

Unstable Waves in Doped Semiconductors and Their Theoretical Investigations

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Abstract: When 1-valent and 2-valent solids are used as doping into semiconductors, in certain values of external electric field, increment waves occur. Frequency and increment of the wave have been analytically investigated depending on external electric field and charge carrier densities in equilibrium case. Also theoretical and experimental results have been compared.

Key words: Doped semiconductors, Charge carrier density, Current oscillations, Charge mobility,

Katkılı Yarıiletkenlerde Kararsız Dalgalar ve Bunların Teorik İncelemeleri

Özet: Bir ve iki valans değerli katılar yarıiletkene katkı maddesi olarak kullanıldığında dış elektrik alanının bilinen değerlerinde dalga artışı meydana gelir. Frekans ve dalgadaki artış harici elektrik alanına ve yük taşıyıcı yoğunluklarındaki denge durumuna bağlı analitik olarak incelendi. Bu arada teorik ve deneysel sonuçlar karşılaştırıldı.
Anahtar kelimeler: Katkılanmış yarıiletkenler, Yük taşıyıcı yoğunluğu, Akım titreşimleri, Yük devinirliği

Introduction

Existing of doped semiconductors in electric field is important to bring up some of their physical phenomena and helps to develop some experimental and theoretical comments about their physical properties. For example, when a doped semiconductor placed in a homogeneous and constant external electric field inside and outside of the semiconductor different electric fields appear. Inside component directs to environment energy supply and generates a radiation. Utilizing of this application various devices are build up. Occurring an oscillation in current by the application of external electric field to the semiconductor caused to develop Gunn devices [Gunn, 1963]. Oscillations of current in doped semiconductors can be causes of recombination and generation of the charge carriers. It is clear that, the energy supplied from external electric field by charge carriers, should be larger than around Coulomb potentials barriers to get over them. That is, the value of the carrier drift velocity produced from the external electric field should be $\mu E_0 \geq S'$. Where E_0 is the external electric field intensity, μ is the mobility of the charge carriers and S' is the sound elasticity wave velocity in the semiconductor medium. Particles having $v_d = \mu E_0$ velocity are called hot charge carriers. After the events of recombination and generation, these carriers in doped semiconductors, that electrons and holes, have their own numbers and these process take place in a very short time. During the recombination and generation process outside of the device an oscillation occurs in current, which is called unstable wave. In fact, this unstable case starts after non equal distributions of electrical field take place inside and outside of the doped semiconductor.

In doped semiconductors, doping materials can have several valent values. For example gold, as a doping material, can possess neutral, 1+, 1-, 2- and 3- valent values in germanium. Again copper atoms can take

neutral, 1+, 2+ and 3+ valences values and etc. [Bruevich et. al, 1972]. Such doping materials can form different energy levels in the forbidden band interval. Formed energy levels by doping atoms can hold electrons or holes and can leave them.

Existing an external effect, such doping centers become active in any case. According to ref. [Bruevich et. al, 1972], 1- valence atom and 2+ valent atom of gold became more active than other. Electrons (and holes) take energy from external electric field as $eE_0\ell$ (e: elementary positive charge, ℓ : mean free space). Electrons with this amount of energy can get over Coulomb potential surrounding 1- value center and be captured by the centers (recombination). However, from doped centers to the conduction band electron can pass by the thermal energy. That means, after recombination and releasing of electrons, the number of electrons in conduction band changes. In the other side, the number of holes increases with doped centers capturing of electron from valence band. Process of capturing and releasing of electron and holes change the number of charge carriers in crystal. Since there are Coulomb barriers around holes, as increasing of electric field intensity, coefficient of holes capturing decreases [Hasanov et al, 2009a; Hasanov et al, 2009b; Hasanov et. al, 2008c], on the contrary, coefficient of electron capturing of by negative centers increases in a large applied electric field ($eE_0\ell \gg k_B T$, k_B is the Boltzmann constant and T is the temperature.). It is experimentally found that there is increasing in electron capturing coefficient and decreasing in hole capturing coefficient [Iglitsin et. al, 1960]. From these experimental results frequencies of internal waves have been calculated [Iglitsin et. al, 1960] for 1- valent and 2- valent values. During inside waves increase, current of outside device oscillates. For this reason we will investigate inside wave increments.

