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RESEARCH ARTICLE

Determining the Effectiveness of the Bus Lines in Urban Transportation using Data Envelopment Analysis

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ABSTRACT

This study aims to determine the efficiency of urban public transport lines and to offer solutions for inefficient lines. In order to determine the efficiency of urban bus lines, a model was created using the data envelopment analysis method. In this model, the input variables were vehicle capacities, distances traveled and number of trips on weekdays and weekend lines. The output variables were the number of passengers carried in each line at peak hour, morning and evening. In addition, taking into account the number of passengers carried and the capacities of the line, capacity utilization rates (CURs) were determined for each line on weekdays and at weekends. CURs were used for the first time in the efficiency analysis of public transportation lines. As the study area, the bus lines used in urban public transportation in Erzurum were examined. As a result of the study, the efficiency and super efficiency degrees of the bus lines were determined. Analysis of the results determined that the highest limit is the K9 line and the lowest is the B4 line. The activity limit are the B2A, B7, G3, G7/A, G9, K3 and K8 lines closest to 100%.

Keywords: Public Transport, Efficiency Score, Efficiency Analysis, Data Envelopment Analysis

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Introduction

Along with the increase in car ownership day by day, there are a number of problems in cities, such as traffic jams, loss of time in traffic, environmental and air pollution, and energy loss. The development of public transportation systems is one of the basic ways of reducing these problems. Using public transportation systems correctly and effectively is important for the environment, health, and the economy. For this reason, accurate evaluation and analysis of the public transport operation and finding a solution to the significant existing problems are important issues that are useful for the planning, design, and operation of the public transport system.

Optimization of the lines should be done well in order to use public transport lines efficiently and effectively. The public transportation system has some inputs and outputs. The inputs of the system are variables such as line lengths, the number of vehicles on the line, capacity of the vehicles, the number of trips, and even the number of employees. The outputs of the system are the number of passengers carried and fuel consumption of the line. In order to determine whether the lines are working efficiently, the variables of the lines should be evaluated together.

The best use of resources is defined as efficiency. Effectiveness is related to how products and services should be produced as well as which products and services should be offered (Karlaftıs and Tsamboulas, 2012). It has been stated that efficiency is related to the savings to be achieved by increasing resource savings and / or output amount (Golany and Tamir, 1995). In the light of these definitions, developments such as coving more kilometers with less fuel cost in urban passenger transportation, reducing the total capacity despite the same number of passengers remaining, in other words, decreasing the operating cost can be described as a productivity and efficiency increase.

Productivity and effectiveness are generally used interchangeably, but they differ in meaning. According to this, it refers to the relationship between output expressed by efficiency, goods, services and other results and the resources used to produce them (Gülcü et al., 2004). Effectiveness is a concept that shows how well output can be produced by using production resources or inputs. It is interpreted as producing the most output using the existing input or obtaining the current output with less input (Budak, 2010). Efficiency measure associated with input and output variables is the ratio of the weighted sum of outputs to the weighted sum of inputs (Bektaş, 2007).

Data envelopment analysis (DEA) is a typical benchmark analysis that has been widely used in econometrics to estimate the efficiency of production units, and has been widely adopted in fields such as health, education, finance, local public transport, information communication technologies and macroeconomics of industry research.

In the literature, DEA has applications in all areas of transportation and areas of interest. Marchese et al (Marchese, Ferrari, and Benacchio, 2000) studied the efficiency of container port terminal operators. Cullinane et al., (Cullinane, Ping, Teng- Fei, 2004) performed DEA windows analysis to determine the efficiency of the world's leading container ports over time. Oll and Hayuth (1993) have defined a theoretical rate for port efficiency. Martinez-Budria et al. (1999) analyzed the efficiency of the ports in relation to the efficiency of a single port. Tongzon (2001) has paid particular attention to the number of cranes and docks for container ships and waiting times by examining the factors that affect the efficiency performance of a port.

DEA was also used for airport analysis: for example, the efficiency of Italian airports was measured by Curi, Gitto, and Mancuso (2008) following the sector's privatization. In a study by Adler and Berechman (2001), airline companies were analyzed to determine airport quality using DEA. Brian, Diarmuid, and Shane (2013) investigated the most efficient transport solution for the Dublin city center-airport route.

Hermans et al. (2009) proposes a computational model based on DEA, created on the model output, to identify the positive and negative aspects of road safety in each country analyzed. Shen et al., (2012) evaluated whether to use DEA as a performance measurement technique that provides an overview of a country's road safety, and whether the road safety results identified in a country correspond to the expected numbers depending on the exposure level. In the study, three models were used: the DEA based road safety model, cross efficiency method and categorical DEA model. In their study, Fancello, Uccheddu and Fadda (2013) used DEA years to compare urban road systems in different cities and evaluate road network performance using various indicators.

DEA began to be used in local public transport in the 1990s to compare results with other analytical techniques. DEA has been used not only to assess the efficiency of companies, but also to assess the impact of policies implemented in the local public transport sector (Gagnepain, Ivaldi, 2002; Piacenza, 2006). Chang and Kao (1992) used DEA to determine the effectiveness of five different bus companies in their study. Levaggi (1994) used parametric and nonparametric approaches to analyze the effectiveness of urban transport in Italy. Viton (1997) used DEA to determine the effectiveness of public and private bus companies. Karlaftis (2004) used DEA to determine the efficiency and effectiveness of transportation companies. Buzzo Margari et al. (2007) examined regulatory and environmental impacts on public transport efficiency using a mixed approach. The most frequently used inputs in the performance measurement of urban passenger transport are the number of employees (workforce), the number of vehicles (capital) and the amount of fuel (energy) (Karlaftıs and Tsamboulas, 2012; Chang Kao, 1992; Karlaftis, 2004; Nolan, 1996; Boame, 2004; Sanchez, 1999). Zhang (2016) evaluated the performance of public transit systems based on a combined evaluation method (CEM) consisting of information entropy theory and super efficiency data envelopment analysis (SE-DEA).

