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On The Non-Existence of Weakly Symmetric Nearly Kenmotsu Manifold with Semi-Symmetric Metric Connection

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Abstract

The object of this paper is to study the non-existence of weakly symmetric nearly Kenmotsu manifold with semi-symmetric metric connection.

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1. Introduction

In 1989, L. Tamassy and T. Q. Binh introduced the notions of weakly symmetric and weakly Ricci-symmetric manifolds [13]. A non flat differentiable manifold M^{2n+1} is called weakly symmetric if there exist a vector field P and 1—forms α , β , γ , δ on M^{2n+1} such that

$$(\nabla_X R)(Y, Z)V = \alpha(X)R(Y, Z)V + \beta(Y)R(X, Z)V + \gamma(Z)R(Y, X)V + \delta(V)R(Y, Z)X + g(R(Y, Z)V, X)P, \tag{1.1}$$

holds for all vector fields $X,Y,Z,V\in\chi(M^{2n+1})$ ([1], [13], [14]). A weakly symmetric manifold (M^{2n+1},g) is pseudo symmetric if $\beta=\gamma=\delta=\frac{1}{2}\alpha$ and P=A, locally symmetric if $\alpha=\beta=\gamma=\delta=0$ and P=0. A weakly symmetric manifold is said to be proper if at least one of the 1-forms α,β,γ and δ is not zero or $P\neq 0$ holds for all vector fields $X,Y,Z\in\chi(M^{2n+1})$.

A non-flat differentiable manifold M^{2n+1} is called weakly Ricci-symmetric if there exist 1-forms ρ , μ , ν such that the condition

$$(\nabla_X S)(Y,Z) = \rho(X)S(Y,Z) + \mu(Y)S(X,Z) + \upsilon(Z)S(X,Y),$$

holds for all vector fields X,Y,Z $\chi(M^{2n+1})$. If $\rho=\mu=\upsilon$ then M^{2n+1} is called pseudo Ricci-symmetric. If M^{2n+1} is weakly symmetric, from (1.1), we have

$$(\nabla_X S)(Z, V) = \alpha(X)S(Z, V) + \beta(R(X, Z)V + \gamma(Z)S(X, V) + \delta(V)S(X, Z) + \rho(R(X, V)Z)$$

$$\tag{1.2}$$

where p is defined by p(X) = g(X, P) for all $X \in \chi(M^{2n+1})$ ([11], [13]). On the other hand, the notion of a semi-symmetric connection on a differentiable manifold were introduced in [8]. A linear connection ∇ is called a semi-symmetric connection if it is not torsion free and satisfies the expression $T(X,Y) = \eta(Y)X - \eta(X)Y$. It is known that if $\nabla g = 0$, then the connection which satisfies the semi-symmetric condition, is called semi-symmetric metric connection, otherwise it is non-metric [12]. Hayden and Yano improved this concept and obtained several important results in Riemannian manifolds ([9], [15]). In recent years, there have been many studies on weakly symmetric and semi-symmetric metric connection ([1], [2], [5], [6]).

In this paper we have investigated weakly symmetric nearly Kenmotsu manifolds with respect to the semi-symmetric metric connection. Firstly we give some brief information about the nearly Kenmotsu manifolds admitting semi-symmetric metric connection. Then we obtain necessary conditions of the non-existence of weakly symmetric nearly Kenmotsu manifold with semi-symmetric metric connection.

2. Preliminaries

An n-dimensional differentiable manifold M^{2n+1} is called an almost contact Riemannian manifold if there is an almost contact structure (φ, ξ, η) consisting of a (1, 1) tensor field φ , a vector field ξ and 1-form satisfying

$$\eta(\xi) = 1 \quad \varphi \xi = 0, \tag{2.1}$$

$$\varphi^2 X = -X + \eta(X)\xi,\tag{2.2}$$

Let g be the Riemannian metric with the almost contact structure, that is

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y),$$

$$\eta(X) = g(X, \xi),$$
(2.3)

for any vector fields X, Y on M^{2n+1} , then the manifold is said to be almost contact metric manifold ([7]). If moreover

$$(\nabla_X \varphi)Y = -g(X, \varphi Y)\xi - \eta(Y)\varphi X, \nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

where ∇ denotes the Riemannian connection of g, then $(M^{2n+1}, \varphi, \xi, \eta, g)$ is called a Kenmotsu manifold [10]. An almost contact manifold $(M^{2n+1}, \varphi, \xi, \eta, g)$ is called nearly Kenmotsu manifold by with the following relation ([3], [4]):

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{2.5}$$

$$R(\xi, X)Y = -g(Y, X)\xi + \eta(X)Y, \tag{2.6}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X,\tag{2.7}$$

$$S(X,\xi) = -2n\eta(X), \ Q\eta = -2n\xi, \tag{2.8}$$

$$S(\phi X, \phi Y) = S(X, Y) + 2n\eta(X)\eta(Y), \tag{2.9}$$

for all vector fields X, Y, Z in which R denotes the Riemannian curvature tensor and S denotes the Ricci tensor.