Theoretical Work

The densities of electron and hole in unit volume are n_- and n_+ respectively. In doped semiconductor let us assume that 1- valence atoms and 2- valence atoms concentrations are N and N_- respectively. Their total is a constant, that is

$$N_0 = N + N_- = \text{Constant} \quad (1)$$

In doped semiconductor the continuity equation is as following.

$$\frac{\partial n_-}{\partial t} + \text{div} J_- = \left(\frac{\partial n_-}{\partial t} \right)_{\text{recom.}} \quad (2)$$

Where $J_- = -n_- \mu_- (E) E - D_- \nabla n_-$ is the current density of electrons, μ_- is the electron mobility and D_- is the diffusion constant. In the right side of

equation (2) the term of $\left(\frac{\partial n_-}{\partial t} \right)_{\text{recom.}}$ is the capturing

and releasing time derivative of electrons and characterizes the number changes of them.

Releasing of the electrons increases the number of free electron inside the semiconductor sample and on the contrary capturing of electrons decreases it. Thus we can write

$$\left(\frac{\partial n_-}{\partial t} \right)_{\text{recom.}} = \varphi_-(0) n_{1-} N_- - \varphi_-(E) n_- N \quad (3)$$

where $\varphi_-(0) n_{1-} N_-$ term is the releasing of electrons from 2-valence of doped and $\varphi_-(E) n_- N$ term is the electron capturing of 1-valent of doped. $\varphi_-(0)$ is the temperature dependent releasing coefficient and $\varphi_-(E)$ is the electric field dependence capturing coefficient. When $E=0$ $\varphi_-(E)$ must be equal to $\varphi_-(0)$ and according to this concept following equations can be written. At the same time equation (3) contains n_{1-} density and in nonexistent of electric field, stable equilibrium case, equation (4) can be written.

$$\left(\frac{\partial n_-}{\partial t} \right)_{\text{recom.}} = 0; \quad \varphi_-(E) = \varphi_-(0) \quad (4)$$

If we replace equation (4) into equation (3) we obtain following result.

$$n_{1-} = \frac{n_-^0 N_0}{N_-^0} \quad (5)$$

Where n_-^0 is the density of electrons in equilibrium state, N_0 is the 1- valent doping density and N_-^0 is the 2- valence doping density. For holes continuity equation (2) can be written as follow.

$$\frac{\partial n_+}{\partial t} + \text{div} J_+ = \varphi_+(E) n_+ N - \varphi_+(0) n_+ N \left(\frac{\partial n_+}{\partial t} \right)_{\text{recom.}} \quad (6)$$

$$J_+ = n_+ \mu_+(E) E - D_+ \nabla n_+$$

Where J_+ , $\mu_+(E)$ and D_+ are the current density, mobility and diffusion coefficient of holes respectively. When $E=0$, $\varphi_+(E) = \varphi_+(0)$. In stable equilibrium state hole density is as following form.

$$n_{1+} = \frac{n_+^0 N_1^0}{N_0} \quad (7)$$

Where n_+^0 is the density of holes in the stable state. Time dependent change of the doping with 2-valence determines change of the doping with 1- valence. According to this case, time dependent doping with charge of 2- values changing depending on time is as following equation.

$$\left(\frac{\partial N_-}{\partial t} \right) = \left(\frac{\partial n_+}{\partial t} \right)_{\text{recom.}} - \left(\frac{\partial n_-}{\partial t} \right)_{\text{recom.}} \quad (8)$$

During the time of non existent of recombination and releasing events according to conservative of particles it should be written

$$\Delta n_+ = \Delta n_- \quad (9)$$

In the existence of recombination and releasing process, conservative of free charge carriers [4] can be written as

$$\text{div} J = e \text{div} (J_+ - J_-) = 0 \quad (10)$$

Equation (10) shows that the density of the current (J) is not the function of the coordinates but only time. It is clear that when there is no current in outside circuit, inside device is in the case of unstable and $J_0 = \text{Constant}$. Thus the current density is as follow.

$$J = J_0 + \Delta J, \quad \Delta J = 0 \quad (11)$$

inside the crystal declined or increased waves form by the effect of the electric field and they are called unstable waves. Inside the crystal relations between unstable wave time, the density of the charge carriers and the electric field can be written as follow.

$$(\Delta n_+, \Delta n_-, \Delta N_-, \Delta E) \cong e^{i(kr - \omega t)} \quad (12)$$

In this particle wave (monochromatic) form k is the wave vector, r is the radius vector and ω is the forming wave frequency. Obtained frequency from the dispersion formula is complex

$$\omega = \omega_0 + ij \quad (13)$$

and the wave vector is real.