It is seen in the literature that cost-based inputs are also used. Operating costs, maintenance and repair costs, general administrative expenses, supply expenses (Chu, Fielding and Lamar, 1992; Boile, 2001), and cost per bus (Bhagavath, 2006) are some of these. It is also seen that variables related to the level of service are included in the model as input. Line length (Hahn, Kım and Kho, 2009), the number of stops (Hahn, Kım and Kho, 2009; Lao and Lıu, 2009; Hawas, Khan and Basu, 2012), service frequency (Hahn, Kım and Kho, 2009; Bhagavath, 2006), and operation time (Lao and Lıu, 2009; Hawas, Khan and Basu, 2012) are some of the variables related to determining the level of service (Güner, 2014).

On the execution of No.5393 and 5216 Metropolitan Municipality Law with the municipal public transport services, Sarı (2010) determined the effectiveness of the 16 metropolitan municipalities in Turkey using the DEA method.

In addition, the operational analysis of Adana Metropolitan Municipality Bus Branch Directorate, where some of the public transportation services of Adana are carried out, has been made and various suggestions have been made for the business to operate more efficiently. The analysis method in the study, data envelopment analysis model for input was used. A questionnaire form about transportation to the municipalities was prepared and eight different models were created with the data obtained.

In the first model, the input variables were bus numbers and staff numbers. The output variables were bus line number and the daily number of voyages per line. In the second model, the input variables were city population, combat area, number of settlement units. The output variables were the number of bus lines, number of daily flights per line, and occupancy rates in the lines. In the third model, the input variables were number of residential units and number of buses. The output variable was the number of bus lines. In the fourth model, the input values were the number of bus lines, number of buses, passenger capacities of buses and number of personnel working in the bus services. The output variable was the number of passengers carried. In the fifth model, the input variables were input values, the daily number of flights per line, and passenger capacity of buses. The output value was the occupancy rates of the lines. In the sixth model, the input variable were city population and the number of lines. The output variables were the number of daily flights per line and passenger capacity of buses. In the seventh model, the input variables were the number of lines and personnel expense line cost. The output variable was the revenue per line. In the eighth model, the input variables were the city population and number of residential units squared. The output variables were daily number of expeditions per line, passenger capacity of buses, average number of residential units passed by one line, and number of bus lines.

Güner and Coşkun (2016) determined the operational efficiency and service efficiency for public transport lines and compared the two efficiencies. While determining the operational efficiency, the input variables of the model were the number of buses, line length and daily fuel consumption. The output variable was the number of trips per day. In the service efficiency model, the input variables were frequency, service time, number of stops per kilometer, deviation from the shortest distance, travel time. The output variable was the number of trips without connection.

This research contributes to the literature on urban bus line performance in the following ways. (1) This paper takes into account Capacity Usage Ratios (CURs) of urban bus systems, and constructs an evaluation indicator system based on satisfaction and efficiency to measure public transit service performance. (2) This paper takes into account capacity, length of lines and number of passenger indicators. This research provides a comprehensive framework with the ability to account for various indicators.

The purpose of this study is to determine the activities of bus lines used in public transportation. Unlike previous studies in the literature, the input variable is the average travel distance on weekdays and weekends determined as the capacity of vehicles on weekdays and weekends. The output variable is the number of passengers carried at peak hour on weekdays and weekends determined as the weekly and weekend capacity utilization rate. Determining the capacity utilization rate for each line has not been used in previous studies. In addition, the variable of the number of passengers carried during peak hours has not been used in previous studies. Efficiency Measurement System-Version 1.3.0 (EMS) program was used to analyze the data envelopment analysis models.

2. Research Material

The material of this study is the data of bus lines used in public transportation in Erzurum (Table 1). The buses used in public transportation have two different capacities, 58 and 100 seats. Table 1 shows the capacities and frequencies of buses operating on the public transportation lines. In addition, in Table 1, the average number of passengers carried daily on weekdays and at weekends is indicated.

Using the number of passengers carried and the capacity information of the buses on the line, the capacity utilization rates for each line were determined according to the correlation in equation 1.

$$CUR = \left(\frac{p_i}{c_i}\right) * 100 \qquad (1)$$

CUR: The capacity utilization ratio

pi : Average number of passengers carried per day for i. the line

ci : Daily total capacity for i. the line

In Equality 1, the daily total capacity of the line is determined by multiplying the number of buses serving and the capacities of the buses. If there were two different capacity buses on one line, the total daily capacity of the line was determined as the result of the number of trips, diameter and total of each bus.

Table 1. Bus and transported passenger information on urban public transport lines in Erzurum