Definition 2.1. Let M^{2n+1} be an (2n+1)-dimensional nearly Kenmotsu manifold. A connection $\tilde{\nabla}$ in M^{2n+1} is called semi-symmetric connection if its torsion tensör [12]

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y],\tag{2.10}$$

satisfies

$$T(X,Y) = \eta(Y)X - \eta(X)Y. \tag{2.11}$$

Further, a semi-symmetric connection is called semi-symmetric metric connection if

$$(\tilde{\nabla}_X g)(Y, Z) = g(\tilde{\nabla}_X Y, Z) + g(Y, \tilde{\nabla}_X Z). \tag{2.12}$$

On a (2n+1)—dimensional nearly Kenmotsu manifold with semi-symmetric metric connection some basic curvature properties as follows [12]:

$$\tilde{R}(X,Y)Z = R(X,Y)Z + 3[g(X,Z)Y - g(Y,Z)X] + 2[g(X,Y)\eta(X) - g(X,Z)\eta(Y)]\xi + 2[\eta(Y)X - \eta(X)Y]\eta(Z), \tag{2.13}$$

$$\tilde{S}(Y,Z) = S(Y,Z) + (2 - 6n)g(Y,Z) + 2(2n - 1)\eta(Y)\eta(Z),\tag{2.14}$$

$$\tilde{S}(Y,\xi) = -4n\eta(Y),\tag{2.15}$$

$$\tilde{R}(X,Y)\xi = 2[\eta(X)Y - \eta(Y)X],\tag{2.16}$$

$$\tilde{R}(\xi, Y)Z = 2[-g(Y, Z)\xi + \eta(Z)Y],$$
(2.17)

$$\tilde{R}(X,Y)Z = -\tilde{R}(Y,X)Z,\tag{2.18}$$

$$\tilde{Q}Y = -4nY, \tilde{Q}\xi = -4n\xi, \tag{2.19}$$

$$\tilde{S}(\phi Y, \phi Z) = S(Y, Z) + (2 - 6n)g(Y, Z) + 2(2n - 2)\eta(Y)\eta(Z), \tag{2.20}$$

$$\tilde{r} = r - 12n^2 + 2n,\tag{2.21}$$

$$\tilde{\nabla}_X \xi = 2(X - \eta(X)\xi = -2\phi^2 X,\tag{2.22}$$

$$\eta(\tilde{R}(X,Y)Z) = 2[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)], \tag{2.23}$$

$$\tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y = 0. \tag{2.24}$$

3. Main Results

Theorem 3.1. There is no weakly symmetric nearly Kenmotsu manifold with semi-symmetric metric connection, unless $\alpha + \gamma + \delta$ is everywhere zero.

Proof. Assume that M^{2n+1} is a weakly symmetric nearly Kenmotsu manifold. By the covariant differentiation of the semi-symmetric tensor \tilde{S} with respect to X we have

$$(\tilde{\nabla}_X \tilde{S})(Z, V) = \tilde{\nabla}_X \tilde{S}(Z, V) - \tilde{S}(\tilde{\nabla}_X Z, V) - \tilde{S}(Z, \tilde{\nabla}_X V), \tag{3.1}$$

