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Covid-19 cases in Morocco: A comparative analysis

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Abstract

Covid-19 is a highly infectious disease caused by novel Corona virus SARS-CoV-2, affecting the whole world. In this paper, we introduce and apply two iterative methods, RMsDTM and R2KM, to obtain approximate values of Covid-19 cases in Morocco. We also compare the approximations of both methods and see that the solution of RMsDTM is more accurate.

Keywords: Morocco Covid-19 Infection Transform function Comparison
2020 MSC: 92C60, 92D30, 60G25

1. Introduction

Covid-19 is a highly infectious disease caused by novel Corona virus SARS-CoV-2. In the past, the novel coronavirus has emerged twice - SARS in 2003 and MERSCoV in 2012. However, the spread of the disease, unlike Covid-19, was limited to a smaller area. The first case of Covid-19 was observed in Wuhan, China, in December 2019. Since then, the disease has spread over almost all the parts of the world and was declared a pandemic by WHO on 11th March, 2020 [17].

Usually, Corona virus affects the respiratory tracts. The incubation period of this disease is 14 days. This virus is transmitted through aerosols or droplets from the nose or mouth of the infected person when the person sneezes,

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coughs or exhales. Also, a healthy individual may get infected through surface contamination. Social distancing, hand hygiene and use of mask are some important measures to prevent the virus from spreading.

At present, the COVID-19 disease is one of the biggest challenges in the world. It has affected more than 200 million (241,485,698) human beings and more than 40 million (4,914,336) people have died all over the world [18]. Different variants of the virus like alpha, beta, gamma, delta; have been seen since the time it was first detected. It has been seen that delta variant spreads very fast and is more fatal [5].

Kingdom of Morocco is the north-westernmost country in Africa. It spans an area of 446,550 km^2 with a population of roughly 37 million. As the Corona pandemic affected the world, Morocco was not left behind. First two cases were confirmed on the same day, 2nd March, 2020, [15] both being Moroccan citizens and had returned from Italy a few days back. After 8 days i.e. on 10th March, third case was reported. On the same day one of the two initial patients, an 89 year old lady died while the other patient recovered subsequently. The disease started spreading in Morocco. As the number of cases started increasing, the government put many restrictions like closing educational institutions and suspending international flights. On 19th March, 2020, a state of medical emergency was declared in Morocco. The situation was brought under control through restrictions imposed by the government. June 2020 onwards the cases started to decline and some restrictions were removed. By the first half of July 2020, daily new infection cases dropped substantially and it was believed that the disease was successfully contained in Morocco. But in the month of August 2020, the numbers spiked again and active Covid cases reached the highest level in the month of November 2020. As a result, the government imposed some restrictive measures again like night curfew, which helped in reducing the number of active cases substantially until the end of June 2021. It was seen that July 2021 onwards, the cases started rising again. The spike could be attributed to various reasons like relaxation on restrictions on air-travel and new Delta variant of the virus. The government again imposed restrictions like night curfew, besides enforcing the use of mask and improving the medical facilities. The country suffered major economic setback due to lockdowns. In order to avert an economical disaster, the country boosted its vaccination strategy. As a result, the vaccination rate in Morocco was the highest in Africa in the first phase. By the end of August 2021, more than 50% population had received single dose of the vaccine and more than 40% were fully vaccinated. Whereas, in the beginning of July 2021, the time when cases had started rising again, around only 27% had received single dose and only 25% were fully vaccinated [16].

Epidemics cause loss of human lives and livelihoods. Sometimes, medical systems are not well prepared to handle huge number of patients, leading to worsening of the situation. Various studies have been undertaken to understand the causes and effects of epidemics. Many mathematical models have also been developed to understand how epidemics spread over a period of time. SIS, SIR, SEIR and SEIRD are few such models that approximate the number of people getting infected at a given time [1, 2, 6, 9]. Such studies help in better preparedness to handle the medical situation. In this paper, we use the SIR model to approximate the parameters of Covid-19 cases in Morocco for the period of 60 days: from June 1st, 2021 to July 30th, 2021.

In SIR compartmental model, s stands for the number of susceptible, which may get infected to move to the i compartment - representing the active number of infected. Infected get removed (recoveries or deaths) from i to move to the r compartment, from where there is no exit.