$$k = \frac{\pi}{L} m \quad (m = 1, 2, 3, \dots) \quad (14)$$

Where L is the length of the crystal. Wave vector given in equation (14) is the statement that there is fixed widening wave. Here we will investigate for the cases of $N \gg N_-$, $n_+ \ll (N, N_-)$ and $n_- \ll (N, N_-)$. Equations (2, 6, 8, 19) conclude inside unstable wave's frequencies increment in ω_0

and j . Non linear dependent of electric field terms in equations (3, 6, 8, 10) are as follow.

$$\begin{aligned} n_{\pm}(x,t) &= n_{\pm}^0 + \Delta n_{\pm}(x,t) \\ \Delta N_{-}(x,t) &= N_{-}^0 + \Delta N_{-}(x,t) \\ E &= E_0 + \Delta E(x,t) \end{aligned} \quad (15)$$

Using equation (13) we can form them linear and can obtain following equations system

$$\Delta N_{-} = \frac{\Omega_{-}(\omega_{ed} + i\omega)}{\omega_{ed}^2 + \omega^2} \Delta n_{-} - \frac{\Omega_{+}(\omega_{ed} + i\omega)}{\omega_{ed}^2 + \omega^2} \Delta n_{+} \quad (16)$$

$$-i\omega \Delta n_{-} - \frac{\partial}{\partial x} \left(v_{-} \Delta n_{-} + \frac{\sigma_{-}}{e} \Delta E \right) = \quad (17)$$

$$\frac{\Delta N_{-}}{\tau_e} - \frac{\Delta n_{-}}{\tau_{-}} - \frac{n_{1+}}{\tau_{-}} \beta_{-} \frac{\Delta E}{E_0}$$

$$-i\omega \Delta n_{+} + \frac{\partial}{\partial x} \left(v_{+} \Delta n_{+} + \frac{\sigma_{+}}{e} \Delta E \right) = \quad (18)$$

$$- \frac{\Delta N_{-}}{\tau_p} - \frac{\Delta n_{+}}{\tau_{+}} + \frac{n_{1+}}{\tau_{+}^E} \beta_{+} \frac{\Delta E}{E_0}$$

$$\Delta E = \frac{1}{\sigma_d} (\Delta J - ev_{+} - ev_{-} \Delta n_{-}) \quad (19)$$

Note that when equations (3, 6, 8, 10) made linear, as equations (16, 17, 18, 19), external electric field $E_0 = E_{0x}$ goes toward x-axis and these characteristic terms are obtained;

$$eE_0 \ell \gg k_0 T; \omega_{ed} = \frac{\tau_e + \tau_d}{\tau_e \tau_d};$$

$$\frac{1}{\tau_e} = \varphi_{-}(E_0) n_{-}^0 + \varphi_{-}(0) n_{1-}$$

$$\frac{1}{\tau_d} = \varphi_{+}(0) n_{+}^0 + \varphi_{+}(E) n_{1+} \quad \text{Where } \tau_e \text{ and } \tau_d$$

are the recombination and releasing time of electrons and holes respectively, that they are produced by unbalanced doping.

$$v_{\pm} = \mu_{\pm} \left[1 + \frac{d \ln \mu_{\pm}(E_0)}{d \ln(E_0^2)} \right] E_0 \text{ is the drift}$$

velocity of the electrons and holes in the electric field.

$$\beta_{\pm} = 2 \frac{d \ln \varphi_{\pm}(E_0)}{d \ln(E_0^2)} \text{ is a constant which has no}$$

unit. τ_{-} , $\left[\frac{1}{\tau_{-}} = \varphi_{-}(E_0) N_0 \right]$ is the capturing time of

the electrons. τ_{+} , $\left[\frac{1}{\tau_{+}} = \varphi_{+}(0) N_{-}^0 \right]$ is the capturing

time of the holes. τ_{+}^E , $\left[\frac{1}{\tau_{+}^E} = \varphi_{+}(E_0) N_0 \right]$ is the releasing time of holes.

$$\Omega_{-} = \frac{1}{\tau_{-}} \left[1 - \frac{\sigma_{-}}{\sigma_d} \left(\beta_{-} + \frac{n_{1+}}{n_{-}^0} \frac{\tau_{-}}{\tau_{+}^E} \beta_{+} \right) \right] \quad \text{and}$$

$$\Omega_{+} = \frac{1}{\tau_{+}} \left[1 + \frac{\sigma_{+}}{\sigma_d} \left(\frac{\tau_{+} n_0}{\tau_{-} n_0^0} \beta_{-} + \frac{n_{1+}}{n_{+}^0} \frac{\tau_{+}}{\tau_{+}^E} \beta_{+} \right) \right] \text{ are}$$

the characteristic frequencies. Where

$$\sigma_d = \sigma_{-} + \sigma_{+}; \sigma_{\pm} = en_{\pm} \mu_{\pm} \left[1 + \frac{d \ln \mu_{\pm}(E_0)}{d \ln(E_0^2)} \right]$$

