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Research Article

ANALYTICAL AND NUMERICAL STUDY OF MICROPOLAR FLUID FLOW IN A POROUS PLATE DUE TO LINEAR STRETCHING

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ABSTRACT

In this research, micropolar fluid flow of a porous plate due to Linear stretching is analyzed. The basic partial differential equations are reduced to nonlinear ordinary differential equations which are solved using Homotopy Perturbation Method (HPM). Comparison between results of Flex-PDE software and analytical method of the issue illustrates excellent precision in solving the nonlinear differential equation. Furthermore, impact of injection and suction velocity (∅), coupling parameter between velocity field and micro-rotation field (ε), vortex viscosity parameter (β) on micro-rotation, and fluid velocity profiles are examined. Conclusions indicate that: by increasing the ε parameter, the f'(η) value decreases. Also, the shear stress $F''(0)$ values are gradually reduced with increasing β, while the opposite trend is observed in $H'(0)$ variations.
Keywords: Micropolar fluid flow, injection and suction, homotopy perturbation method (HPM), porous plate.

1. INTRODUCTION

Micropolar fluids are a model of standard ordinary fluid with nonsymmetric and microstructure stress tensor. In fact, deformation of the fluids is neglected and includes rigid which related to spherical particles in a cohesive medium. Eringen [1] was the first to introduce the micro plastic fluid model. This model is an important simplified model of Navier–Stokes equations in two areas of application and theory, which offers a lot of scientific phenomena than ordinary states. The field of micropolar fluid is a vital field that has many applications, including

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laboratory on chip, oil extraction, refining industry, heat transfer issues, paper industry, lubrication, surface coating, material science, bio sciences, and so on. Considering these important applications, the world's most acclaimed universities, including MIT, Cornell, and others, have set up an independent laboratory of the same name. Even in some countries, such as Germany, an independent institute of the same name has been created. Many researchers have considered various problems in micropolar fluids, such as Hassanien and Gorla [2] investigated on non-isothermal stretching plate heat transfer using micropolar fluid flow with blowing and suction. Their results showed that heat transfer rate and friction parameter were affected by mass transfer rate. Vajravelu [3] studied viscous fluid heat transfer rate and convection fluid flow on a vertical, infinite and porous stretching sheet. His research is compatible with this truth that by increment in Prandtl number the thermal boundary layer will reduce. Salleh et al. [4] carried out a mathematical theory for a micropolar fluid flow boundary layer because of moving flat sheet. The outcomes of this study illustrated that in the range of $0 \leq n \leq 1$ by the increment in the n, wall shear stress decreases and gyration factor increases, where *k* is constant and n is a ratio of wall shear stress of fluid flow. There are many studies that have been used mathematical and numerical solutions to model nanofluid flow [5-14]. Also, effect of several parameters of micropolar fluid flow including magneto hydrodynamic convective flow and heat transfer rate on moving vertical porous sheet studied by Rahman and Sattar [15]. Their results indicated that by increasing in coupling factor (K), Richardson factor (γ), prandtl number (Pr) and suction factor (F_w), heat transfer rate will increase steadily. Rahman and Sultana [16] studied the micropolar fluid flow radiative heat transfer rate with variable heat flux passing porous flat sheet. Their results showed that Darcy factor increases the angular velocity and temperature, while it causes to decrease in velocity. Sheikholeslami et al. [17] used Fe3O4 –water ferrofluid flow in a porous cavity in order to examine the effects of non-uniform magnetic field on it. Their outcomes indicated that heat transfer rate and velocity of nanofluid decreased by increasing in Hartmann number. The factors of micropolar fluid flow including the non-uniform sink and electric conductivity passing an inclined flat sheet with heat flux of the surface investigated by Rahman et al. [18]. They illustrated that the effect of Newtonian fluid is greater than non-uniform heat production and electric conductivity in a micropolar fluid flow. Alomari et al. [19] demonstrated the micropolar fluid flow heat transfer rate and uniform boundary layer flow on an isothermal moving sheet surface. The outcomes obtained from the research indicated that HAM method has a very high accuracy and is also very useful for solving nonlinear equations. Sheikholeslami also applied HAM method in his investigations [20,21]. Investigation on nanoparticles behavior around the heated cylinder is conducted by using Response Surface Methodology (RSM) in order to discover the best profile of wavy-wall for an enclosure by Hatami [22]. According to the conclusions, it was observed that Nusselt number will decrease by increasing in cylinder diameter of more than 1.0, while heat transfer will improve when the diameter is less than 1.0. Using THAM method for solving nonlinear systems in the semi-infinite domain for micropolar fluid flow passing porous stretching plate studied by Kazem and Shaban [23]. Their results showed that in THAM method, in contrary with HAM method the rule of solution explanation and ergodicity are out of work. Ahmad et al. [24] investigated on heat transfer rate passing a nonlinearly stretching sheet by considering viscous loss for micropolar fluid. They showed that by increasing in K and keeping constant of n, the local Nusselt number and skin friction coefficient will decrease. In addition, these parameters will increase with the nonlinear stretching parameter of n. By considering Lorentz forces, nanofluid flow behavior under the effect of melting heat transfer has been studied by Sheikholeslami et al. [25]. Based on their conclusions, increasing in melting parameter results in an increase in velocity and decrease in temperature. Also, by increasing in melting and porosity parameters, Nusselt number will be increased. The heat transfer rate of micropolar fluid using AGM method in a permeable channel studied by Mirgolbabaee et al. [26]. Their outcomes represented the fact that Péclet number has inverse relation with Sherwood number and Nusselt number, while Reynolds number has direct relation with them. Hatami and

Ganji [27] applie d differential quadrature method (DQM) and differential transformation method (DTM) in order to study particle movement coupled equations in a fluid enforced vortex. Their results showed that although the particle angular velocity increases, its radial velocity decreases when the particle recedes from the vortex center. Recently, Sui et al. [28] Studied the viscoelasticity based on micropolar fluid in a slip flow. They illustrated that the influence of particle thermophoresis on convection heat transfer and mass transfer. Moreover, Kataria et al. [29] Studied the unsteady natural convective flow of a micropolar fluid between two vertical walls under the effect of magnetic field. There are lots of investigations that have been conducted about heat transfer in porous media [30-37].