If $\beta,\ \gamma$ are the rate of transmission of the disease and rate of recovery respectively, then the following constitute the system of differential equations for this model [4]:

$$\begin{cases} \dfrac{ds}{dt} = -\beta si, \\[2ex] \dfrac{di}{dt} = \beta si - \gamma i, \\[2ex] \dfrac{dr}{dt} = \gamma i. \end{cases} \tag{1}$$

This paper is organized as follows: In Section 2, we describe three numerical methods, namely the Differential Transformation Method (DTM), Multistage Differential Transformation Method (MsDTM) and Runge-Kutta Method (RKM). In this section, after a brief explanation of these methods, we introduce Repeated Multistage Differential

Transform Method (RMsDTM) and Repeated Runge-Kutta Method (R2KM). We present a case study in Section 3 wherein we apply the SIR model to obtain the values of s, i and r using repeated MsDTM and repeated RKM. We also make a comparison of values obtained using both methods with the actual data. Conclusion of the study is given in Section 4.

2. Known Numerical Methods

2.1. Differential Transform Method (DTM)

Zhou [13] introduced and applied the method of Differential Transform to solve linear and nonlinear initial value problems in electric circuit analysis. This method has since been applied to solve systems of linear and non-linear ordinary differential equations as well as systems of partial differential equations, with initial conditions [8, 12]. We briefly describe the method.
For an analytical function f defined on some open interval containing 0, the Taylor series about 0 is given by

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k.$$

The kth coefficient of this series is called the kth differential transform of f and is denoted by $F(k)$ i.e.

$$F(k) = \frac{f^{(k)}(0)}{k!}.$$

Then, f is the inverse differential transform of F and is defined as

$$f(x) = \sum_{k=0}^{\infty} F(k) x^k. \tag{2}$$

In Table 1, we enlist some basic properties of the transform function that are used to solve the system of differential equations.

Table 1:

	Original Function	**Transformed Function**
1.	$f(x) = u(x) \pm v(x)$	$F(k) = U(k) \pm V(k)$
2.	$f(x) = u(x)v(x)$	$F(k) = \sum_{m=0}^{k} U(m)V(k-m)$
3.	$f(x) = \alpha u(x)$	$F(k) = \alpha U(k)$
4.	$f(x) = \alpha x^m$	$F(k) = \alpha\delta(k-m)$ where $\delta(k-m) = \begin{cases} 1 & \text{if } k = m \\ 0 & \text{if } k \neq m \end{cases}$
5.	$f(x) = \frac{du(x)}{dx}$	$F(k) = (k+1)U(k+1)$
6.	$f(x) = \frac{d^{(m)}u(x)}{dx^m}$	$F(k) = (k+1)(k+2)\ldots(k+m)U(k+m)$

In Differential Transform method, the function f is approximated for $x \in [0, T]$ by a finite degree polynomial

$$f(x) = \sum_{k=0}^{K} F(k) x^k \quad \text{for some } k \in \mathbb{N} \tag{3}$$

obtained from the series in (2) upto the Kth degree term. The remaining terms represent the error in the above approximation which is negligible.

Using the properties of the transform function as stated in Table 1, the system of differential equations for SIR model, given in (1), are transformed as:

$$\begin{cases} S(k+1) = \frac{1}{k+1}\left(-\beta \sum_{m=0}^{K}(S(m)I(k-m))\right), \\ I(k+1) = \frac{1}{k+1}\left(\beta \sum_{m=0}^{K}(S(m)I(k-m) - \gamma I(k))\right), \\ R(k+1) = \frac{1}{k+1}\gamma I(k). \end{cases}$$

The above equations give recursive formulae for finding the transform functions $S(k)$, $I(k)$ and $R(k)$ of $s(x)$, $i(x)$ and $r(x)$ respectively. Using these transform functions, values of Susceptible, Infected and Recovered for various days are obtained from inverse differential transforms of S, I and R as given in (2).

2.2. *Multistage Differential Transform Method (MsDTM)*

It was seen that the approximation of a function by a polynomial, as given in equation (3), using DTM, was not very accurate in case the time period is long. In order to obtain a better approximation, Multistage Differential Transform Method was introduced [12]. In this method, the domain under consideration is partitioned into sub-intervals of equal length, say the interval $[0, T]$ is subdivided into n sub-intervals $[t_{i-1}, t_i]$, $i = 1, \ldots, n$, where $t_0 = 0$ and $t_n = T$. There are n sub-intervals of length $h = T/n$ each. DTM is applied on each interval, using the values obtained in the previous step as the initial conditions of the present step. Such consideration of changing the initial conditions within the process, leads to a better approximation of the solution as compared to the solution obtained in DTM.

