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Boundary Layer Flow of Viscous Incompressible Fluid Over a General Exponential Stretching Plate with Suction and Heat Transfer with Convective Surface Boundary Condition

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Abstract

The purpose of this article is to generalize the laminar boundary layer flow past an exponential stretching sheet, studied by Swati Mukhopadhyay [16], to general exponential stretching sheet. Thus, we study the heat transfer with convective surface boundary condition.

Keywords: Exponential stretching; Heat transfer; Boundary layer equations; Viscous incompressible fluid.

1. INTRODUCTION

Fluid dynamics is one of the important branches of science which deals with the study of fluid in motion and subsequent effect of fluid motion on the boundaries, which may be either solid surface or the interface of two immiscible fluids. The main beliefs of fluid dynamics are based on Newton's law of motion, conservation of momentum and conservation of energy. The base of fluid mechanics or fluid dynamics is fluid, so we define the fluid as substance which tends to flow due to the action of some force(s).

Many authors such as Gupta and Gupta [10], Dutta et al. [9], Chen and Char [5], extended the work of Crane [8], that is, investigated the boundary layer flow caused by the stretching sheet with heat transfer and mass transfer under different physical situations. Most of the literature deals with the study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed to be linear and proportional to the distance from the origin. Kumaran and Romanaiah [14] very beautifully dealt with boundary layer flow over a general quadratic stretching sheet. There after many authors such as Cortell [6, 7], Hayat and Sajid [11], Ali [2] investigated the thermal boundary layer flow by considering the non-linear stretching surface.

In all these above studies, the stretching character of plate is governed by linear function, nonlinear function or exponential function ex. In this chapter, we generalize the exponentially stretching sheet to general exponential stretching sheet. We also discuss the thermal boundary layer flow for nano-fluids Cu-water and Ag-water, Sanjayanand and Khan [21], studied the visco-elastic boundary layer flow and heat transfer to an exponentially stretching sheet. The problem of boundary layer flow and heat transfer of an incompressible viscous fluid with thermal radiation due to an exponential stretching sheet is investigated numerically by Bidin and Nazar. The influence of thermal radiation on the boundary layer flow in case of an exponential stretching sheet investigated by Hayat and Sajid [11]. Recently, numerical solution of flow and heat transfer of Powell-Eyring fluid over an exponential stretching sheet with variable conductivity studied by Khader and Megahed [13] and MHD stagnation point flow towards an exponential stretching sheet with prescribed wall temperature and heat flux have been studied by S.Q. Alvi.

1.1 Boundary Layer Theory

The boundary layer theory began with Ludwig Prandtl's paper on the motion of a fluid with very small viscosity, which was presented at the Third International Congress of Mathematicians in August, 1904 at {Heidelberg} and published in the Proceedings of the Congress in the following year.

1.2 Stretching Plate

A plate or sheet immersed in a fluid at rest. When the plate starts moving such that its velocity in a direction is directly proportional to the distance from a reference point or orifice, then the plate is called stretching plate. Every stretching plate is equivalent to the moving plate.

2. BOUNDARY LAYER FLOW PROBLEM

The governing equations for steady boundary layer flow of viscous incompressible fluid past a stretching plate are:

Continuity equation $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

Momentum equation $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}\quad , \quad (2)$$

where u and v are the velocity components along x and y axes, respectively, and ν is the kinematic viscosity of the fluid.

The appropriate boundary conditions for flow problem are:

$$y=0,\ u=U(x),\ v=-V(x) \quad \text{and} \quad u=0 \ \ as\, y\to\infty.$$

Here we have $U(x)=U_0a^{x/L}$ is the stretching velocity, where U_0 is the reference velocity, $V(x)>0$ is suction velocity and we assume a special type of velocity at the wall as $V(x)=V_0a^{x/L}$, where V_0 is the initial strength of suction.