so replacing V with ξ in (3.1) and using (2.2), (2.4) and (2.15), we obtain

$$\begin{split} &(\tilde{\nabla}_{X}\tilde{S})(Z,\xi) = \tilde{\nabla}_{X}\tilde{S}(Z,\xi) - \tilde{S}(\tilde{\nabla}_{X}Z,\xi) - \tilde{S}(Z,\tilde{\nabla}_{X}\xi) \\ &= \tilde{\nabla}_{X}(-4n\eta(Z)) + 4n\eta(\tilde{\nabla}_{X}Z) + 2\tilde{S}(Z,\varphi^{2}X) \\ &= -4n\tilde{\nabla}_{X}g(Z,\xi) + 4n\eta(\tilde{\nabla}_{X}Z) - 2\tilde{S}(Z,X) - 8n\eta(X)\eta(Z) \\ &= -4ng(\tilde{\nabla}_{X}Z,\xi) - 4ng(Z,\tilde{\nabla}_{X}\xi) + 4n\eta(\tilde{\nabla}_{X}Z) - 2\tilde{S}(Z,X) - 8n\eta(X)\eta(Y) \\ &= 8ng(Z,\varphi^{2}X) - 2\tilde{S}(Z,X) - 8n\eta(X)\eta(Y) \\ &= -8ng(Z,X) + 8n\eta(X)\eta(Y) - 2\tilde{S}(Z,X) - 8n\eta(X)\eta(Z) \end{split}$$

and

$$(\tilde{\nabla}_X \tilde{S})(Z, \xi) = -8ng(Z, X) - 2\tilde{S}(Z, X). \tag{3.2}$$

On the order hand replacing V with ξ in (1.2) and by the use of

$$\begin{split} &(\tilde{\nabla}_{X}\tilde{S})(Z,\xi) = \alpha(X)\tilde{S}(Z,\xi) + \beta(\tilde{R}(X,Z)\xi) + \gamma(Z)\tilde{S}(X,\xi) + \delta(\xi)\tilde{S}(X,Z) + p(\tilde{R}(X,\xi)Z) \\ &= -4n\alpha(X)\eta(Z) + \beta(2[\eta(X)Z - \eta(Z)X] - 4n\gamma(Z)\eta(X) + \delta(\xi)\tilde{S}(X,Z) + p(2[g(X,Z)\xi - \eta(Z)X]) \\ &= -4n\alpha(X)\eta(Z) + 2\eta(X)\beta(Z) - 2\eta(Z)\beta(X) - 4n\gamma(Z)\eta(X) + \delta(\xi)\tilde{S}(X,Z) + 2g(X,Z)p(\xi) - 2\eta(Z)p(X). \end{split}$$
(3.3)

Hence, comparing the right hand sides of the equations (3.2) and (3.3) we have

$$-8ng(Z,X) - 2\tilde{S}(Z,X) = -4n\alpha(X)\eta(Z) + 2\eta(X)\beta(Z) - 2\eta(Z)\beta(X) - 4n\gamma(Z)\eta(X) + \delta(\xi)\tilde{S}(X,Z) + 2g(X,Z)p(\xi) - 2\eta(Z)p(X).$$
(3.4)

Therefore putting $X = Z = \xi$ in (3.4) and using (2.15) and (2.1), we get

$$-8n + 8n = -4n\alpha(\xi) - 4n\gamma(\xi) - 4n\delta(\xi), \tag{3.5}$$

$$0 = \alpha(\xi) + \gamma(\xi) + \delta(\xi), \tag{3.6}$$

holds on M^{2n+1} . Now we will show that $\alpha + \gamma + \delta = 0$ hols for all vektor fields on M^{2n+1} . In (1.2) taking $Z = \xi$ similar to the previous calculations it follows that

$$(\tilde{\nabla}_X \tilde{S})(\xi, V) = \alpha(X)\tilde{S}(\xi, V) + \beta(\tilde{R}(X, \xi)V) + \gamma(\xi)\tilde{S}(X, V) + \delta(V)\tilde{S}(X, \xi) + P(\tilde{R}(X, V)\xi),$$

$$-8ng(V,X) - 2n\tilde{S}(V,X) = -4n\alpha(X)\eta(V) + \beta[2g(X,V)\xi - \eta(V)X] + \gamma(\xi)\tilde{S}(X,V) - 4n\delta 8V)\eta(X) + p(2[\eta(X)V - \eta(V)X]),$$

$$-8ng(V,X) - 2\tilde{S}(V,X) = -4n\alpha(X)\eta(V) + 2g(X,V)\beta(\xi) - 2\eta(V)\beta(X) + \gamma(\xi)\tilde{S}(X,V) - 4n\delta(V)\eta(X) + 2\eta(X)p(V) - 2\eta(V)p(X).$$
(3.7)

Putting $V = \xi$ in (3.7) by virtue of (2.1) and (2.15), we get

$$-8ng(\xi,X) - 2\tilde{S}(\xi,X) = -4n\alpha(X)\eta(\xi) + 2g(X,\xi)\beta(\xi) - 2\eta(\xi)\beta(X) + \gamma(\xi)\tilde{S}(X,\xi) - 4n\delta(\xi)\eta(X) + 2\eta(X)p(\xi) - 2\eta(\xi)p(X),$$

$$0 = -4n\alpha(X) + 2\eta(X)\beta(\xi) - 2\beta(X) - 4n\eta(X)\gamma(\xi) - 4n\delta(\xi)\eta(X) + 2\eta(X)p(\xi) - 2p(X). \tag{3.8}$$