Line Name	Line Length (Km)	Weekdays						Weekend					
		Number of buses for 58 people (Piece)	Number of buses for 100 people (Piece)	Number of voyages (Piece)	Daily total capacity of the line (Piece)	Average number of passengers carried per day (Piece)	The capacity utilization rate	Number of buses for 58 people (Piece)	Number of buses for 100 people (Piece)	Number of Flights (Piece)	Daily total capacity of the line (Piece)	Average number of passengers carried per day (Piece)	The capacity utilization rate
B1	46.4	-	9	83	8300	6773,6	81,61	-	8	76	7600	4522.0	59.50
B2A	42.7	4	-	31	1798	1339,0	74,47	4	-	31	1798	893.8	49.71
B2	31.5	7	4	115	8560	6288,2	73,46	7	4	108	7146	4197.6	58.74
B3	30.8	7	4	114	8376	7270,4	86,80	7	4	109	7582	4853.2	64.01
B4	50.1	-	2	16	1600	97,6	6,10	-	2	16	1600	65.1	4.07
B5	47.5	-	1	7	700	370,1	52,87	-	1	7	700	247.0	35.29
B6	45.9	-	4	33	3300	2874,0	87,09	-	4	34	3400	1918.6	56.43
B7	50.3	-	6	45	4500	4410,0	98,00	-	6	45	4500	2943.9	65.42
B8	28.2	-	4	55	5500	2263,3	41,15	-	3	55	5500	1510.9	27.47
D1	35.3	3	-	32	1856	1350,1	72,74	3	-	32	1856	901.3	48.56
G1	19.4	11	-	172	9976	9339,5	93,62	10	-	137	7946	6234.4	78.46
G2	27.2	-	2	25	2500	497,3	19,89	2	-	25	1450	331.9	22.89
G3	26.3	12	1	136	8392	8146,1	97,07	10	-	116	6728	5437.6	80.82
G4	44.5	9	-	69	4002	5488,7	137,15	9	-	56	4046	3664.1	90.56
G4A	67.2	-	5	31	3100	2483,7	80,12	-	5	31	3100	1657.9	53.48
G5	21.6	8	2	110	7304	4342,2	59,45	8	-	96	5568	2898.7	52.06
G6	33	5	-	44	2552	1660,1	65,05	5	-	44	2552	1108.3	43.43
G7	20.8	10	-	124	7192	3599,6	50,05	10	-	124	7192	2402.8	33.41
G7A	19.6	6	-	61	3538	2173,4	61,43	9	-	61	3538	1450.9	41.01
G8	32.2	3	-	30	1740	923,1	53,05	3	-	30	1740	616.1	35.41
G9	28.9	14	4	170	10658	13445,1	126,15	18	-	153	8874	8975.2	101.14
G10	27.2	6	-	56	3248	2934,9	90,36	5	-	47	2726	1959.2	71.87
K1	25.4	2	1	38	2792	1637,8	58,66	2	-	32	1856	1093.4	58.91
K2	31.2	12	-	104	6032	8245,1	136,69	10	-	92	5336	5503.6	103.14
K3	41.7	10	-	74	4764	5370,0	112,72	8	-	62	3596	3584.5	99.68
K4	36.3	-	4	52	5200	2628,1	50,54	-	4	52	5200	1754.5	33.74
K5	28.5	2	-	21	1218	589,6	48,41	1	-	13	754	393.6	52.20
K6	29.3	2	-	23	1334	740,1	55,48	2	-	23	1334	494.0	37.03
K7	46.7	-	6	41	4100	3605,1	87,93	-	6	41	4100	2406.3	58.69
K7/A	42	-	5	36	3600	2424,6	67,35	-	5	36	3600	1618.6	44.96
K8	53.7	-	2	15	1500	1051,2	70,08	-	2	15	1500	701.7	46.78
K9	20.1	1	-	15	870	495,1	56,91	1	-	15	870	330.5	37.99
K10	30.3	5	-	61	3538	1984,5	56,09	5	-	61	3538	1324.6	37.44

2.1. Research Method

In the literature, it is seen that there are three basic approaches to measuring operational effectiveness (Yolalan, 1993). First of all, ratio analysis is calculated by determining the ratio of a single input to single output and performance indicators are also calculated. Then decision making units (DMUs) are compared according to these indicators. The second is that the parametric approach is associated with one output and multiple inputs. In this approach, it is assumed that the output function of the DMU, whose effectiveness will be measured, has an analytical structure. Third, a nonparametric approach is used. This method does not predict the existence of any analytical form of the output function of the DMU. Due to these features, they are more flexible than parameter methods. The nonparametric approach has a structure suitable for multi-input and multi-output efficiency measurement (**Figure 1**) (Yeşilyurt, 2003).

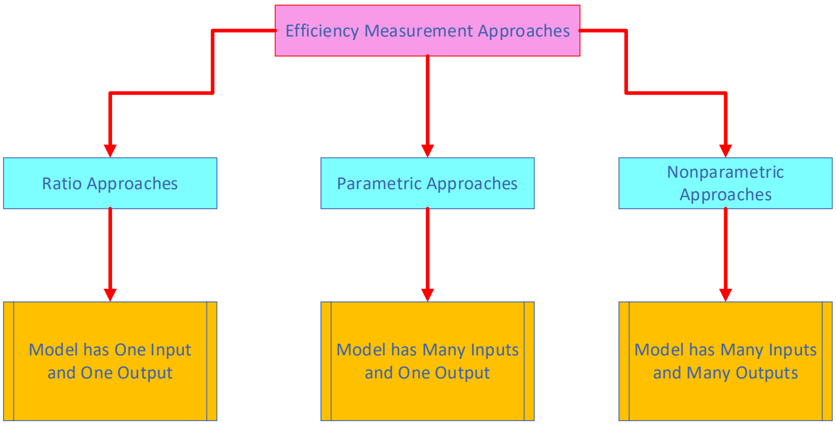

Figure 1. Approaches used to measure efficiency

Data Envelopment Analysis (DEA), which is one of the non-parametric efficiency measurement methods, is an efficiency measurement method developed to measure the relative effectiveness of homogeneous production units. Data envelopment analysis, which is capable of processing a large number of inputs and outputs, is an approach that can produce highly valid and meaningful results compared to other efficiency measurement methods by using mathematical programming (Bektaş, 2007). DEA is a non-parametric method based on linear programming that calculates the relative activities of units with common features and similar purposes (Anderson, Sweeney, and Williams, 1991).