If we replace equations (15, 18) into equations (16, 17), using equation (12) condition, we can obtain following equations.

$$\left\{ -i\omega + \frac{ik\sigma_{-}v_{-}}{\sigma_d} \left[\frac{1 + \frac{\mu_{+}}{\tau_{+}} \frac{1}{\tau_{-}} (1 - \Omega_{-} \tau_{-})}{\tau_{+}(\omega_{ed}^2 + \omega^2)} \right] \frac{\sigma_{-}(\omega_{ed} + i\omega)}{\tau_{-}(\omega_{ed}^2 + \omega^2)} + \frac{1}{\tau_{-}} + \frac{n_{1+} ev_{+}}{\tau_{-}^E \sigma_{+} E_0} \beta_{+} \right\} \Delta n_{-} \quad (20)$$

$$+ \left\{ \frac{\sigma_{+} v_{+}}{\sigma_d} - \frac{\Omega_{+}(\omega_{ed} + i\omega)}{\tau_{+}(\omega_{ed}^2 + \omega^2)} + \frac{n_{1+} ev_{-}}{\tau_{+}^E \sigma_{-} E_0} \beta_{-} \right\} \Delta n_{+} = 0$$

$$\left\{ \frac{ik\sigma_{-}v_{-}}{\sigma_d} + \frac{\left[\frac{1 + \frac{\mu_{+}}{\tau_{+}} \frac{1}{\tau_{-}} (1 - \Omega_{-} \tau_{-}) (\omega_{ed} + i\omega)}{\tau_{+}(\omega_{ed}^2 + \omega^2)} \right]}{\tau_{-}(\omega_{ed}^2 + \omega^2)} - \frac{n_{1+} ev_{+}}{\tau_{-} \sigma_{+} E_0} \beta_{+} \right\} \Delta n_{-} \quad (21)$$

$$+ \left\{ -i\omega - \frac{ik\sigma_{+}v_{+}}{\sigma_d} - \frac{\Omega_{+}(\omega_{ed} + i\omega)}{\tau_{+}(\omega_{ed}^2 + \omega^2)} + \frac{1}{\tau_{+}} - \frac{n_{1+} ev_{-}}{\tau_{+} \sigma_{-} E_0} \beta_{-} \right\} \Delta n_{+} = 0$$

Transforming formula (19, 20) to dispersion equation as following forms

$$A_{+} \Delta n_{+} + A_{-} \Delta n_{-} = 0$$

$$B_{+} \Delta n_{+} + B_{-} \Delta n_{-} = 0 \quad (22)$$

$$A_{+} B_{-} - A_{-} B_{+} = 0$$

following equation is obtained.

$$\begin{aligned} \omega^2 - \frac{\omega_{ed} + i\omega}{\omega_{ed}^2 + \omega^2} \\ \left[\frac{i\omega(1+x)}{\tau_{+} \tau_{-}} + \frac{ik\sigma_{-}v_{-}}{\sigma_d \tau_{+} \tau_{-}} \left(1 + \frac{\tau_{-}^2}{\tau_{+}^2} x \right) + \frac{\mu_{+}}{\mu_{-}} \frac{1}{\tau_{+} \tau_{-}^2} \left(1 + \frac{\mu_{+}^2}{\mu_{-}^2} x \frac{\tau_{-}^3}{\tau_{+}^3} \right) \right] \\ + \frac{\omega k \sigma_{+} v_{+}}{\sigma_d} (1-x) + \frac{ik\sigma_{-}v_{-}}{\sigma_d \tau_{+}} \left(1 + \frac{\tau_{-}}{\tau_{+} x} \right) \end{aligned} \quad (23)$$

This dispersion equation is obtained from following values of β_{\pm} parameters and electric field in $\beta_{+} < 0$ condition.

$$|\beta_{\pm}| = \frac{\tau_{+}^E \sigma_d n_{+}^0}{\tau_{+} \sigma_{+} n_{1+}}; \beta_{-} = \frac{\sigma_d n_{1-}^0}{\sigma_{-} n_{1-}}; \beta_{\pm} \gg 1;$$

$$E_0 = \frac{\sigma_d}{k(2\sigma_{-} \sigma_{+} \mu_{-} \mu_{+} \tau_{+} \tau_{-})^{1/2}}; x = \frac{n_{-}^0}{n_{+}^0} \quad (24)$$

Since equation (23) is a fourth degree of ω frequency it is difficult to adopt it to a geometrical statement. For that reason, to solve this equation we will take some conditions of the parameters x and

$\frac{\tau_+}{\tau_-}$ These conditions must be convenient for experimental results.