Now taking $X = \xi$ in (3.7), we have

$$-8ng(V,\xi) - 2\tilde{S}(V,\xi) = -4n\alpha(\xi)\eta(V) + 2g(\xi,V)\beta(\xi) - 2\eta(V)\beta(\xi) + \gamma(\xi)\tilde{S}(\xi,V) - 4n\delta(V)\eta(\xi) + 2\eta(\xi)p(V) - 2\eta(V)p(\xi),$$

$$0 = -4n\alpha(\xi)\eta(V) - 4n\gamma(\xi)\eta(V) - 4n\delta(V) + 2p(V) - 2\eta(V)p(\xi).$$
(3.9)

Replacing V with X in (3.9) and summing with (3.8), in view of (3.5), we find

$$0 = -4n\alpha(\xi)\eta(X) - 4n\eta(X)\gamma(\xi) - 4n\delta(X) + 2p(X) - 2\eta(X)p(\xi) - 4n\alpha(X) + 2\eta(X)\beta(\xi) - 2\beta(X) - 4n\eta(X)\gamma(\xi) - 4n\delta(\xi)\eta(X) + 2\eta(X)p(\xi) - 2p(X),$$

$$0 = -4n\alpha(X) + 2\eta(X)\beta(\xi) - 2\beta(X) - 8n\eta(X)\gamma(\xi) - 4n\eta(X)\alpha(\xi) - 4n\delta(\xi)\eta(X) - 4n\delta(X). \tag{3.10}$$

Now putting $X = \xi$ in (3.4), we have

$$-8ng(Z,\xi)-2\tilde{S}(Z,\xi)=-4n\alpha(\xi)\eta(Z)+2\eta(\xi)\beta(Z)-2\eta(Z)\beta(\xi)-4n\gamma(Z)\eta(\xi)+\delta(\xi)\tilde{S}(\xi,Z)+2g(\xi,Z)p(\xi)-2\eta(Z)p(\xi),$$

$$0=-4n\eta(Z)\alpha(\xi)+2\beta(Z)-2\eta(Z)\beta(\xi)-4n\gamma(Z)-4n\eta(Z)\delta(\xi)+2\eta(Z)p(\xi)-2\eta(Z)p(\xi).$$

$$0 = -4n\eta(Z)\alpha(\xi) + 2\beta(Z) - 2\eta(Z)\beta(\xi) - 4n\eta(Z) - 4n\eta(Z)\delta(\xi)$$
(3.11)

Replacing Z with X in (3.11) and taking the summation with (3.10), we have

$$\begin{array}{l} 0 = -4n\eta(X)\alpha(\xi) + 2\beta(X) - 2\eta(X)\beta(\xi) - 4n\gamma(X) - 4n\eta(X)\delta(\xi) - 4n\alpha(X) \\ + 2\eta(X)\beta(\xi) - 2\beta(X) - 8n\eta(X)\gamma(\xi) - 4n\eta(X)\alpha(\xi) - 4n\eta(X)\delta(\xi) - 4n\delta(X), \end{array}$$

$$0 = -8n\eta(X)\alpha(\xi) - 8n\eta(X)\gamma(\xi) - 8n\eta(X)\delta(\xi) - 4n\alpha(X) - 4n\gamma(X) - 4n\delta(X),$$

$$0 = -8n\eta(X)[\alpha(\xi) + \gamma(\xi) + \delta(\xi)] - 4n[\alpha(X) + \gamma(X) + \delta(X)].$$

So in view of (3.5), we obtain $\alpha(X) + \gamma(X) + \delta(X)$ for all X on M^{2n+1} . This completes the proof of the theorem.

Theorem 3.2. There is no weakly symmetric nearly Kenmotsu manifold with semi-symmetric metric connection, unless $\rho + \mu + \upsilon$ is everywhere zero.