DEA is an important method that provides the opportunity to measure the effectiveness of organizations compared to other organizations they are competing with, determines the sources and amounts of ineffectiveness of ineffective organizations and determines the rate of improvement that these enterprises need in order to be effective. Charnes, Cooper and Rhodes (1978) named organizations such as institutions, departments, businesses, and administrative units that produce similar outputs using similar inputs as DMU. DEA emerged for the first time in an article prepared by Farrell in 1957 as a multi-dimensional and non-parametric efficacy measurement method (Farrel, 1957). In this article, Mouse has examined the activities of units with multiple inputs and a single output, and for the first time used linear programming in efficiency measurement (Çağlar, 2003).

Based on this study in 1978, the efficiency of schools was predicted with many inputs and outputs, and it created the DEA proportional formula known as the Charnes, Cooper, Rhodes (CCR) model (Charnes, Cooper and Rhodes, 1978). In this study, Charnes, Cooper and Rhodes (1978) assume fixed return status according to the scale. Later, in Banker et al. studies, they dealt with the variable return by scale and this was called the BCC model (Banker, Charnes, and Cooper, 1984). Two separate formats for input and output have been installed for each of the CCR and BCC models. This situation not only increased the ability to interpret the results of DEA studies but also expanded the application area (Yeşilyurt and Alan, 2003).

DEA, as mentioned earlier, can be used for input and output in two ways. Input-oriented DEA models investigate how the most appropriate input combination should be used in order to produce a particular output composition in the most efficient way. Output-oriented DEA models, on the other hand, investigate how much output composition can be obtained with a given input composition. In this study, the effectiveness measurement of public transport services provided in metropolitan municipalities was made using the DEA model for input (Sarı, 2010).

2.2. Model Selection

Which of the models for input and output is chosen or what model to build depends on whether the inputs and outputs can be controlled. In other words, if the decision-maker has control over the input, the models for the input are preferred. If the control over the output is concerned, output-oriented models are preferred. Those who run the city have control and power over the inputs of public transport lines, but control and power over the outputs is quite difficult. For this reason, it was appropriate to use the input-oriented CCR model, which aims to minimize inputs, based on the constant return assumption according to the scale. The purpose of this model is to determine the best amount of input required to achieve a certain amount of output.

2.2.1. Input Oriented CCR Models

The CCR model, consisting of the initials of the names Charnes, Cooper and Rhodes, was created based on Farrell's previous work (Cooper, Seiford and Tone, 2007). Charnes, Cooper and Rhodes (Bircik, 2019) generalize the known effectiveness rate for single input and single output and for multiple inputs and multiple outputs. Considering the data, there is a need for optimization of each number of DMUs and for measuring the effectiveness of each DMU. As o = 1,2,…, n, the DMUs to be evaluated are indicated by DMUo and the others by DMUj.

For the variables, input "weight" (vi) for i= 1,2,…,m, and output "weight" (ur), for r=1,2,…,s, were obtained with the help of equations (1) and (2) (Cooper, Seiford and Tone, 2007).

$$max\theta = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} = \frac{u_1 y_{10} + u_2 y_{20} + \cdots + u_s y_s}{v_1 x_{10} + v_2 x_{20} + \cdots + v_m x_{m0}} \quad (1)$$

Constraints

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{i=1}^{m} v_i x_{i_j}} = \frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \leq 1, \tag{2}$$

$$j = 1,2 \dots, n;$$

$$u_r \geq 0;\ \ r = 1,2 \dots, s;$$

$$v_i \geq 0;\ \ \ i = 1,2, \dots, m$$

Identified here, θ is the effectiveness of oth DMU, ysj is the sth output produced by the jth DMU, xmj the mth input produced by the jth DMU, us is the weight given to the sth output by DMU, vm is the weight given to mth input by DMU. It shows n are the number of DMUs, s are number of outputs, and m are number of inputs.

For each DMU, the “virtual output” / “virtual input” ratio should not exceed 1 in restrictions. The aim is to evaluate the DMUs by maximizing the ratio of the input and output weights of the DMU. Instead of the fractional programming model above, when the following linear programming model (Cooper, Seiford and Tone, 2007) is created, the input direction CCR primal model is obtained;

$$\sum_{r=1}^{s} u_r y_{r_0} = u_1 y_{10} + u_2 y_{20} + \cdots + u_s y_s \tag{3}$$

Constraints

$$v_1 x_{10} + v_2 x_{20} + \cdots + v_m x_{m0} = 1$$

$$u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj} \leq v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}$$

$$j = 1,2 \dots, n;$$

$$u_r \geq 0;\ \ r = 1,2 \dots, s;$$

$$v_i \geq 0;\ \ \ i = 1,2, \dots, m \tag{4}$$

Here u_r shows the rth output weight, where r = 1,2,…, s. In the input CCR model, we tried to maximize the weighted sum of the outputs of each DMU, respectively (Equation 3). In the constraints, the weighted sum of the inputs of the interested DMU was equalized to 1, so that the weighted average of the inputs was 1 for each of the DMUs (Equation 4). The second constraint requires that the weighted sum of the outputs be smaller than the weighted sum of the inputs. In this way, the output / input ratio can be at most 1 for each of the DMUs, which means that the decision-making unit is active. The effectiveness value will be less than 1 for inactive DMUs (Bektaş, 2007).

3. Model Established In The Study

It has been explained in the introduction that the method suitable for the purpose of the study is DEA. Data envelopment analysis can determine the efficiency of systems with many inputs and many outputs. In this study, a system model with four inputs and four outputs was created. The inputs of the system were the average daily capacities of the working vehicles, and the travel distance. The outputs of the vehicles on the line were the number of passengers carried at peak hour and the capacity utilization rates of each line (**Figure 2**).