At the center of all engineering sciences, various phenomena show themselves in a mathematical relationship modeled in the form of differential equations. Most of these mathematical relationships are in nonlinear form. Among the nonlinear relationships used in fluid mechanics, micropolar fluid problems are interesting. Because of the nonlinear nature of micropolar fluids, we need a strong analytical tools. For this reason, resolving these difficult problems has been a controversial issue for mathematicians, physicists and engineers. In recent years, some newly developed manners have been proposed to achieve an approximate solution of nonlinear equations, such as Adomian decomposition method (ADM) [38,39], Differential Transformation Method (DTM) [40-42], Optimal Homotopy Asymptotic Method (OHAM) [43,44]. Since there are lots of semi-analytical methods for solving nonlinear equations, consideration of all of them in this study is so extensive. One of the semi-analytical methods which does not need small parameters is Homotopy Perturbation Method (HPM). This method in most cases provides fast convergence to solve series. It is worth mentioning that the small number of errors in this method leads to an achievement to high precision solutions. Sheikholeslami [45,46] also used HPM method in his studies.

In this study, we have used HPM to find the approximate solutions of nonlinear problem governing the Micropolar fluid flow of porous plate due to lineal stretching. As well as, the effect of injection and suction velocity (∅), coupling parameter between velocity field and micro-rotation field (ε), vortex viscosity parameter (β) on micro-rotation, and fluid velocity profiles are analyzed. It should be noted that comparison between results of Flex-PDE software and analytical method of the issue illustrates excellent precision in solving the nonlinear differential equation.

2. PROBLEM DESCRIPTION

As illustrated in figure 1, an incompressible micropolar fluid in two-dimensional steady motion which is passing over a flat sheet and is linearly stretching away from a fixed point has been considered. The micro-rotation is σ and the velocity components (u, v) are in (x, y) directions, respectively. Governing boundary layer expressions for present flow characteristics are:[47]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(\kappa+\nu)\frac{\partial^2 u}{\partial y^2}+\kappa\frac{\partial \sigma}{\partial y}$$
$$\kappa(2\sigma+\frac{\partial u}{\partial y})=\gamma\frac{\partial^2 \sigma}{\partial y^2} \tag{2}$$

$$u = \alpha x,\ \ v = -\Phi,\ \sigma = 0 \quad \text{at} \quad y = 0$$
$$\begin{cases} u \to 0 \\ \sigma \to 0 \end{cases} \qquad \text{as} \quad y \to \infty \tag{3}$$

Where κ, ν, Φ, α and γ are skew-symmetric deformation, kinematic viscosity coefficients to joined with the rates of symmetric deformation, suction velocity through the porous surface, the stretching parameter and rotation, respectively. Eq. (2) can be rephrased as follows:

$$\eta = \alpha^{\frac{1}{2}}(\kappa+\nu)^{-\frac{1}{2}} y,\ u = \alpha x f'(\eta),\ v = -\alpha^{\frac{1}{2}}(\kappa+\nu)^{\frac{1}{2}} f(\eta),$$
$$\sigma = \alpha^{\frac{3}{2}}(\kappa+\nu)^{-\frac{1}{2}} x h(\eta) \tag{4}$$
$$\varepsilon = \kappa(\kappa+\nu)^{-1},\ \beta^2 = 2\kappa(\kappa+\nu)(\alpha\gamma)^{-1}$$
$$\phi = \alpha^{-\frac{1}{2}}(\kappa+\nu)^{-\frac{1}{2}}\Phi$$

As systems of nonlinear ordinary differential equation are given as [47]:

$$F''' + FF'' - F'^2 = \varepsilon H',$$
$$H'' - \beta^2 H = \frac{1}{2}\beta^2 F''$$

(5)

$$F(0) = \phi, \qquad F'(0) = 1, \qquad F'(\infty) = 0,$$
$$H(0) = 0, \qquad H(\infty) = 0, \tag{6}$$

Where 'prime' denotes differentiation with respect to *x*; the similarity coordinate measuring distances normal to the sheet *F'*, *H*, ε, β and ∅ are velocity, micro-rotation, coupling parameter, a constant characteristic of the fluid and suction velocity through the porous surface.

3. MATHEMATICAL PROCEDURES

In this section HPM method have been investigated:

3.1. Default Homotopy Perturbation Method (HPM)

To explain the basic ideas of this manner, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0,\ \ r \in \Omega, \tag{7}$$

With the boundary condition of:

$$B(u, \frac{\partial u}{\partial n}),\ \ r \in \Gamma, \tag{8}$$

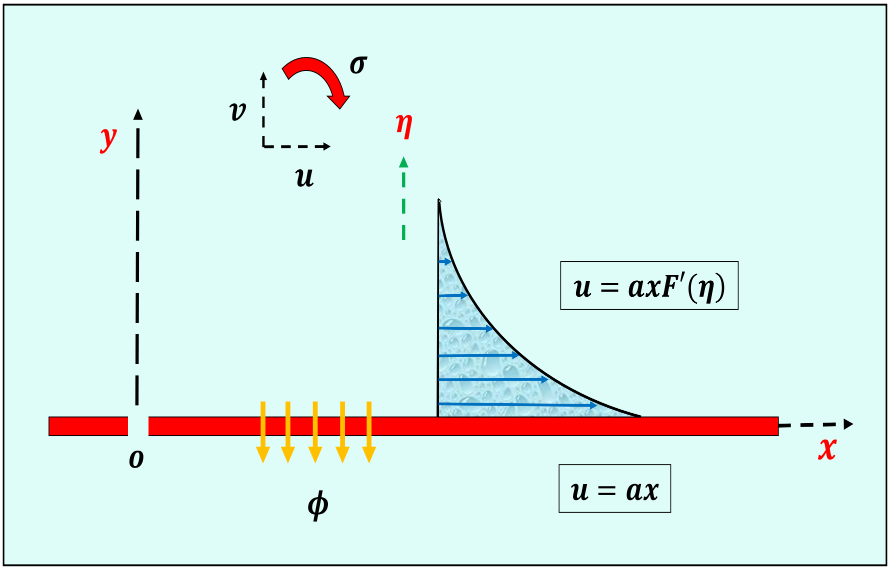

Figure 1. Geometry of the considered problem.