2.1 Method of Solution

Introducing the suitable dimensionless transformation as

$$\eta=\sqrt{\frac{U_0}{2L\nu}}a^{x/L}y,\quad u=U_0a^{\frac{x}{L}}f'(\eta) \quad \text{and} \quad v=-\sqrt{\frac{U_0\nu}{2L}}a^{\frac{x}{2L}}\log a\,\{f(\eta)+\eta f'(\eta)\}. \quad (4)$$

Substituting (4) in equation (2), the momentum equation transforms to

$$f'''-\log a\left(2f'^2-ff''\right)=0 \quad (5)$$

and the boundary conditions becomes:

$$f'(0) = 1, \qquad f(0) = S/\log a, \qquad f' \to 0 \ \text{ as } \ \eta \to \infty \tag{6}$$

where prime (') denotes the derivative with respect to the η and the suction is given by $S = \frac{V_0}{\sqrt{U_0 \nu / 2L}} > 0$.

3. HEAT TRANSFER PROBLEM

The energy equation with convective surface boundary condition is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{7}$$

with relevant boundary conditions

$$T(x,0) = T_w = T_\infty + T_0 a^{x/2L} \tag{8a}$$
$$T \to T_\infty \ \text{ as } \ y \to \infty \tag{8b}$$

where α is the thermal diffusivity of the fluid, T_w is temperature of the wall and T_∞ is the ambient fluid temperature, that is, the temperature of the fluid far away from the plate, T_0 is the reference temperature.

Referring Rosseland, S. [20] and Siegel R., Howell J. R. [22], the radiative heat flux may be considered as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{9a}$$

where σ^* and k^* are the Stefan-Bltzmann constant and the mean absorption coefficient, respectively. Here we use the approximation as it is being used by Battler [3], [4], Pal [18], Pal and Mondal [19], Mukhopadhyay and Layek [17], Ishak [12] and N. Ahmad and Ravins [1], as

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{9b}$$

Using (9a) and (9b) in equation (7), we get

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\alpha}{K_0}\frac{\partial^2 T}{\partial y^2}, \tag{10}$$

where $K_0 = \frac{3N}{3N+4}$ with $N = \frac{Kk^*}{4\sigma * T_\infty^3}$ is the radiation parameter.

Defining the dimensionless temperature as

$$\theta(\eta) = \frac{T(\eta) - T_\infty}{T_0 a^{\frac{x}{2L}}}, \tag{11}$$

which further implies that

$$T(\eta) = T_\infty + T_0 a^{\frac{x}{2L}} \theta(\eta) \quad \text{and} \quad \eta = \sqrt{\frac{U_0}{2L\nu}} a^{x/L} y. \tag{12}$$

Substituting the value of u, v and the similarity transformation (12) into equation (10), this equation reduces to

$$\theta'' = PrK_0 \log a \,(f'\theta - f\theta')\,, \tag{13}$$

where $Pr = \nu/\alpha$, is the Prandtl number and boundary conditions (8a) and (8b) take the following form:

$$\theta(0) = 0 \quad \text{and} \quad \theta \to 0 \;\text{ as }\; \eta \to \infty. \tag{14}$$

The equations (5) and (13) along with the boundary conditions (6) and (14), respectively, are solved by converting them to the initial value problem.

The boundary value problem (5) with (6) is non-linear two-point boundary value problem in an infinite domain. To solve numerically we apply the shooting method to convert the boundary value problem into initial value problem. By shooting method we get $f''(0) = -0.9811$ correct to 10^{-6}. Similarly, by shooting technique, we get $\theta'(0) = -4.2902$ correct to 10^{-6}. Finally, we apply Runge-Kutta method of order four using MATLAB R2009a, we get the different graphs and by reading these graphs, we reach to the conclusions.

4. DISCUSSION AND RESULTS

Generalization of boundary layer flow problem and heat transfer over an exponential stretching sheet to a general exponential stretching sheet were solved numerically by applying shooting technique and Runge-Kutta fourth order method.

The present flow and heat transfer problem becomes a three-parameter boundary value problem. Here we discuss three cases in a general exponential stretching sheet $a = e, a < e$, and $a > e$.