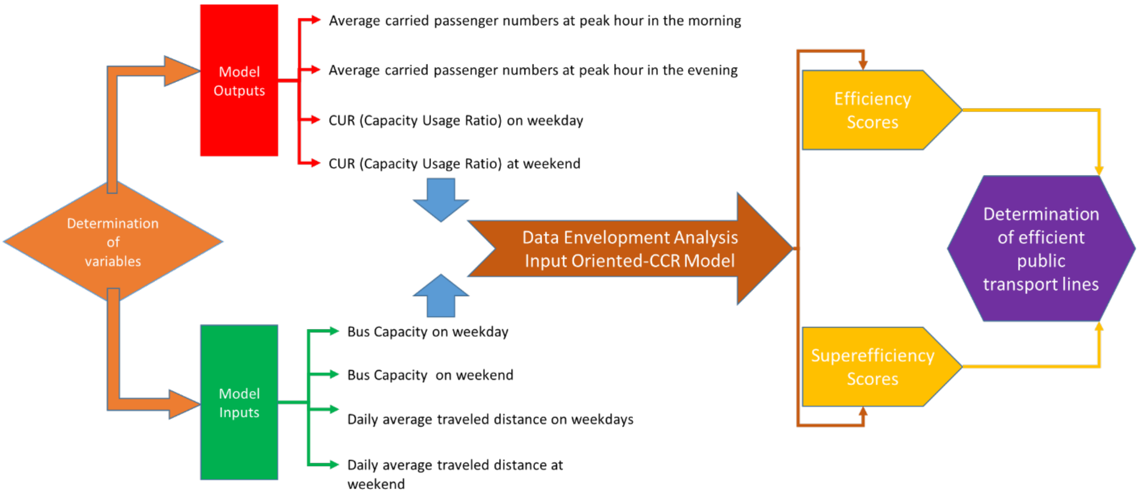

Figure 2. The stages of the model in the study

Municipal and public buses and minibuses are used in urban public transportation. In the Erzurum Metropolitan Municipality inventory, there are 205 buses on 33 different lines with two different types and capacities. The line length of the buses is quite different from each other and varies between 15 and 55 km (**Figure 3**). On the public transportation line with bus capacity of 58 and 100 people, the capacity utilization rate was obtained by proportioning the number of passengers carried or the total capacity of the vehicles used (Charnes et al., 1990).

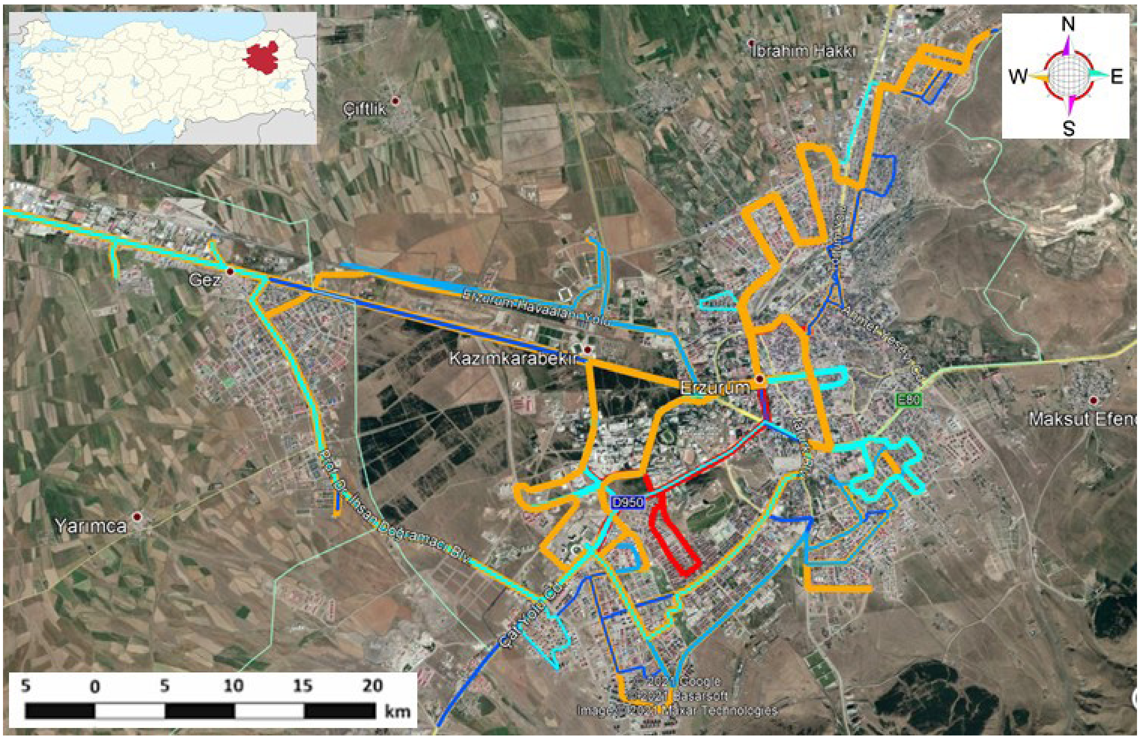

Figure 3. Bus lines in Erzurum

Efficiency results are highly dependent on the selection of inputs and outputs (Zhang 2016). In this study, the capacity of the lines and the length of the traveled lines were determined for both weekdays and weekends as inputs. The number of passengers