Where *f* (r) is a known analytical function, *A* is a general differential operator, *B* is a boundary operator, (∂u/∂n) denotes differentiation along the normal drawn outwards from (Ω) and (Γ) is the boundary of the domain (Ω).

A can be divided into two parts which are *L* and *N*, where *L* is linear part and *N* is nonlinear part. Eq. (7) can therefore be rewritten as follows:

$$L(u)+N(u)-f(r)=0, \tag{9}$$

Homotopy perturbation structure is shown as follows:

$$H(v,p)=L(v)+L(u_0)+pL(u_0)+p(N(v)-f(r))=0, \tag{10}$$

Where

$$v(r,p):\quad \Omega\times[0,1]\rightarrow R, \tag{11}$$

In Eq. (10), u_0 is the first approximation that satisfies the boundary condition and p∈ [0, 1] is an embedding parameter. We can assume that the solution of Eq. (10) can be written as a power series in *P*, as following:

$$v=v_0+pv_1+p^2v_2+... \tag{12}$$

and the best approximation for solution is:

$$u=\lim_{p\rightarrow 1} v=v_0+v_1+v_2+... \tag{13}$$

4. APPLICATION OF DESCRIBED MANNERS IN THE ISSUE

4.1. Homotopy Perturbation method

In this section, we will apply the HPM to nonlinear ordinary differential Eq. (5). According to the HPM, we have created a homotopy assumption for the solution of Eq. (5):

$$\begin{aligned}&H(f,p):=(1-p)(f'''(x)+f''(x))+p(f'''(x)+f(x)f''(x)-f'^2(x)+\varepsilon h'(x),\\&H(h,p):=(1-p)(h''(x)-\beta^2h(x))+p(h''(x)-\beta^2h(x)-0.5\beta^2f''(x)),\end{aligned} \tag{14}$$

We consider *h*(*x*), *f(x)* as Following:

$$\begin{aligned}&f(x):=f_0(x)+pf_0(x)+p^2f_2(x),\\&h(x):=h_0(x)+ph_1(x)+p^2h_2(x),\end{aligned} \tag{15}$$

By substituting Eq. (15) into Eq. (14) and some simplification and rearranging on powers of *P* terms, we have:

$$p^0\begin{cases}\dfrac{d^3}{dx^3}f_0(x)+\left(\dfrac{d^2}{dx^2}f_0(x)\right)=0\\\dfrac{d^2}{dx^2}h_0(x)-\beta^2h_0(x)=0\end{cases} \tag{16}$$

The boundary conditions for Eqs. (16) are:

$$\begin{cases}f_0(x)=0,\\f_0'(0)=1,\\f_0'(\infty)=0,\end{cases} \tag{17}$$

$$\begin{cases}h_0(0)=0,\\h_0(\infty)=0,\end{cases} \tag{18}$$

For ε=0.1, β=1 and ∅=0

$$\begin{cases}f_0(x)=-\dfrac{1}{e^{-10}-1}+\dfrac{e^{-10}x}{e^{-10}-1}+\dfrac{e^{-x}}{e^{-10}-1},\\h_0(x)=0,\end{cases} \tag{19}$$

$$p^1\begin{cases}\left(\dfrac{d^2}{dx^2}f_0(x)\right)f_0(x)-1.\left(\dfrac{d}{dx}f_0(x)\right)^2+0.1\left(\dfrac{d}{dx}h_0(x)\right)\\-1.\left(\dfrac{d^2}{dx^2}f_0(x)\right)+\dfrac{d^3}{dx^3}f_1(x)+\dfrac{d^2}{dx^2}f_1(x)=0\\\dfrac{d^2}{dx^2}h_1(x)-1.h_1(x)=0\end{cases} \tag{20}$$

The boundary conditions for Eq. (20) are

$$\begin{cases} f_1(x)=0, \\ f_1'(0)=1, \\ f_1'(6)=0, \end{cases} \tag{21}$$

$$\begin{cases} h_1(0)=0, \\ h_1(6)=0, \end{cases} \tag{22}$$

For ε=0.1, β=1 and ∅=0

$$\begin{cases} f_1(x)=45.004.e^{-10.-1.x}+7.0006.e^{-10.-1.x}x+1.0306^{-9}.x^2 \\ -0.50.e^{-10.-1.x}\left(-10.-1.x\right)^2-0.99.e^{-1.x}-0.00004.x+1.0001 \\ \\ h_1(x)=0.28.e^{-0.6x}-0.000001.e^{0.6x}-0.28.e^{-1.x} \end{cases} \tag{23}$$

In the same manner, the rest of components are obtained by using the Maple package and we obtained 15 parameters of it. According to HPM, we can conclude:

$$\begin{aligned} f(x) &= \sum_{i=0}^{j} f_j(x) = f_1+f_2+f_3+f_4+f_5+\ldots\ldots\ldots \\ h(x) &= \sum_{i=0}^{j} h_j(x) = h_1+h_2+h_3+h_4+h_5+\ldots\ldots\ldots \end{aligned} \tag{24}$$

Maple is used to solving the linear equations up to first few orders. Also, j must be sufficiently large. In practice, we decided to stop the calculations at **j = 5** (at 5th-order) having realized that a sufficiently small tolerance has been met.