In this section, our concentration for the variation of parameters on the temperature only because of all these parameters is free from the velocity components in all three cases. We summarize the results in the following paragraphs:

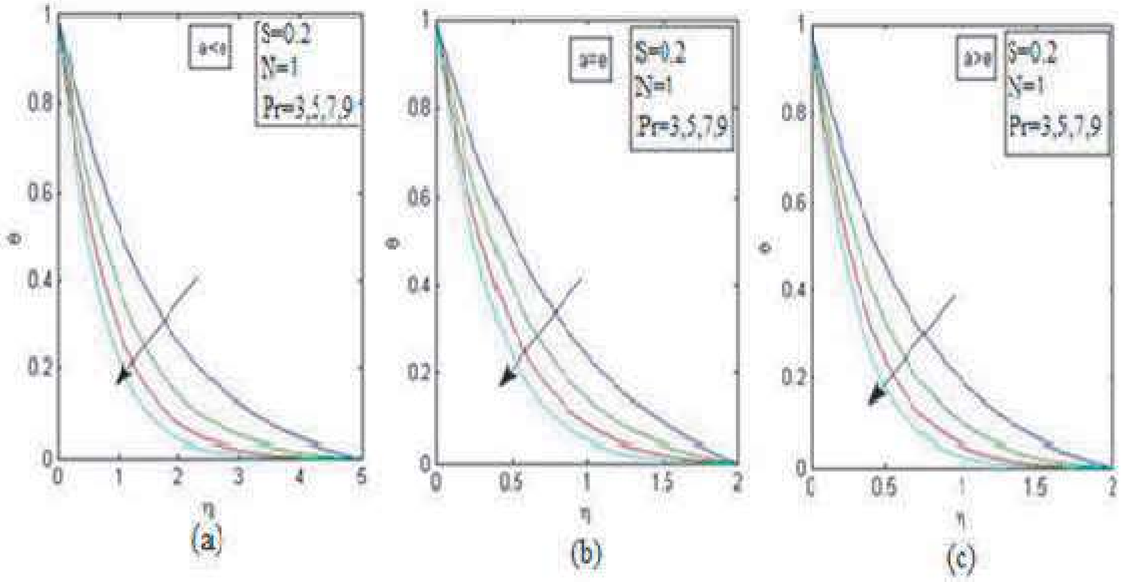

Figure 1. Temperature profile for different values of Prandtl numbers, keeping radiation parameter $N = 1$ and parameter $S = 0.2$ fixed in all three cases $a = e, a < e$, and $a > e$.

General exponential function a^x where $a > 0, a \neq 1$ and x is any real number. To maintain the stretching, we take $a > 1$, so that the physically boundary condition $u = U_0 a^{\frac{x}{L}}$ represents the stretching character of the plate. To see the effect of Prandtl number Pr on temperature field, we take suction parameter $S = 0.2$ and radiation parameter $N = 1$, that is, suction and radiation become fixed. We vary the Prandtl number, Pr, randomly. In all three cases $a = e, a < e$, and $a > e$, the temperature field decreases as Prandtl number increases. Since, Pr increases, if the thermal conductivity K decreases. Therefore, the temperature field decreases. This trend is agreed with the work done by Swati Mukhopadhyay [16].