Solution of Multistage DTM [11] using the initial conditions $f_i(t_{i-1}) = f_{i-1}(t_{i-1})$ at the ith step with $f_0(t_0) = f(t_0)$, is of the form

$$f(x) = \begin{cases} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{cases} \quad \text{where} \quad f_i(x) = \sum_{k=0}^{K} F(k)(x - t_{i-1})^k, \ x \in [t_{i-1}, t_i]$$

for $i = 1, \ldots, n$.

2.3. *Runge-Kutta Method (RKM)*

In this section, we briefly describe the 4th order Runge-Kutta Method, a widely used iterative method [3]. In this method the nth solution, say y_n, is dependent on the $(n-1)$th solution, y_{n-1}. For a given initial value problem given as

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0,$$

the solution at nth iteration is given by

$$y_{n+1} = y_n + (h/6)(k_1 + 2k_2 + 2k_3 + k_4),$$

where

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ k_2 &= f(t_n + h/2, y_n + k_1 h/2), \\ k_3 &= f(t_n + h/2, y_n + k_2 h/2), \\ k_4 &= f(t_n + h, y_n + k_3 h). \end{aligned}$$

Now, we extend this method to solve system of equation (1). Then, equation (1) can be written as

$$\begin{cases} \dfrac{ds}{dt} = f_1(t,s,i,r) = -\beta si, \\ \dfrac{di}{dt} = f_2(t,s,i,r) = \beta si - \gamma i, \\ \dfrac{dr}{dt} = f_3(t,s,i,r) = \gamma i \end{cases}$$

where β is the rate of infection and γ is the rate of recovery. By using 4th order Runge-Kutta Method, we obtain solution of the form:

$$\begin{aligned} s_{n+1} &= s_n + (h/6)(k_1 + 2k_2 + 2k_3 + k_4), \\ i_{n+1} &= i_n + (h/6)(g_1 + 2g_2 + 2g_3 + g_4), \\ r_{n+1} &= r_n + (h/6)(l_1 + 2l_2 + 2l_3 + l_4) \end{aligned}$$

where

$$\begin{aligned} k_1 &= f_1(t_n, s_n, i_n, r_n) = -\beta s_n i_n, \\ g_1 &= f_2(t_n, s_n, i_n, r_n) = \beta s_n i_n - \gamma i_n, \\ l_1 &= f_3(t_n, s_n, i_n, r_n) = \gamma i_n; \end{aligned}$$

$$\begin{aligned} k_2 &= f_1(t_n + h/2, s_n + k_1h/2, i_n + g_1h/2, r_n + l_1h/2) \\ &= -\beta(s_n + k_1h/2)(i_n + g_1h/2), \\ g_2 &= f_2(t_n + h/2, s_n + k_1h/2, i_n + g_1h/2, r_n + l_1h/2) \\ &= \beta(s_n + k_1h/2)(i_n + g_1h/2) - \gamma(i_n + g_1h/2), \\ l_2 &= f_3(t_n + h/2, s_n + k_1h/2, i_n + g_1h/2, r_n + l_1h/2) \\ &= \gamma(i_n + g_1h/2); \end{aligned}$$

$$\begin{aligned} k_3 &= f_1(t_n + h/2, s_n + k_2h/2, i_n + g_2h/2, r_n + l_2h/2) \\ &= -\beta(s_n + k_2h/2)(i_n + g_2h/2), \\ g_3 &= f_2(t_n + h/2, s_n + k_2h/2, i_n + g_2h/2, r_n + l_2h/2) \\ &= \beta(s_n + k_2h/2)(i_n + g_2h/2) - \gamma(i_n + g_2h/2), \\ l_3 &= f_3(t_n + h/2, s_n + k_2h/2, i_n + g_2h/2, r_n + l_2h/2) \\ &= \gamma(i_n + g_2h/2); \end{aligned}$$

$$\begin{aligned} k_4 &= f_1(t_n + h, s_n + k_3h, i_n + g_3h, r_n + l_3h) = -\beta(s_n + k_3h)(i_n + g_3h), \\ g_4 &= f_2(t_n + h, s_n + k_3h, i_n + g_3h, r_n + l_3h) \\ &= \beta(s_n + k_3h)(i_n + g_3h) - \gamma(i_n + g_3h), \\ l_4 &= f_3(t_n + h, s_n + k_3h, i_n + g_3h, r_n + l_3h) = \gamma(i_n + g_3h). \end{aligned}$$