Table 2. Input and output variables of the model in the study

Line Name	Weekday Capacity {I}	Weekend Capacity {I}	Daily average weekdays {I}	Weekend average daily distance {I}	Average Number of Passengers Carried At Peak Hour In The Morning {O}	Average Number of Passengers Carried In The Evening At Peak Hour {O}	Weekdays CUR {O}	Weekend CUR {O}
B1	8300	7600	3851	3526	430	679	81.61	59.5
B2A	1798	1798	1324	1324	94	120	74.47	49.71
B2	8560	7146	3623	3402	470	533	73.46	58.74
B3	8376	7582	3511	3357	472	572	86.8	64.01
B4	1600	1600	802	802	0	0	6.1	4.07
B5	700	700	333	333	10	62	52.87	35.29
B6	3300	3400	1515	1561	201	220	87.09	56.43
B7	4500	4500	2264	2264	365	360	98	65.42
B8	5500	5500	1551	1551	203	365	41.15	27.47
D1	1856	1856	1130	1130	89	148	72.74	48.56
G1	9976	7946	3337	2658	613	1093	93.62	78.46
G2	2500	1450	680	680	19	69	19.89	22.89
G3	8392	6728	3577	3051	407	930	97.07	80.82
G4	4002	4046	3071	2492	308	500	137.15	90.56
G4A	3100	3100	2083	2083	113	77	80.12	53.48
G5	7304	5568	2376	2074	462	411	59.45	52.06
G6	2552	2552	1452	1452	108	120	65.05	43.43
G7	7192	7192	2579	2579	346	394	50.05	33.41
G7A	3538	3538	1196	1196	132	324	61.43	41.01
G8	1740	1740	966	966	95	0	53.05	35.41
G9	10658	8874	4913	4422	508	1071	126.15	101.14
G10	3248	2726	1523	1278	154	231	90.36	71.87
K1	2792	1856	965	813	159	158	58.66	58.91
K2	6032	5336	3245	2870	564	745	136.69	103.14
K3	4764	3596	3086	2585	308	227	112.72	99.68
K4	5200	5200	1888	1888	102	400	50.54	33.74
K5	1218	754	599	371	4	39	48.41	52.2
K6	1334	1334	674	674	69	43	55.48	37.03
K7	4100	4100	1915	1915	242	319	87.93	58.69
K7/A	3600	3600	1512	1512	200	173	67.35	44.96
K8	1500	1500	806	806	94	87	70.08	46.78
K9	870	870	302	302	47	50	56.91	37.99
K10	3538	3538	1848	1848	76	161	56.09	37.44

transported during peak hours and capacity utilization rates were determined as output variables. Output variables measure the yield or level of activity of services. The outputs used to measure efficiency in public transit are usually vehicle-km, seat-km, passenger-km, passengers and prime operating revenue (Karlaftis, 2004; Zhang, 2016). In Table 2, the names of the bus lines are in the first column. In the second and third columns, there are daily capacities based on the number of vehicles working on weekdays and weekend lines, and the number of trips. In the fourth and fifth columns, the total number of distances covered by bus times and line lengths of the buses in each line is determined in **Table 1**. In the sixth and seventh columns, there are the number of passengers carried in the morning and evening peak hours. The number of passengers carried at the peak hour was determined for morning and evening on weekdays (**Table 2**). In the eighth and ninth columns, the capacity utilization rates (CUR) were determined by using the number of passengers carried on the lines and capacities and equation 1. In **Table 2**, the input variables of the model in this study are specified with {I} and output variables with {O}.

4. Results

In this study, the findings presented that DEA will be used to determine the efficiency of public transport lines. Zhang (2016) used DEA in a case study to evaluate the performance of 13 transit operators in YDR of China. Caulfield, Bailey and Mullarkey (2013) showed how the DEA methodology can be used to compare different modes of public transport. The DEA analysis conducted in this research allowed the determination of the most efficient solutions for the airport route.

According to the results obtained after the analysis made in this study, the active lines were compared with the inactive lines. Within the scope of the study, efficiency values related to 33 lines used in urban transportation in Erzurum are given in **Table 3**. The data used in this study are the variables of the number of passengers carried on the lines, line lengths and capacity utilization rates. Variables were obtained separately for both weekdays and weekends. In addition, in this study, capacity utilization rate values were used for each specially determined line. Zhang (2016) used GDP, number of passengers per year, public transport price, population density, and total length of bus lines. Some studies also focused on the social outcomes, such as frequency, stops per km, safety, population density etc. of transit services (Chu, Fielding, and Lamar, 1992; Lao and Liu, 2009).

The efficiency score of lines B5. G1. G4. G5. K1. K2. K5 and K9 was 100%. The lines closest to the efficiency limit were B2A with an efficiency score of 80.10%. B7 with an efficiency score of 92.37%. G3 with an efficiency score of 99.62%. G7/A with an efficiency score of 90.74%. G9 with an efficiency score of 86.82%. K3 with an efficiency score of 96.17% and K8 with an efficiency score of 92.47%. These lines were not efficient. However. since they are very close to the efficiency limit. the improvements they need in order to be effective will be easier to achieve than the lines that are far or too far from the efficiency limit.