4.2. Solution with *Flex-PDE* software

In this research, at first we introduce the *Flex-PDE* software. *Flex-PDE* software is simple modeling software based on finite element method for coding. This means that *Flex-PDE* converts written codes and partial differential equations to a finite element model. This software is a powerful tool for making connection among mathematical model, numerical solution and graphical results. Also, this software has ability to analyze the wide range of engineering problems such as tension, chemical reaction kinetic and modeling of real mathematical problems. The steps of solving the problem in this software are as below:

- Initial analysis of equations.
- Formation of derivations, integrals and functions with Galerkin finite element method.
- Construction of coupling matrix and solving it.
- Response graphical presentation.

In this paper, we compare the results of HPM with obtained results from *Flex-PDE* software.

5. RESULTS AND DISCUSSIONS

In this section, we illustrate the analytical study of the boundary layer flow of a micropolar fluid with suction and injection due to a linearly stretching of porous surface by HPM (see figure

1). Comparison between the HPM results with numerical software (*Flex-PDE*) for different values of active parameter are shown in Table (1). The slight error in this tables indicates that HPM is a high accuracy method to solve these issues.

Table 1. Comparison between the HPM and *Flex-PDE* results for *F'* (left) and *H* (Right).

	β=0.4, ∅=0.0, ε=0.1			β=1, ∅=0.1, ε=0.2		
η	HPM	Flex-	Error	HPM	Flex-	Error
0.0	1.0000	1.0000	0.0002	0.0000	0.0000	0.0000
1.0	0.3646	0.3541	0.0105	0.9902	0.9832	0.0070
2.0	0.1208	0.1107	0.0101	0.6918	0.6922	0.0004
3.0	0.0301	0.0311	0.0010	0.3280	0.3292	0.0012
4.0	0.0001	0.0001	0.0000	0.0295	0.0284	0.0011
5.0	0.0000	0.0000	0.0000	0.0009	0.0009	0.0000
6.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

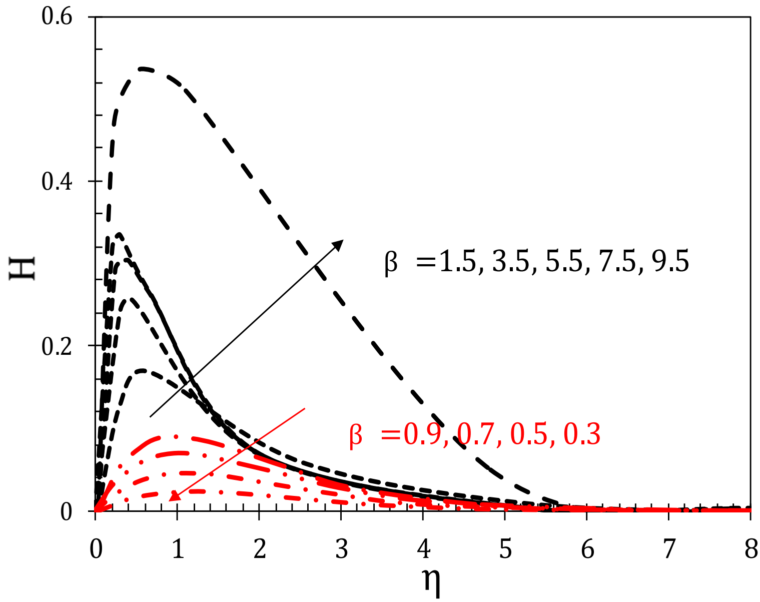

Figure 2. Effect of β on micro-rotation (*H*(η)).

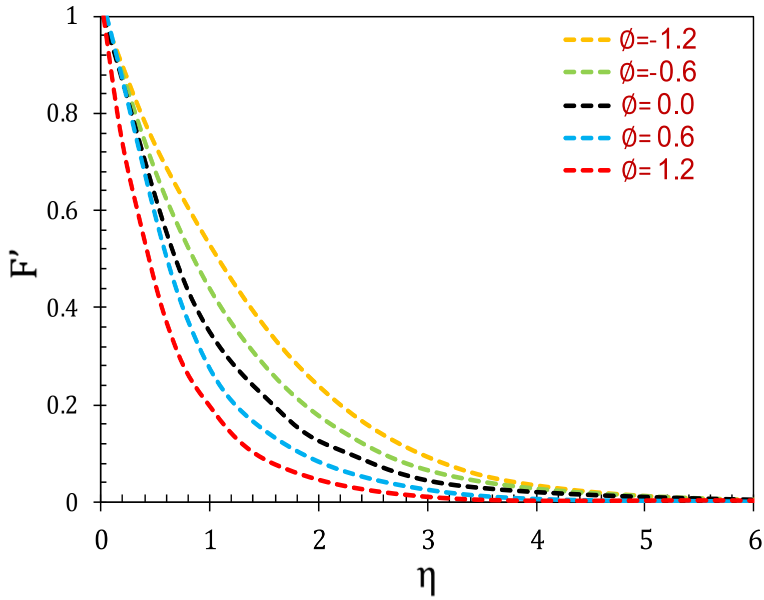

Figure 3. Effect of ∅ on velocity ($F'(\eta)$) profile.

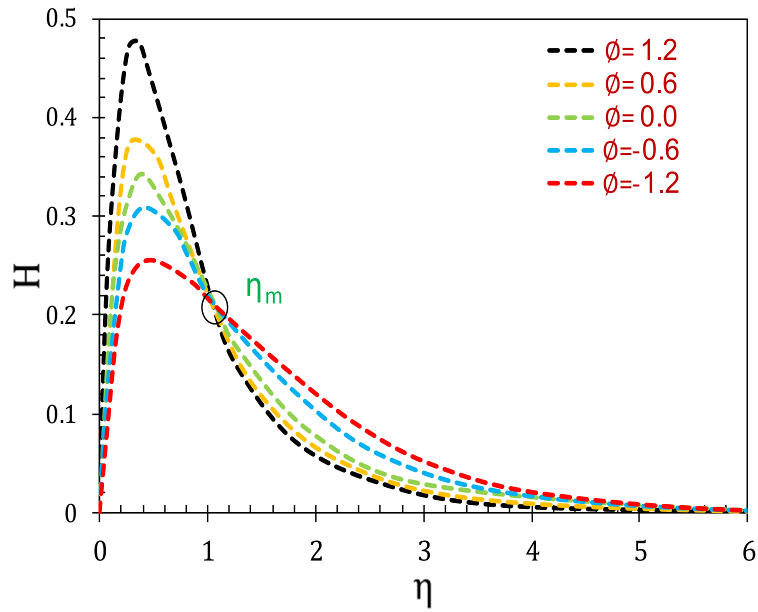

Figure 4. Effect of ∅ on micro-rotation ($H(\eta)$).

