equation

If $r_{n-1}(\xi)$ and $r_n(\xi) = r_n(r_{n-1}(\xi))$ are the solutions of Eq. (6), we have

$$\frac{dr_n(\xi)}{d\xi}=\frac{dr_n(\xi)}{d\xi}\frac{dr_{n-1}(\xi)}{d\xi} \qquad (17)$$
$$=\frac{dr_n(\xi)}{dr_{n-1}\xi}(ar_{n-1}^{2-s}+br_{n-1}+cr_{n-1}^{s}),$$

nam ely

$$\frac{dr_n(\xi)}{ar_n^{2-s}+br_n+cr_n^s}=\frac{dr_{n-1}(\xi)}{ar_{n-1}^{2-s}+br_{n-1}+cr_{n-1}^s}. \qquad (18)$$

Integrating equation (17) once with respect to ξ, we acquire

$$r_n(\xi)=\left(\frac{-cA_1+aA_2(r_{n-1}(\xi))^{1-s}}{bA_1+aA_2+aA_1(r_{n-1}(\xi))^{1-s}})\right)^{\frac{1}{1-s}}, \qquad (19)$$

here A_1 and A_2 are arbitrary constants.

3. IMPLEMENTATION OF THE METHOD

We transform Eq. (1) to the ODE below:

$$-Vu(\xi)+\frac{\gamma}{2}u(\xi)^2+u''(\xi)-\varepsilon u(\xi)u''(\xi)-\frac{k}{2}u'(\xi)^2-\upsilon u(\xi)-\sigma u(\xi)u'(\xi)=0. \qquad (20)$$

If we replace Eqs. (6) and (7) in Eq. (20), then we get

$$-6abu(\xi)^2+2absu(\xi)^2+2a\upsilon u(\xi)^2+2ab\beta u(\xi)^3+2a\delta u(\xi)^3+6ab\varepsilon u(\xi)^3 \qquad (21)$$
$$-2abs\varepsilon u(\xi)^3-4a^2u(\xi)^{3-s}+2a^2su(\xi)^{3-s}+a^2\beta u(\xi)^{4-s}+4a^2\varepsilon u(\xi)^{4-s}$$
$$-2a^2s\varepsilon u(\xi)^{4-s}-2bcu(\xi)^{2s}-2bcsu(\xi)^{2s}+2c\upsilon u(\xi)^{2s}+c^2\beta u(\xi)^{3s}$$
$$+2c^2s\varepsilon u(\xi)^{3s}-2b^2u(\xi)^{1+s}-4acu(\xi)^{1+s}+2b\upsilon u(\xi)^{1+s}+2Vu(\xi)^{1+s}+b^2\beta u(\xi)^{2+s}$$
$$+2ac\beta u(\xi)^{2+s}-\gamma u(\xi)^{2+s}+2b\delta u(\xi)^{2+s}+2b^2\varepsilon u(\xi)^{2+s}$$
$$+4ac\varepsilon u(\xi)^{2+s}+2bc\beta u(\xi)^{1+2s}+2c\delta u(\xi)^{1+2s}+2bc\varepsilon u(\xi)^{1+2s}$$
$$+2bcs\varepsilon u(\xi)^{1+2s}-2c^2su(\xi)^{-1+3s}=0,$$

setting $s=0$ in Eq. (21), we obtain

$$-2bc+c^2\beta+2c\upsilon-2b^2u(\xi)-4acu(\xi)+2bc\beta u(\xi)+2b\upsilon u(\xi)+2Vu(\xi)+2c\delta u(\xi)+$$
$$2bc\varepsilon u(\xi)-6abu(\xi)^2+b^2\beta u(\xi)^2+2ac\beta u(\xi)^2+2a\upsilon u(\xi)^2-\gamma u(\xi)^2+2b\delta u(\xi)^2+$$
$$2b^2\varepsilon u(\xi)^2+4ac\varepsilon u(\xi)^2-4a^2u(\xi)^3+2ab\beta u(\xi)^3+2a\delta u(\xi)^3+6ab\varepsilon u(\xi)^3+a^2\beta u(\xi)^4+$$
$$4a^2\varepsilon u(\xi)^4=0, \qquad (22)$$

setting each $u^i (i=0,1,2,3,4)$ to zero in Eq. (22), we have

$$c(-2b+c\beta+2\upsilon)=0, \qquad (23)$$
$$2(-b^2-2ac+n+c\delta+b(\upsilon+c(\beta+\varepsilon)))=0, \qquad (24)$$
$$(-\gamma+2b\delta+b^2(\beta+2\varepsilon)+2a(-3b+c\beta+\upsilon+2c\varepsilon))=0, \qquad (25)$$
$$2a(-2a+\delta+b(\beta+3\varepsilon))=0, \qquad (26)$$
$$a^2(\beta+4\varepsilon)=0. \qquad (27)$$