Table 3. Results of the model established in the study

Line order no	**Line Name**	**Efficiency Score**	**Super efficiency Score**	**Bus Line Capacity on weekdays {I}**	**Bus Line Capacity at weekend {I}**	**Daily average distance traveled on weekdays {I}**	**Daily average distance traveled at weekend {I}**	**Average carried number of passengers at peak hour in the morning {O}**	**Average carried passenger numbers at peak hour in the evening {O}**	**CUR (Capacity Usage Ratio) on weekdays {O}**	**CUR (Capacity Usage Ratio) at weekend {O}**	**References**
1	B1	69.07%	69.07%	0.731	0	0.269	0	0	1	0	0	G1 (0.209) K2 (0.605)
2	B2A	80.10%	80.10%	1	0	0	0	0.403	0.155	0.443	0	G1 (0.087) K2 (0.034) K9 (1.016)
3	B2	71.15%	71.15%	0.171	0	0.829	0	0.934	0.066	0	0	G1 (0.003) G5 (0.445) K2 (0.465)
4	B3	73.94%	73.94%	0.172	0	0.828	0	0.93	0.07	0	0	G1 (0.079) G5 (0.371) K2 (0.448)
5	B4	5.05%	5.05%	0.866	0.134	0	0	0	0	1	0	B5 (0.115)
6	**B5**	**100.00%**	**126.88%**	**0**	**1**	**0**	**0**	**0**	**0.359**	**0.577**	**0.064**	**7**
7	B6	77.52%	77.52%	0.161	0	0.839	0	0.913	0	0.087	0	G5 (0.035) K2 (0.254) K9 (0.884)
8	B7	92.37%	92.37%	0.149	0	0.851	0	0.944	0	0.056	0	G5 (0.063) K2 (0.572) K9 (0.282)
9	B8	73.66%	73.66%	0	0	1	0	0	0.898	0.102	0	G1 (0.325) K9 (0.188)
10	D1	79.41%	79.41%	0.956	0.044	0	0	0.271	0.277	0	0.452	B5 (0.379) G4 (0.186) K2 (0.012) K9 (0.450)
11	**G1**	**100.00%**	**143.97%**	**0**	**0**	**0**	**1**	**0.287**	**0.713**	**0**	**0**	**12**
12	G2	44.28%	44.28%	0	0.766	0.234	0	0	0.696	0	0.304	6 (0.567) G1 (0.025) K2 (0.009)
13	G3	99.62%	99.62%	0	0.967	0	0.033	0	1	0	0	G1 (0.350) K2 (0.735)
14	**G4**	**100.00%**	**106.57%**	**1**	**0**	**0**	**0**	**0**	**0.847**	**0.153**	**0**	**3**
15	G4A	52.71%	52.71%	1	0	0	0	0.604	0	0.396	0	K2 (0.104) K9 (1.158)
16	**G5**	**100.00%**	**106.14%**	**0**	**0.172**	**0.828**	**0**	**1**	**0**	**0**	**0**	**7**
17	G6	57.55%	57.55%	1	0	0	0	0.643	0	0.357	0	K2 (0.120) K9 (0.854)
18	G7	71.41%	71.41%	0.196	0	0.804	0	0.934	0.066	0	0	G1 (0.101) G5 (0.461) K2 (0.126)
19	G7A	90.74%	90.74%	0.097	0	0.903	0	0	0.84	0.16	0	B5 (0.049) G1 (0.266) K9 (0.596)
20	G8	72.32%	72.32%	1	0	0	0	0.66	0	0.34	0	K2 (0.113) K9 (0.660)
21	G9	86.82%	86.82%	0	0.956	0.044	0	0	1	0	0	G1 (0.281) K2 (1.026)
22	G10	79.80%	79.80%	0	0.568	0	0.432	0.166	0.448	0.034	0.353	B5 (0.223) G1 (0.027) K2 (0.196) K5 (0.442) K9 (0.490)
23	**K1**	**100.00%**	**106.59%**	**0**	**0.252**	**0**	**0.748**	**0.783**	**0**	**0**	**0.217**	**1**
24	**K2**	**100.00%**	**132.77%**	**0.126**	**0.872**	**0.002**	**0**	**0.869**	**0.131**	**0**	**0**	**22**
25	K3	96.17%	96.17%	0.128	0.872	0	0	0.602	0	0	0.398	K1 (1.038) K2 (0.218) K9 (0.422)
26	K4	69.87%	69.87%	0.68	0	0.32	0	0	0.936	0.064	0	B5 (0.243) G1 (0.308) K2 (0.065)
27	**K5**	**100.00%**	**137.32%**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**1**	**1**
28	K6	78.77%	78.77%	1	0	0	0	0.574	0	0.426	0	K2 (0.051) K9 (0.851)
29	K7	73.30%	73.30%	0.118	0	0.882	0	0.862	0.067	0.071	0	G1 (0.037) G5 (0.013) K2 (0.319) K9 (0.704)
30	K7/A	74.66%	74.66%	0.174	0	0.826	0	0.931	0	0.069	0	16 (0.142) K2 (0.190) K9 (0.579)
31	K8	92.47%	92.47%	1	0	0	0	0.592	0	0.408	0	K2 (0.080) K9 (1.039)
32	**K9**	**100.00%**	**167.00%**	**0**	**0**	**1**	**0**	**0.375**	**0**	**0.625**	**0**	**15**
33	K10	40.38%	40.38%	0.857	0	0.143	0	0	0.823	0.177	0	B5 (0.634) G4 (0.006) K2 (0.160)