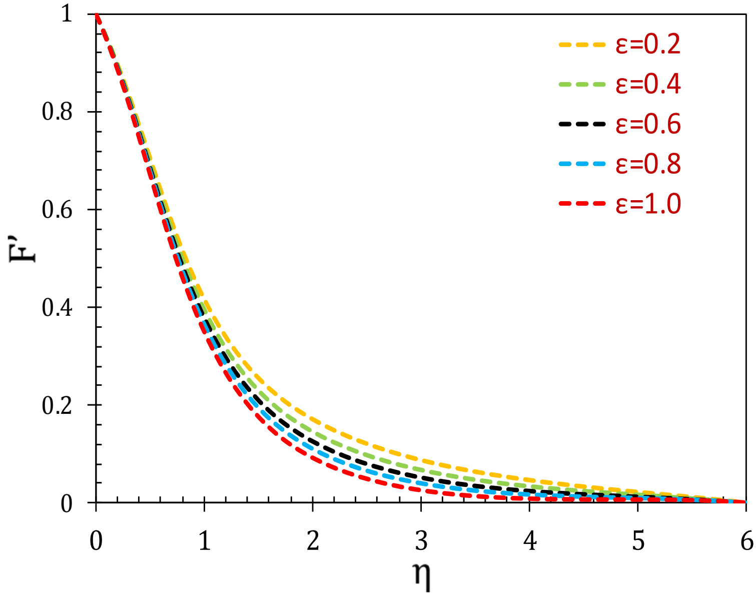

Figure 5. Effect of ε on velocity ($F'(\eta)$) profile.

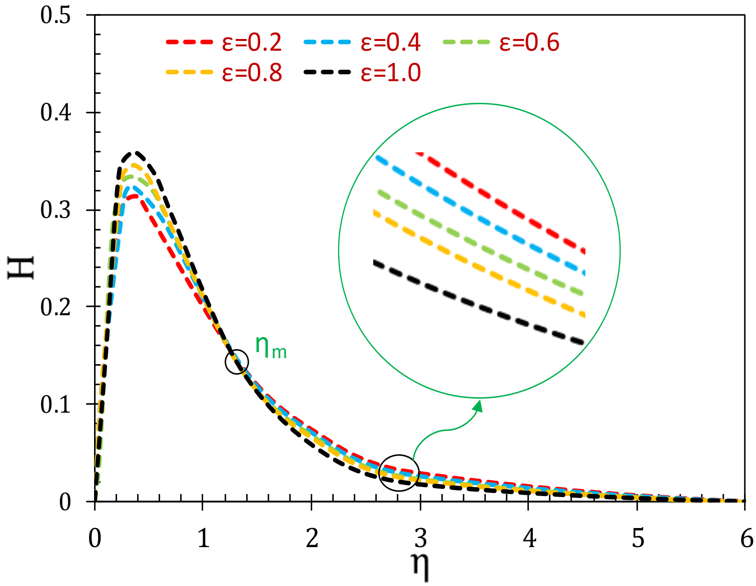

Figure 6. Effect of ε on micro-rotation ($H(\eta)$).