Solving Eqs. (23)-(27) using Mathematica 9, we get

- $\beta=-4\varepsilon$, $a=\frac{1}{8}\left(2\delta-2\upsilon\varepsilon\pm\sqrt{16\gamma\varepsilon+(-2\delta+2\upsilon\varepsilon)^2}\right)$, $b=\frac{2a\upsilon-\gamma}{2a}$, $c=0$, $n=b(b-\upsilon)$,

which produces the following soliton solutions

$$u_1(x,t)=(-\frac{2a^2}{2a\upsilon-\gamma}+Ce^{-\frac{2a\upsilon-\gamma}{2a}(\xi)})^{-1}. \qquad (28)$$

$$u_2(x,t)=\frac{-2a\upsilon-\gamma}{4a^2}+\frac{\gamma-2a\upsilon}{4a^2}\tan(\frac{\gamma-2a\upsilon}{2}(\xi+C)), \qquad (29)$$

and

$$u_3(x,t) = \frac{-2a\upsilon-\gamma}{4a^2} + \frac{2a\upsilon-\gamma}{4a^2}\cot(\frac{\gamma-2a\upsilon}{2}(\xi+C)). \tag{30}$$

$$u_4(x,t) = \frac{-2a\upsilon-\gamma}{4a^2} + \frac{2a\upsilon-\gamma}{4a^2}\coth(\frac{\gamma-2a\upsilon}{2}(\xi+C)), \tag{31}$$

and

$$u_5(x,t) = \frac{-2a\upsilon-\gamma}{4a^2} + \frac{2a\upsilon-\gamma}{4a^2}\tanh(\frac{\gamma-2a\upsilon}{2}(\xi+C)). \tag{32}$$

$$u_6(x,t) = (-\frac{1}{a(\xi+C)} + 2a\upsilon + \gamma), \tag{33}$$

where $\xi = x - ((\frac{2a\upsilon-\gamma}{2a})^2 - \frac{2a\upsilon^2-\gamma\upsilon}{2a})t$.

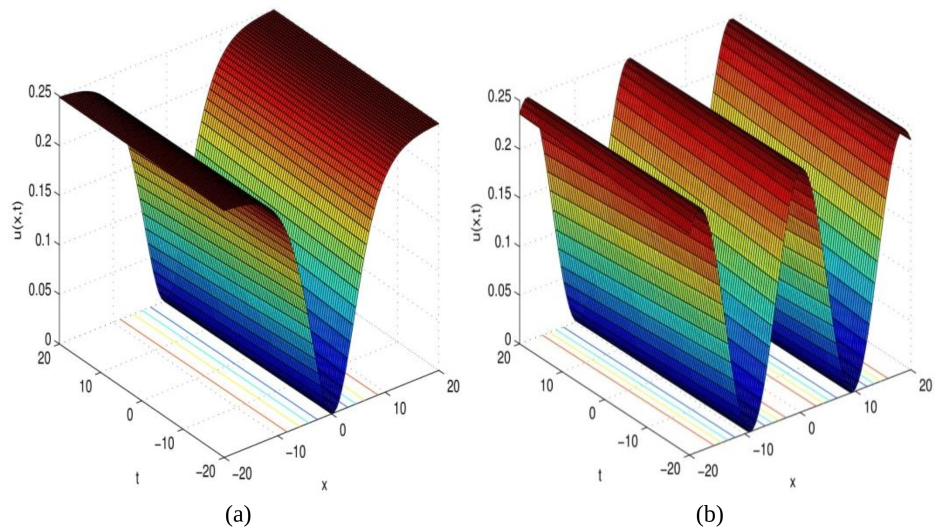
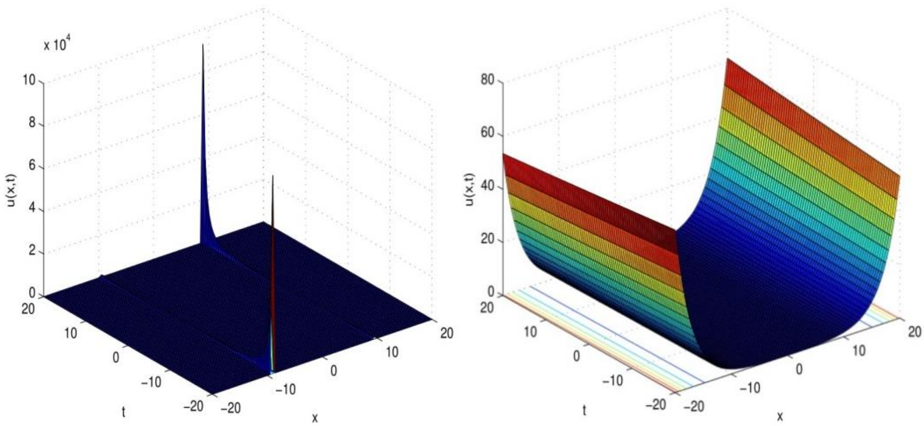


Figure 1. (a) Numerical simulation of Eq. (32) when $\upsilon = 2$, $\gamma = 1.3$, $\delta = 3$, $\varepsilon = 1$.
(b)Numerical simulation of Eq. (31) when $\upsilon = 5$, $\gamma = 1.3$, $\delta = 13$, $\varepsilon = 1$.



(c) (d)

Figure 2. (c) Numerical simulation of Eq. (29) when $\upsilon = 2, \gamma = 3, \delta = 5, \varepsilon = 1.8$.
(d) Numerical simulation of Eq. (30) when $\upsilon = 1.8, \gamma = 13, \delta = 1, \varepsilon = 2$.

4. CONCLUDING REMARK

In this study, we discussed and investigated the new soliton solutions for the KS equation in mathematical physics. The proposed method gave a new infinite sequence of solutions. These solutions were expressed by trigonmetric, hyperbolic, algebraic and exponential structures. Some phyiscal features of the obtained solutions are presented.

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