2.4. Repeated MsDTM and Repeated RKM

The solution of Multistage DTM, though better than the approximation of DTM solution has a limitation. In case there are parameters in the function, this method uses the same values of the parameters throughout the solution. Even though, at each step, new initial conditions on the function are used, the parameters are assumed to have same values as in the previous step. This assumption about parametric values is not in coherence with practical situations, for example parameters in SIR model that represent rates of transmission and recovery respectively, keep changing with time. Keeping this in view, we are introducing a method that takes this limitation into consideration, thereby making the solution better.

Repeated Multistage Differential Transform Method (RMsDTM). In RMsDTM, we divide the time period under consideration into suitable smaller time periods, say $[t_{j-1}, t_j]$, $j = 1, \ldots, m$. For each sub-time-interval, we apply the MsDTM as described in Section 2.2, where initial conditions on the function are dynamically updated. Further, we also update the values of the parameters of the function at each step in accordance with the prevalent conditions.

Repeated Runge-Kutta Method (R2KM). In R2KM also, we divide the time period under consideration into suitable smaller time periods, say $[t_{j-1}, t_j]$, $j = 1, \ldots, m$. For each sub-time-interval, we apply RKM as described in Section 2.3, where initial conditions on the function are dynamically updated, besides using the changed values of the parameters at each step according to that time.

3. Morocco: Covid-19

The Covid-19 pandemic has been modelled in equation (1) as the SIR model. We have used repeated MsDTM and repeated RKM introduced in Section 2.4 to find the number of susceptible (s), infected (i) and recovered (r) of Morocco Covid cases for a period of 60 days, starting from June 1st, 2021. The period of 60 days was divided into 4 parts of length 15 days each.

The initial conditions, as obtained from the actual data [14], used for both methods are as follows:

$$s(0) = 36825571; i(0) = 2944; r(0) = 167.$$

The values of β and γ used at the 4 steps of the method are enlisted in the Table 2. These values were obtained as averages of β and γ in each fortnight from day $1-15$, $16-30$, $31-45$ and $46-60$ respectively.

Table 2:

Step	β	γ
I	3.020791×10^{-9}	0.09995997
II	3.246358×10^{-9}	0.10176529
III	4.830442×10^{-9}	0.11046297
IV	5.161926×10^{-9}	0.08704035

3.1. *Repeated MsDTM for Morocco*

For the study of Covid-19 cases in Morocco through RMsDTM, we used the tools of Mathematica [10]. In this study, MsDTM is applied 4 times, at an interval of 15 days each, to obtain the values of the parameters susceptibles (s), active infections (i) and recovered (r) for the period of 60 days, starting from June 1st, 2021. At each iteration, the values of β and γ are updated as per Table 2. Also, the final s, i, r values obtained from preceding iteration are assumed as new initial conditions at each step. The number of infected and recovered thus obtained are plotted along-with the actual data in Figure 1.

We observe from the graphs that graph of actual infected is mostly increasing but is also decreasing in some portions. Whereas, the graph of values obtained using RMsDTM, with updated values of β and γ, is increasing throughout. However, the solutions obtained using this method are convergent to the actual data.

3.2. *R2KM for Morocco*

In this case also, we used the tools of Mathematica [10] to study the Covid-19 cases of Morocco through R2KM. At each step, 4th order RKM is applied. The period of 60 days, starting from 1st June, 2021, is divided into 4 blocks of a fortnight each and the method applied to each block. The initial values of s, i, r are updated using the output from the previous step. Besides, updated values of β and γ as given in Table 2 are used for each block. The graphs