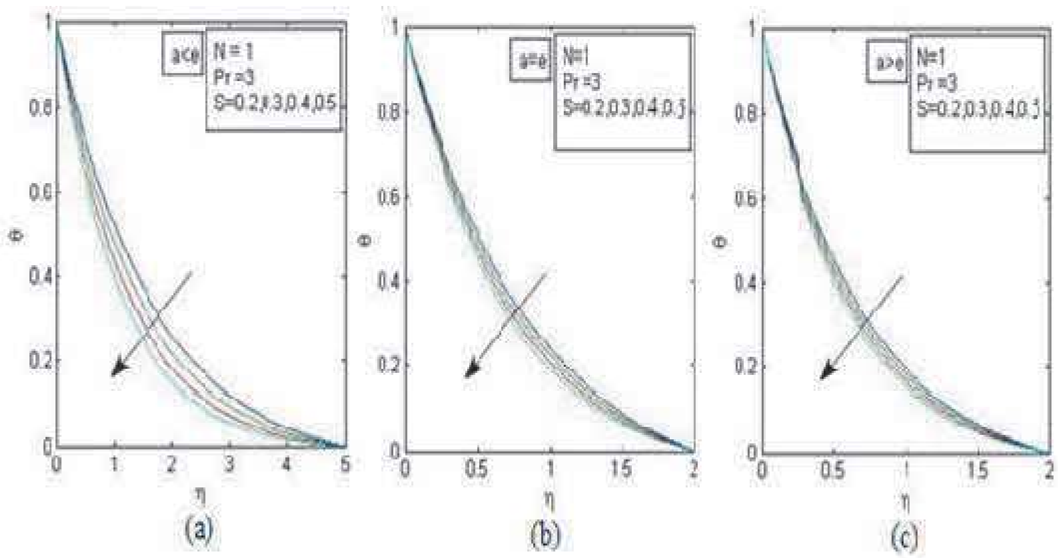

Figure 2. Temperature profile for different values of suction parameter $S = 0.2,0.3,0.4,0.5$, keeping radiation parameter $N = 1$ and Prandtl numbers $Pr = 3$ fixed in all three cases $a = e, a < e$, and $a > e$.

The effect of suction parameter S has been shown on temperature. As suction parameter S increases, the temperature field decreases in all the three cases (a), (b) and (c) of Figure 2. It is well known fact that as S increases, the temperature decreases because the suction has the cooling effect.

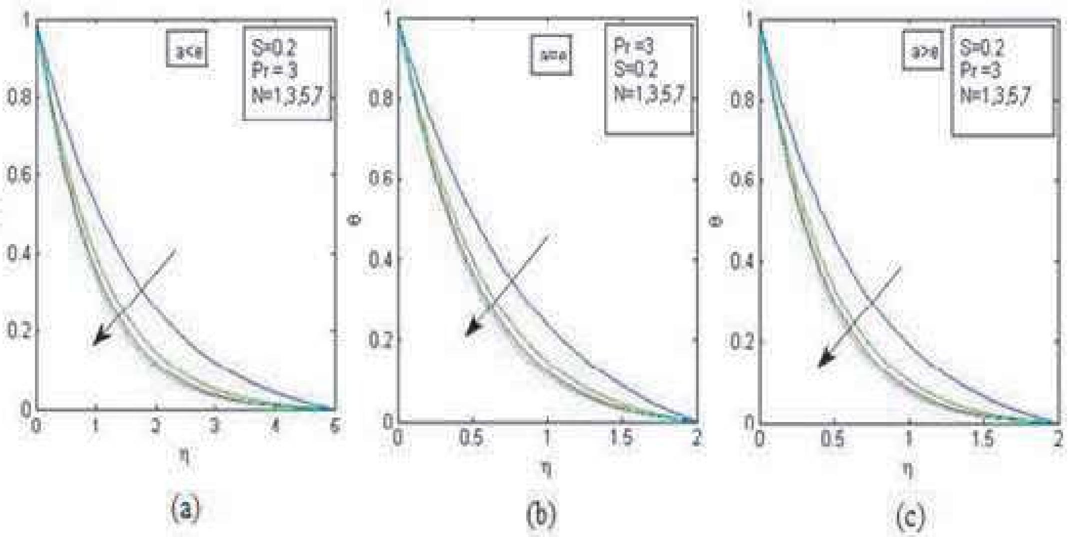

Figure 3. Temperature profile for different values of radiation parameter $N = 1,3,5,7$, keeping suction parameter $S = 0.2$ and Prandtl numbers $Pr = 3$ fixed in all three cases $= e, a < e$, and $a > e$.

From all the graphs given in Figure 3, we see that the temperature field is maximum when $N = 1$. Thus, the radiation of unit magnitude contributes well to increase the temperature field. As the magnitude of radiation increases, the process of heat transfer from fluid to atmosphere starts, hence temperature field decreases as radiation increases.

5. CONCLUSION

This paper is the generalization of the work done on boundary layer flow over an exponentially stretching sheet. The pattern of heat transfer is almost same as considered for the stretching governed by e^x.

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