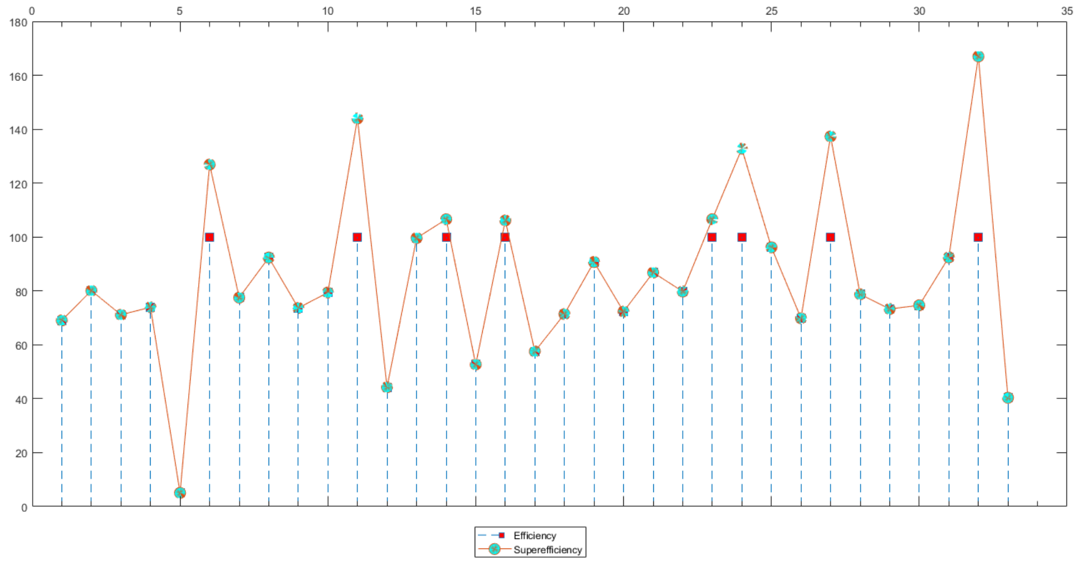

Figure 4. Efficiency and super efficiency scores

Among the errors that are effective in DEA analysis. the ones which are more effective are determined from the super activity score. In Table 3. the highest score according to the super efficiency score was K9 line with 167.0%. The second most effective line was the G1 line with 143.97%. G4. G5 and K1 line's super efficiency scores were 106.57%. 106.14% and 106.59%. respectively (**Figure 4**).

The capacity of the B2 / A line affected the efficiency score 100% negatively from the input variables. To bring the B2A line to its effective limit. weekday capacity needs to be reduced. B2A line was similar to G4. K2 and K9 lines: 8.7%. 3.4% and 101.6%. respectively. If improvements are made with reference to similar lines. the efficiency limit will be reached.

The daily average weekly distance traveled from the input variables of the B7 line's effectiveness score negatively affected 85.10%. The distance traveled on weekdays must be reduced to bring the B7 line to its effective limit. Line B7 was similar to lines G5. K2 and K9 at 6.3%. 57.2% and 2.82%. respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

The efficiency score of the G3 line affected 96.70% of capacity over the weekend from the input variables. To bring the G3 line to its effective limit. capacity over the weekend should be reduced. The G3 line was similar to the G1 and K2 lines at the rates of G1 35% and 73.5% respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

The distance traveled on weekdays from the input variables affected the efficiency score of the G7 / A line by 90.30%. In order to bring the G7/A line to its effective limit. the distance traveled on weekdays must be reduced. G7 / A line B5. G1 and K2 lines were 4.9%. 26.6% and 59.6% respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

The capacity of the G9 line affected the efficiency score 95.6% of the input variables over the weekend. To bring the G9 line to its effective limit. the distance traveled over the weekend should be reduced. The G9 line was similar to the G1 and K2 lines at the rates of 28.1% and 102.6% respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

The efficiency score of the K3 line affected 87.20% of the capacity negatively from the input variables. In order to bring the K3 line to its effective limit. the distance traveled on weekdays must be reduced. The K3 line was similar to the K1. K2 and K9 lines at the rates of 103.8%. 21.8% and 42.2%. respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

The weekly capacity negatively affected the efficiency score of the K8 line from the input variables. To bring the K8 line to its effective limit. weekday capacity reduction is required. The K8 line was similar to the K2 and K9 lines at 8% and 103.9% respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

The line that is the farthest from the event score was the B4 line with 5.05% activity score. The activity score of the B4 line affected the weekly capacity 86.60% negatively from the input variables. To bring the B4 line to its effective limit. capacity should be reduced on weekdays. Line B4 was similar to line B5 with a rate of 11.5%. If improvements are made with reference to line B5. the efficiency limit will be reached.

Another line that is farthest from the event boundary was the K10 line with an 40.38% event score. The weekly capacity negatively affected the efficiency score of the K10 line from the input variables. To bring the K10 line to its effective limit. capacity should be reduced on weekdays. Line K10 was similar to lines B5. G4 and K2 at the rates of 63.4%. 0.6% and 16%. respectively. If improvements are made with reference to these lines. the efficiency limit will be reached.

Conclusions

In this study. the efficiency of 33 bus lines used in Erzurum urban public transportation was determined. The DEA method was used to determine the efficiency of the lines. While creating input and output variables in the DEA. the most appropriate variables were determined to ascertain public transport efficiency. In this study. depending on the input variables. vehicle capacities on weekdays and weekend lines and number of trips on weekday and weekend capacities were taken into account separately. Using the line lengths and the number of trips of the vehicles on the lines. the distances traveled separately on weekdays and weekends of each line were determined. The output variables of the system were determined as the number of passengers carried at peak hour in each line for morning and evening. In addition. taking into account the number of passengers carried and the capacities of the line. capacity utilization rates were determined for each line on weekdays and at weekends.

The aim of DEA is to calculate an efficiency limit measured according to the distance to the ideal class created using the observed input and output data of different units called

DMU. In this study. the lines formed DMUs. Input and output data of DMUs were used. As a result of the analysis. it was determined that the highest limit was the K9 line and the lowest was the B4 line. The activity limits closest to 100% were the B2A. B7. G3. G7/A. G9. K3 and K8 lines.

In this study. the efficiency values of the lines used in urban public transportation were determined. It was determined which lines are active and which lines are not. It was determined which input variable values of inactive lines will be reduced. It was determined which lines are similar to other lines. Due to these determinations. DEA analysis will be beneficial for decision makers in the assessment of urban public transportation lines.

In this study. the shortening of the existing line as a solution for the lines in which line length is reduced is impossible for public transportation. which is a social service. For example. it has been determined that the distance traveled on lines such as B7. G1. G9 and K3 should be reduced. In these lines. the distance can be reduced by reducing the number of trips. In order to reduce the capacity of lines such as B2/A. G3 and K8. instead of reducing the number of vehicles working on the lines. the number of voyages of the working vehicles can be reduced and a solution can be provided.

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