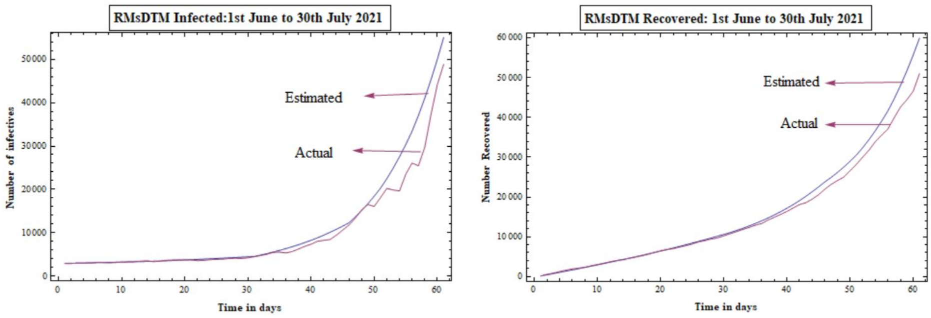

Figure 1: Infected and Recovered: RMsDTM

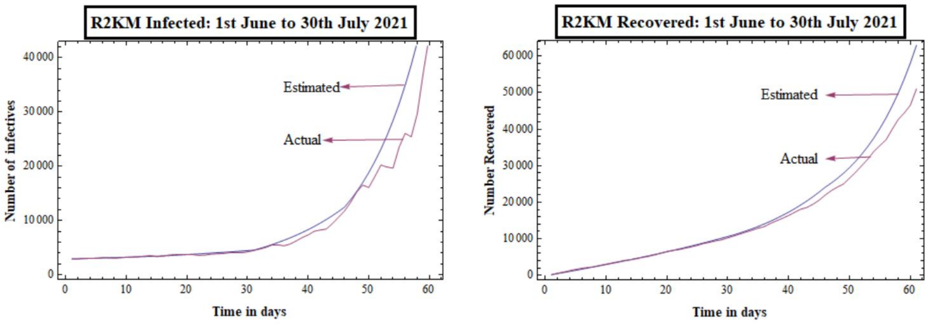

Figure 2: Infected and Recovered: R2KM

in Figure 2 depict the values of infected and recovered obtained using R2KM against the actual values. These graphs clearly show that the solution obtained using R2KM is close to the actual data. This justifies the use of updated values of β and γ in different blocks of the entire time period.

3.3. Comparison

Both Repeated MsDTM and Repeated RKM give solutions convergent to the actual data. A comparison of the solutions of both the methods to see which method is more accurate, is also attempted. For this, we visualised error in the estimations of both the methods, as shown in Figure 3. It can be seen from the graphs that the estimations

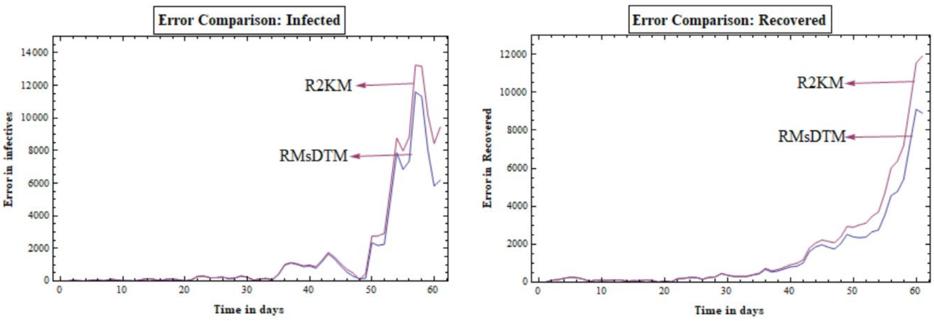

Figure 3: Error Comparison: Infected: RMsDTM & R2KM and Recovered: RMsDTM & R2KM

of RMsDTM are better than R2KM as the errors in the solutions of RMsDTM are lesser. The errors in number of infected and number of recovered are tabulated in Table 3 and Table 4, respectively.