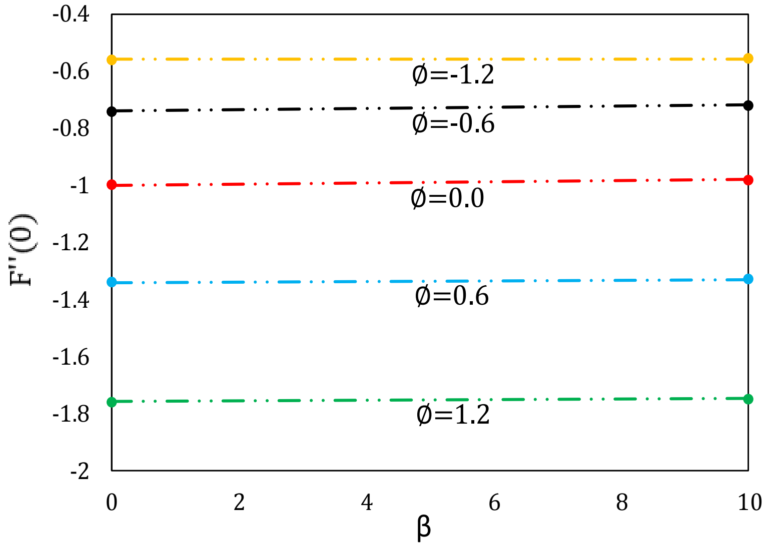

Figure 7. Variation of the F''(0) with $\emptyset$ *and* β for ε =0.1

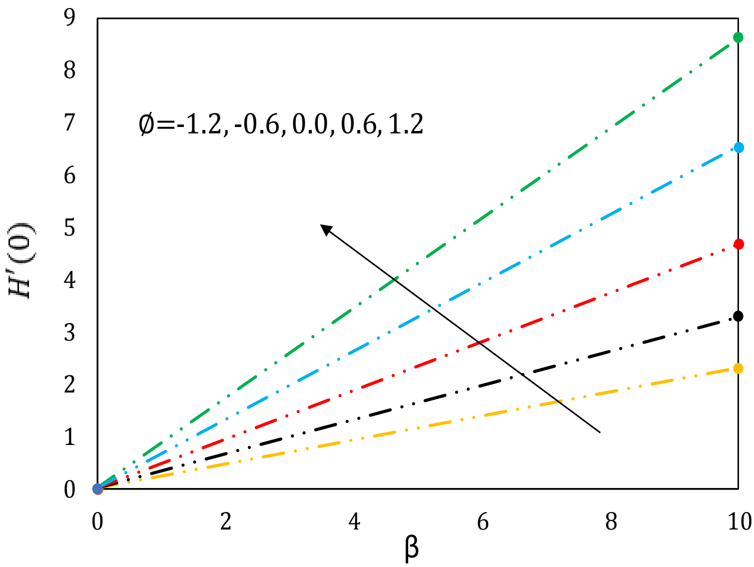

Figure 8. Variation of the H'(0) with $\emptyset$ *and* β for ε =0.5

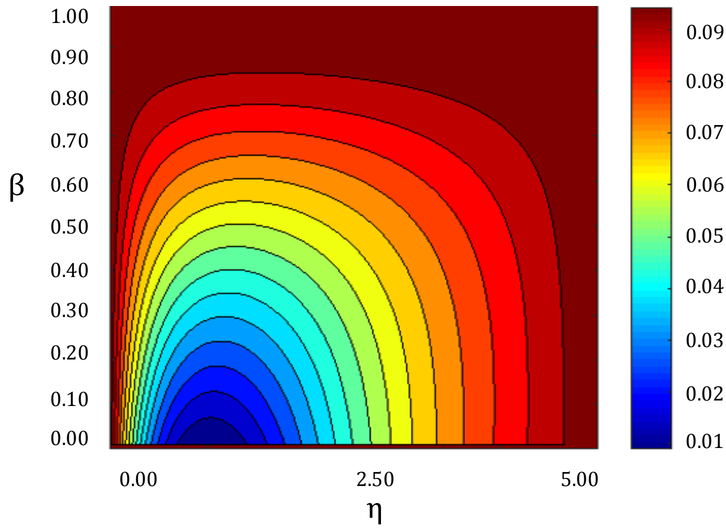

Figure 9. Contour plots of H (η) for various amounts of active parameters.

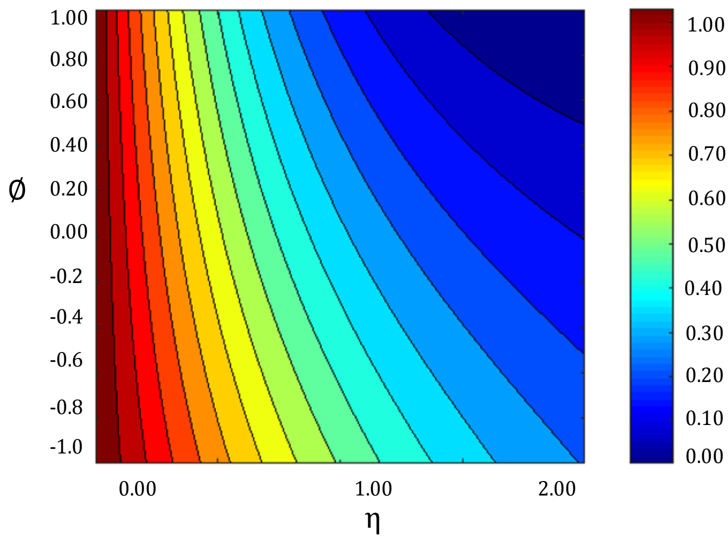

Figure 10. Contour plots of F' (η) for various amounts of active parameters.

Figure 2 plots the effect of vortex viscosity parameter (β) on micro-rotate profile (when β>1 and β<1), respectively. As it is shown, by the increment of the vortex viscosity parameter, the micro-rotate profile increases. An important thing in micro-rotate profile is how it moves near the bottom plate. Also, the micro-rotation profile (*H*(η)) changes near the bottom of the page in β<1 are more than β>1. Figure 3 and 4, illustrate the effect of injection and suction velocity parameter (∅) on the velocity and rotational profiles (when injection (∅<0) and suction (∅>0)). In Fig.3, the velocity (*F'*(η)) profile has an increasing trend in injection ∅<0 and decreasing trend in suction ∅>0. In Figure 4, the micro-rotate profile increases to its peak value and then has a decrease trend.

The changes of the micro-rotate profile in $\eta<\eta_m$ and $\eta_m<\eta$ is one of the most important points. η_m is the location of meeting of curves for different $\emptyset$ at the same time. With increasing in $\emptyset$, the micro-rotational profile in $\eta<\eta_m$ has an increasing trend and then by passing η_m has a decreasing trend. Figures 5 and 6 display $H(\eta)$ and $F'(\eta)$ profiles variations for different micro-rotation field (ε) values. In Figure 5, by increasing the ε parameter, the $f'(\eta)$ value decreases. Unlike that by increasing the ε, the $H(\eta)$ value increases near the bottom plate in $\eta< \eta_m$. In the following by passing the η_m the H profile has a decreasing trend. Plus, As shown in paper, with increase of e the value of wall shear stress and velocity boundary layer thickness decreases. Figures 7 and 8 show the effect of $\emptyset$ and β parameters on $H'(0)$ and shear stress ($F''(0)$). As shown in Fig. 7, the $F''(0)$ values are gradually reduced with increasing β, while in Figure 8 the opposite trend is observed in $H'(0)$ variations. It should also be noted that the injection phenomenon reduces $F''(0)$ and $H'(0)$. This is while the scenario is in reverse in suction state. In this study, in order to better understanding of the $F'(\eta)$ and H profiles, contours plots are illustrated for various values of the active parameter in different intervals.