If $x \gg 1$ and $\tau_+ = \tau_-$ that is, in the equilibrium state the density of free electrons is larger than that of holes and the capturing time of electrons is equal to that of holes. In that case internal waves in mentioned semiconductors quickly decline and their increment is $j < 0$. In the case of $x \gg 1$, $\frac{\tau_+}{\tau_-} \gg 1$

and $\frac{\tau_+}{\tau_-} > x$ it is possible to write equation (23) as follow.

$$\begin{aligned} & (\omega_{ed}^2 + \omega^2) \left(\omega^2 - \frac{\omega_1 \omega}{\mu_-} \mu_+ - i \frac{\omega_1}{\tau_- x} \right) \\ & - (\omega_{ed} + i\omega) \left(\frac{i\omega x}{\tau_- \tau_+} + \frac{i\omega_1 \mu_+ \tau_-}{\tau_+^2 \mu_- \tau_d x} + \frac{\mu_- \tau_-}{\mu_+ \tau_d \tau_+^3 x} \right) = 0 \end{aligned} \quad (25)$$

Where $\omega_1 = \frac{k\sigma_- v_-}{\sigma_d}$ is a characteristic frequency.

If we separately make equal the real and the imaginer parts of equation (25) to zero, we can obtain following equations.

$$\begin{aligned} \omega_0 &= \frac{\omega_1 \mu_+}{\mu_-} \\ j &= \left(\frac{\mu_-}{\mu_+} \right)^2 \frac{1}{\tau_+} \frac{\tau_- \omega_{ed}}{\tau_d} \frac{1}{2(\tau_+ \omega_{ed})^2} \end{aligned} \quad (26)$$

If we replace the value of electric field given in equation (24) into equation (26), according to $\omega_0 \gg j$ condition, we obtain following statement.

$$x \gg \left(\frac{\mu_-}{\mu_+} \right)^4 \frac{\tau_-}{\tau_d} \left(\frac{\tau_-}{\tau_+} \right)^3 \frac{1}{2\omega_{ed} \tau_+} \quad (27)$$

Seeing that equation (26) statement is conservative, the wave including ω_0 , in equation (25), and increment of j , becomes unstable and comes out to outside the device forming oscillation in the current.

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Related with the electric field and the densities, this equation can be written as follow

$$\begin{aligned} (\Delta n_+, \Delta n_-, \Delta E) &\approx e^{i(kx - \omega t)} \\ &= \Phi(x) e^{j\omega t} \cos(\omega_0 t + \delta) \end{aligned} \quad (28)$$

Where δ is the phase of the wave. The wave increases with time and becomes unstable. To calculate the ω_0 frequency and the j increment of the outside devices wave (equation 28), firstly it is necessary to measure the impedance (complex resistance) of the semiconductors having above mentioned model.

Conclusions

The aim of this study is to learn about the unstable waves interior the semiconductor crystal. Manufactured devices, using the principles of the Gunn's effect, work in the range of 10^9 - 10^{11} Hz frequencies. Critical value of the electric field were obtained interval of 200-300 V/cm. If we look at these values for ω_1 frequency, as the length of the crystal

$L \approx 1$ cm, we can obtain it as $\omega_0 \approx 10^8 - 10^9$ rad/s.

The increments of the waves for the same values can be obtained $j \approx 10^8 - 10^7$ Hz, which show that theoretical work is quite well agreement with the experimental studies.

It can be said that, when solids having 1- and 2-valence values are used as doping material into p-type and n-type semiconductors, waves can occur inside the doped semiconductor by an external applied electric field. For the waves, if the time of the electron capturing is longer than that of the holes, the wave is declined, in contrast, the time of the electron is lesser than that of the holes, the wave is increased. In this process the density of the electrons must be larger than the density of holes in equilibrium state. The capturing of the electron is more dominant compared to the capturing of holes. These results are obtainable for the doped semiconductors having the parameters $x \gg 1$ and $\tau_- \ll \tau_+$. Appropriate model semiconductors can be understood from their ω_0 frequencies and j increments. Related apparatus made of such semiconductor material can be more durable.

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