Table 3: Errors in Infected

Day	**RMsDTM**	**R2KM**	**Day**	**RMsDTM**	**R2KM**
1	49.40	49.40	31	116.18	116.47
2	18.83	18.82	32	158.84	160.60
3	2.68	2.66	33	72.14	78.35
4	49.84	49.88	34	407.11	422.33
5	57.28	57.19	35	980.72	1011.31
6	14.94	14.87	36	1108.53	1141.16
7	113.79	113.86	37	1013.12	1049.89
8	65.92	65.98	38	887.48	932.62
9	17.44	17.51	39	921.61	981.66
10	21.35	21.45	40	791.52	875.45
11	7.27	7.20	41	1187.27	1276.61
12	84.46	84.43	42	1661.10	1758.92
13	144.24	144.23	43	1353.02	1465.38
14	101.39	101.40	44	929.02	1065.18
15	22.44	22.46	45	537.10	709.73
16	108.03	108.05	46	296.86	489.08
17	118.40	118.44	47	136.90	90.86
18	76.68	76.70	48	177.83	472.86
19	0.86	0.80	49	2346.04	2757.57
20	78.06	78.27	50	2175.73	2772.35
21	283.39	283.54	51	2256.77	2917.62
22	299.92	300.03	52	5029.59	5784.11
23	200.65	200.76	53	7869.20	8773.05
24	205.58	205.74	54	6843.60	7981.47
25	237.71	238.01	55	7344.78	8833.49
26	150.38	150.58	56	11603.60	13247.61
27	184.37	184.49	57	11321.70	13174.09
28	308.66	308.74	58	8012.20	10169.30
29	236.26	236.37	59	5829.07	8435.17
30	61.18	61.40	60	6193.32	9445.64

From Table 3, it can be seen that the methods RMsDTM and R2KM are comparable as the error in number of infected are almost same in the beginning of the table. We can also observe that on most of the days, approximation by RMsDTM is better.

Table 4: Errors in Recovered

Day	**RMsDTM**	**R2KM**	**Day**	**RMsDTM**	**R2KM**
1	114.06	114.05	31	290.02	300.75
2	130.80	130.75	32	294.46	307.94
3	205.21	205.04	33	375.78	396.89
4	263.31	262.91	34	435.97	472.28
5	238.09	237.30	35	670.04	732.01
6	162.98	162.18	36	531.17	597.16
7	43.37	42.51	37	589.52	663.02
8	112.24	111.25	38	697.08	785.12
9	89.59	88.36	39	806.86	920.23
10	105.44	103.80	40	840.85	994.40
11	108.18	106.52	41	1013.08	1176.92
12	113.20	111.49	42	1613.12	1792.42
13	53.49	55.34	43	1860.95	2065.81
14	79.90	82.00	44	1970.60	2216.39
15	68.03	70.55	45	1840.04	2147.77
16	118.98	121.53	46	1750.42	2075.85
17	89.24	91.91	47	2033.21	2390.00
18	1.81	4.83	48	2517.41	2932.63
19	70.31	66.60	49	2380.03	2895.60
20	12.87	17.73	50	2344.06	3018.40
21	181.05	185.94	51	2377.20	3109.25
22	195.12	200.17	52	2653.22	3468.38
23	248.08	253.53	53	2751.13	3697.15
24	242.94	249.15	54	3526.92	4676.36
25	151.69	159.17	55	4556.59	6009.43
26	248.18	255.70	56	4770.70	6364.76
27	273.20	280.89	57	5412.40	7193.98
28	449.75	457.88	58	7277.71	9330.31
29	360.84	369.79	59	9102.61	11550.94
30	297.45	307.78	60	8900.12	11914.47

Similar to what we observed in Table 3, about the error in the number of infected, we see from Table 4 of error in the number of recovered, that RMsDTM is giving better results in comparison to R2KM. Initially, for a few days R2KM is giving lesser error than RMsDTM, but eventually the error increases as the days proceed.

4. Discussion and Conclusion

The study of Covid-19 cases of Morocco through a compartmental model, solved using two iterative methods, enabled a comparison of accuracy of the methods employed. We introduced an improvement over the existing methods, namely Multistage Differential Transform Method and the 4th order Runge-Kutta Method. In the proposed advancements, termed as Repeated Multistage Differential Transform Method (RMsDTM) and Repeated Runge-Kutta Method (R2KM), the values of parameters used in a function are also updated repeatedly, besides updating the values of the function - a step already being used in the existing methods. Such amendment in the methods was considered keeping in view a possible change in the parametric values. In these methods, first the entire time period is divided according to the phases where parametric values show a variation, in our case we chose 15 days for this purpose. The parametric values are updated at the beginning of each such period. Then, within these phases, there is a further subdivision of the time period in which the values of the variables are calculated in each period and the solutions thus obtained in one step are used as initial conditions for the next.

A comparison of number of infected and number of recovered obtained using both the enhanced iterative methods with the actual data, indicates that both methods are giving solutions that are convergent to the actual data. We can conclude from this that the use of these methods to estimate the solution is reliable, with small error.

It could also be seen from the comparisons of both methods with each other, through comparing the errors in their approximations, that RMsDTM gives better solution than R2KM.

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