Proof. Suppose that M^{2n+1} is a weakly symmetric nearly kenmotsu manifold with semi-symmetric metric connection. Replacing Z with ξ in (1.2) and using (2.15) we have

$$(\tilde{\nabla}_X \tilde{S})(Y,\xi) = \rho(X)\tilde{S}(Y,\xi) + \mu(Y)\tilde{S}(X,\xi) + \nu(\xi)\tilde{S}(X,Y),$$

$$-8ng(Y,X) - 2\tilde{S}(Y,X) = -4n\eta(Y)\rho(X) - 4n\eta(X)\mu(Y) + v(\xi)\tilde{S}(X,Y). \tag{3.12}$$

So in view of (3.12) and (3.2) we obtain

$$-8ng(Y,X) - 2\tilde{S}(Y,X) = -4n\eta(Y)\rho(X) - 4n\eta(X)\mu(Y) + \upsilon(\xi)\tilde{S}(X,Y). \tag{3.13}$$

Taking $X = Y = \xi$ in (3.13) and by the use of (2.15), (2.1) and (2.2) we get

$$-8ng(\xi,\xi)-2\tilde{S}(\xi,\xi)=-4n\eta(\xi)\rho(\xi)-4n\eta(\xi)\mu(\xi)+\upsilon(\xi)\tilde{S}(\xi,\xi),$$

$$-8n + 8n = -4n\rho(\xi) - 4n\mu(\xi) - 4n\nu(\xi),$$

$$0 = -4n[\rho(\xi) + \mu(\xi) + v(\xi)],$$

which gives (since 2n + 1)

$$\rho(\xi) + \mu(\xi) + v(\xi) = 0. \tag{3.14}$$

Now putting $X = \xi$ in (3.13) we have

$$-8ng(Y,\xi)-2\tilde{S}(Y,\xi)=-4n\eta(Y)\rho(\xi)-4n\eta(\xi)\mu(Y)+\upsilon(\xi)\tilde{S}(\xi,Y),$$

$$-8n\eta(Y)+8n\eta(Y)=-4n\eta(Y)\rho(\xi)-4n\mu(Y)+4n\eta(Y)\upsilon(\xi),$$

$$0 = -4n\eta(Y)[\rho(\xi) + v(\xi)] - 4n\mu(Y),$$

so by virtue of (3.14) this yields

$$-4n\eta(Y)[-\mu(\xi)] - 4n\mu(Y) = 0,$$

$$4n[\mu(\xi)\eta(Y)] - 4n\mu(Y) = 0,$$

which gives us (since 2n + 1)

$$\mu(Y) = \mu(\xi)\eta(Y). \tag{3.15}$$

Similary taking $Y = \xi$ in (3.13) we also have

$$-8ng(\xi,X)-2\tilde{S}(\xi,X)=-4n\eta(\xi)\rho(X)-4n\eta(X)\mu(\xi)+\upsilon(\xi)\tilde{S}(X,\xi),$$

$$-8n\eta(X) - 8n\eta(X) = -4n\rho(X) - 4n\eta(X)\mu(\xi) - 4n\eta(X)v(\xi),$$

$$0 = -4n[\rho(X) + \eta(X)\mu(\xi) + \eta(X)\nu(\xi)],$$

$$0 = \rho(X) + \eta(X)[\mu(\xi) + \upsilon(\xi)],$$

hence applying (3.14) into the last equation, we find

$$\rho(X) = \rho(\xi)\eta(X). \tag{3.16}$$

Since $(\tilde{\nabla}_X \tilde{S})(\xi, X) = 0$, then from (1.2) we obtain $X = Y = \xi$ and Z = X

$$(\tilde{\nabla}_X \tilde{S})(\xi, X) = \rho(\xi) \tilde{S}(\xi, X) + \mu(Y) \tilde{S}(\xi, X) + \nu(X) \tilde{S}(\xi, \xi),$$

$$0 = -4n\eta(X)\rho(\xi) - 4n\eta(X)\mu(\xi) - 4n\upsilon(X),$$

$$0 = \eta(X)\rho(\xi) + \eta(Y)\mu(\xi) + \upsilon(X),$$

$$0 = \eta(X)[\rho(\xi) + \mu(\xi)] + v(X).$$

So by making use of (3.14) the last equation reduces to

$$v(X) = v(\xi)\eta(X). \tag{3.17}$$

Therefore changing Y with X in (3.15) and by the summation of the equations (3.15), (3.16) and (3.17), we obtain

$$\rho(X) + \mu(X) + v(X) = \eta(X)[\rho(\xi) + \mu(\xi) + v(\xi)]$$

and so in view of (3.14) it follows that

$$\rho(X) + \mu(X) + \upsilon(X) = 0$$

for all X, which implies $\rho + \mu + \nu = 0$ on M^{2n+1} . Our theorem is thus proved.

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