6. CONCLUSIONS

In this study, we illustrated the analytical study of the boundary layer flow of a micropolar fluid with suction and injection due to a linearly stretching of porous surface by HPM. The comparison of the results of Homotopy Perturbation Method (HPM) and *Flex-PDE* software indicates excellent complying in solving this nonlinear issue. Moreover, the effect of injection and suction velocity ($\emptyset$), the coupling parameter between the velocity field and micro-rotation field (ε), vortex viscosity parameter (β) on micro-rotation ($H(\eta)$), and fluid velocity ($F'(\eta)$) profiles are analyzed. Main outcomes are displayed:

- By the increment of vortex viscosity parameter, the micro-rotate profile increases.
- Velocity profile has an increasing trend in injection of $\emptyset<0$ and a decreasing trend in suction of $\emptyset>0$.
- With increasing $\emptyset$, the micro-rotational profile in $\eta<\eta m$ has an increasing trend and then by passing η_m has a decreasing trend.

Finally, it can be concluded that HPM is an excellent analytical method due to its efficiency in solving different problems.

REFERENCES

[1] Eringen, A. Cemal. "Theory of micropolar fluids." *Journal of Mathematics and Mechanics* (1966): 1-18.

[2] Hassanien, I. A., and R. S. R. Gorla. "Heat transfer to a micropolar fluid from a non-isothermal stretching sheet with suction and blowing." *Acta Mechanica* 84, no. 1-4 (1990): 191-199.

[3] Vajravelu, K. "Convection heat transfer at a stretching sheet with suction or blowing." *Journal of mathematical analysis and applications* 188, no. 3 (1994): 1002-1011.

[4] Salleh, Mohd Zuki, Azizah Mohd Rohni, and Norsarahaida Amin. "Boundary layer flow due to a moving flat plate in micropolar fluid." *Jurnal Teknologi* 43: 67-83.

[5] Zhou, Jiandong, M. Hatami, Dongxing Song, and Dengwei Jing. "Design of microchannel heat sink with wavy channel and its time-efficient optimization with combined RSM and FVM methods." *International Journal of Heat and Mass Transfer* 103 (2016): 715-724.

[6] Hatami, M., M. Sheikholeslami, and G. Domairry. "High accuracy analysis for motion of a spherical particle in plane Couette fluid flow by multi-step differential transformation method." *Powder Technology* 260 (2014): 59-67.

[7] Dogonchi, A. S., M. Hatami, and G. Domairry. "Motion analysis of a spherical solid particle in plane Couette Newtonian fluid flow." *Powder Technology* 274 (2015): 186-192.
[8] Hatami, M., and D. D. Ganji. "Motion of a spherical particle on a rotating parabola using Lagrangian and high accuracy multi-step differential transformation method." *Powder Technology*258 (2014): 94-98.
[9] Tang, Wenhui, M. Hatami, Jiandong Zhou, and Dengwei Jing. "Natural convection heat transfer in a nanofluid-filled cavity with double sinusoidal wavy walls of various phase deviations." *International Journal of Heat and Mass Transfer*115 (2017): 430-440.
[10] Sheikholeslami, M., M. Hatami, and D. D. Ganji. "Numerical investigation of nanofluid spraying on an inclined rotating disk for cooling process." *Journal of Molecular Liquids* 211 (2015): 577-583.
[11] Pourmehran, O., M. Rahimi-Gorji, M. Hatami, S. A. R. Sahebi, and G. Domairry. "Numerical optimization of microchannel heat sink (MCHS) performance cooled by KKL based nanofluids in saturated porous medium." *Journal of the Taiwan Institute of Chemical Engineers* 55 (2015): 49-68.
[12] Hatami, M., Dongxing Song, and Dengwei Jing. "Optimization of a circular-wavy cavity filled by nanofluid under the natural convection heat transfer condition." *International Journal of Heat and Mass Transfer* 98 (2016): 758-767.
[13] Hatami, M., J. Zhou, J. Geng, D. Song, and D. Jing. "Optimization of a lid-driven T-shaped porous cavity to improve the nanofluids mixed convection heat transfer." *Journal of Molecular Liquids* 231 (2017): 620-631.
[14] Hatami, M., and D. Jing. "Optimization of wavy direct absorber solar collector (WDASC) using Al2O3-water nanofluid and RSM analysis." *Applied Thermal Engineering* 121 (2017): 1040-1050.
[15] Rahman, M. M., and M. A. Sattar. "Magnetohydrodynamic convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption." *Journal of Heat Transfer* 128, no. 2 (2006): 142-152.
[16] Rahman, M. M., and Tamanna Sultana. "Radiative heat transfer flow of micropolar fluid with variable heat flux in a porous medium." *Nonlinear Anal. Model. Control* 13, no. 1 (2008): 71-87.
[17] Sheikholeslami, M., D. D. Ganji, and R. Moradi. "Heat transfer of Fe3O4–water nanofluid in a permeable medium with thermal radiation in existence of constant heat flux." *Chemical Engineering Science* 174 (2017): 326-336.
[18] Rahman, M. M., M. J. Uddin, and A. Aziz. "Effects of variable electric conductivity and non-uniform heat source (or sink) on convective micropolar fluid flow along an inclined flat plate with surfaceheat flux." *International Journal of Thermal Sciences* 48, no. 12 (2009): 2331-2340.
[19] Alomari, A. K., Mohd Salmi Md Noorani, and R. Nazar. "Homotopy solution for flow of a micropolar fluid on a continuous moving surface." *International Journal for Numerical Methods in Fluids* 66, no. 5 (2011): 608-621.
[20] Sheikholeslami, M., and D. D. Ganji. "Analytical investigation for Lorentz forces effect on nanofluid Marangoni boundary layer hydrothermal behavior using HAM." *Indian Journal of Physics* 91, no. 12 (2017): 1581-1587.
[21] Sheikholeslami, M., R. Ellahi, H. R. Ashorynejad, G. Domairry, and T. Hayat. "Effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium." *Journal of Computational and Theoretical Nanoscience* 11, no. 2 (2014): 486-496.
[22] Hatami, M. "Nanoparticles migration around the heated cylinder during the RSM optimization of a wavy-wall enclosure." *Advanced Powder Technology* 28, no. 3 (2017): 890-899.

[23] Kazem, S., and M. Shaban. "Tau-homotopy analysis method for solving micropolar flow due to a linearly stretching of porous sheet." *Commun Numer Anal* 2012 (2012): cna-00114.

[24] Ahmad, Kartini, Anuar Ishak, and Roslinda Nazar. "Micropolar fluid flow and heat transfer over a nonlinearly stretching plate with viscous dissipation." *Mathematical Problems in Engineering* 2013 (2013).

[25] Sheikholeslami, M., M. Nimafar, and D. Domiri Ganji. "Analytical approach for the effect of melting heat transfer on nanofluid heat transfer." *The European Physical Journal Plus*132, no. 9 (2017): 385.

[26] Mirgolbabaee, H., S. T. Ledari, and D. D. Ganji. "Semi-analytical investigation on micropolar fluid flow and heat transfer in a permeable channel using AGM." *Journal of the Association of Arab Universities for Basic and Applied Sciences* 24, no. 1 (2017): 213-222.

[27] Hatami, M., and D. D. Ganji. "Motion of a spherical particle in a fluid forced vortex by DQM and DTM." *Particuology* 16 (2014): 206-212.

[28] Sui, Jize, Peng Zhao, Zhengdong Cheng, and Masao Doi. "Influence of particulate thermophoresis on convection heat and mass transfer in a slip flow of a viscoelasticity-based micropolar fluid." *International Journal of Heat and Mass Transfer* 119 (2018): 40-51.

[29] Kataria, Hari R., Harshad R. Patel, and Rajiv Singh. "Effect of magnetic field on unsteady natural convective flow of a micropolar fluid between two vertical walls." *Ain Shams Engineering Journal* 8, no. 1 (2017): 87-102.

[30] Sheikholeslami, M., M. Jafaryar, Davood Domairry Ganji, and Zhixiong Li. "Exergy loss analysis for nanofluid forced convection heat transfer in a pipe with modified turbulators." *Journal of Molecular Liquids* 262 (2018): 104-110.

[31] Sheikholeslami, M., Davood Domairry Ganji, and R. Moradi. "Forced convection in existence of Lorentz forces in a porous cavity with hot circular obstacle using nanofluid via Lattice Boltzmann method." *Journal of Molecular Liquids* 246 (2017): 103-111.

[32] Sheikholeslami, M., and D. D. Ganji. "Influence of electric field on Fe3O4-water nanofluid radiative and convective heat transfer in a permeable enclosure." *Journal of Molecular Liquids* 250 (2018): 404-412.

[33] Hosseini, S. R., M. Sheikholeslami, M. Ghasemian, and D. D. Ganji. "Nanofluid heat transfer analysis in a microchannel heat sink (MCHS) under the effect of magnetic field by means of KKL model." *Powder Technology* 324 (2018): 36-47.

[34] Sheikholeslami, M., and D. D. Ganji. "Numerical analysis of nanofluid transportation in porous media under the influence of external magnetic source." *Journal of Molecular Liquids* 233 (2017): 499-507.

[35] Sheikholeslami, M., and D. D. Ganji. "Numerical approach for magnetic nanofluid flow in a porous cavity using CuO nanoparticles." *Materials & Design* 120 (2017): 382-393.

[36] Sheikholeslami, M., Z. Ziabakhsh, and D. D. Ganji. "Transport of Magnetohydrodynamic nanofluid in a porous media." *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 520 (2017): 201-212.

[37] Sheikholeslami, M., M. Jafaryar, K. Bateni, and D. D. Ganji. "Two phase modeling of nanofluid flow in existence of melting heat transfer by means of HAM." *Indian Journal of Physics* 92, no. 2 (2018): 205-214.

[38] Sheikholeslami, M., D. D. Ganji, H. R. Ashorynejad, and Houman B. Rokni. "Analytical investigation of Jeffery-Hamel flow with high magnetic field and nanoparticle by Adomian decomposition method." *Applied Mathematics and Mechanics*33, no. 1 (2012): 25-36.

[39] Sheikholeslami, M., D. D. Ganji, and H. R. Ashorynejad. "Investigation of squeezing unsteady nanofluid flow using ADM." *Powder Technology* 239 (2013): 259-265.

[40] Sheikholeslami, M., D. D. Ganji, and M. M. Rashidi. "Magnetic field effect on unsteady nanofluid flow and heat transfer using Buongiorno model." *Journal of Magnetism and Magnetic Materials* 416 (2016): 164-173.

[41] Sheikholeslami, Mohsen, and Davood Domiri Ganji. "Nanofluid flow and heat transfer between parallel plates considering Brownian motion using DTM." *Computer Methods in Applied Mechanics and Engineering* 283 (2015): 651-663.

[42] Sheikholeslami, M., and D. D. Ganji. "Nanofluid hydrothermal behavior in existence of Lorentz forces considering Joule heating effect." *Journal of Molecular Liquids* 224 (2016): 526-537.

[43] Sheikholeslami, Mohsen, Hamid Reza Ashorynejad, Davood Domairry, and Ishak Hashim. "Investigation of the laminar viscous flow in a semi-porous channel in the presence of uniform magnetic field using optimal homotopy asymptotic method." *Sains Malaysiana* 41, no. 10 (2012): 1177-1229.

[44] Sheikholeslami, M., and D. D. Ganji. "Magnetohydrodynamic flow in a permeable channel filled with nanofluid." *Scientia Iranica. Transaction B, Mechanical Engineering* 21, no. 1 (2014): 203.

[45] Sheikholeslami, M., and D. D. Ganji. "Heat transfer of Cu-water nanofluid flow between parallel plates." *Powder Technology* 235 (2013): 873-879.

[46] Sheikholeslami, M., H. R. Ashorynejad, D. D. Ganji, and A. Yıldırım. "Homotopy perturbation method for three-dimensional problem of condensation film on inclined rotating disk." *Scientia Iranica* 19, no. 3 (2012): 437-442.

[47] H. Bararnia, E. Ghasemi, G. Domairry, S. Soleimani. "Behavior of micro-polar flow due to linear stretching of porous sheet with injection and suction" *Advances in Engineering Software* 41 (2010) 893–897.