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Research Article

ANALYTICAL MODEL FOR ESTIMATION OF TEMPERATURE DISTRIBUTION IN PARALLEL AND COUNTER FLOW DOUBLE PIPE HEAT EXCHANGERS

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ABSTRACT

Heat exchangers are a widely used as a device for meeting the heat transfer requirement in industrial applications. Many types of innovative heat exchanger designs have been implemented with the aim of increasing heat transfer. Numerous experimental and numerical studies are available in the literature on obtaining heat transfer and temperature distribution these systems. In the present study, an analytical model was developed to solve the nonlinear differential energy equation in order to estimate the temperature distribution in the parallel and counter flow double pipe heat exchanger. These analytical solutions have occurred of Bessel functions. A numerical study was carried out to determine the validity of the analytical results. Moreover, the analytical and the numerical results were compared with each other. Water is used as a fluid and the analyses were carried out for laminar and steady-state flow conditions at a certain Reynolds number (Re=1500). The findings showed that the analytical results for temperature distribution in the radial direction are in good agreement with the numerical results for all the flow conditions. However, the analytical model for the temperature distribution in the axial direction yielded more accurate results in the parallel flow conditions.
Keywords: Counter flow, double pipe heat exchanger, heat transfer, parallel flow.

1. INTRODUCTION

In cooling and heating systems of industrial application, heat exchangers are taken part of important role. There are many numerical, analytical and experimental studies in the literature about heat transfer and temperature distribution in the heat exchanger. For complex geometries, numerical methods are more preferred because exact solutions of analytical equations are complex and difficult. Besides that, numerical solutions have convergence, stiffness, numerical diffusion, and stability problems [1]. Isaza et al. examined analytical solutions of moving bed heat exchangers. The paper presented formulation and non-dimensional analyzation of steady state energy equation for co-current parallel plate systems. Temperature functions for the solids and fluid were formulated [2]. Quintero and Vera studied theoretically and numerically for multilayered, counter flow, parallel-plate heat exchangers. The exact solution for the temperature

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field provides analytical express ions for the interfacial and bulk temperatures, local heat-transfer rates, overall heat-transfer coefficient, Nusselt numbers, and outlet bulk temperatures of both fluids. Same parameters were evaluated with numerical solutions. The results of solutions were seen excellent agreement with studies in the literature [3]. Single phase liquid heated with constant temperature fluid was examined by Yin and Jensen. They calculated liquid temperature with numerical and analytical methods. The results of both solutions were seen that compatible with each other. [4]. Tunnel lining ground heat exchanger was used to solve the freezing damage problem of tunnel in a cold region by Zhang et al. The study presented as an analytical and experimental study for different heat exchanger parameter [5].

Double pipe heat exchanger which is one of the heat exchanger types, is commonly used in chemical, food, oil and gas industries due to their low cost of design and maintenance[6]. In the type of this exchanger cold and hot fluid flow either a counter or parallel direction. So, heat transfer between hot and cold water is occurred [7]. A few analytical study about double pipe heat exchanger were examined in the literature. Lachi et al investigated the behavior of a double pipe or shell and tube heat exchangers on transient phase when a sudden change of the flow rate imposed at the entrance of the two inlets. Analytical and experimental solution of this condition were compared. The results showed that experimental investigations were in a good agreement with theoretical results. [8] Nunge and Gıll developed orthogonal expansion technique for solving a new type of counter-flow in double pipe heat exchanger. The local Nusselt numbers and the temperature changing at the wall between the two streams were identified. Besides that bulk temperature changes in the two streams and mean overall Nusselt numbers were given [9]. Transient response of temperatures along a tubular counter flow heat exchanger was investigated when mass flow rate is subjected to sudden change by Abdelghani-Idrissi et al. [10]

Nomenclature

κR	Radius of inner pipe, mm	α	The thermal diffusivity of water, m^2s^{-1}
L	Length of pipe, mm	**Subscript**	
R	Radius of outer pipe, mm	ave	Average
r	Radius, mm	f	Film
Re	Reynolds number	i	Grid location at the direction of radial
T	Temperature value, ℃	in	Inner pipe
V	Velocity of water in the axial direction, m/s	j	Grid location at the direction of axial
	Greek Symbols	o	Outer pipe
β	Coefficient that includes the pipe dimensions	w	Wall
	and the thermal properties of the fluid	amb	ambient
ξ	The dimensionless axial distance	ins	Insulation
η	The dimensionless radial distance		
θ	Dimensionless temperature		

In this study, an analytical model was developed with the aim of estimating the temperature distribution in parallel and counter-flow double pipe heat exchangers. Analytical solutions of nonlinear differential equations have been performed using the separation method of variables. Numerical analysis was performed to verify analytical solutions and analytical and numerical results were compared.

2. PHYSICAL SKETCH AND MATHEMATICAL MODEL

The geometry used for examining the temperature distribution in a double pipe heat exchanger is given in Fig. 1. The temperature distribution is investigated for cases where the flow is parallel and counter in the inner and outer pipes. Analyses were carried out for the laminar and steady-state flow condition. Water was used as a fluid and it was assumed that the fluid was newtonian, incompressible and the convection properties of fluid were constant. The effect of gravitational acceleration was neglected for the reason that the flow occurred in a horizontal position.

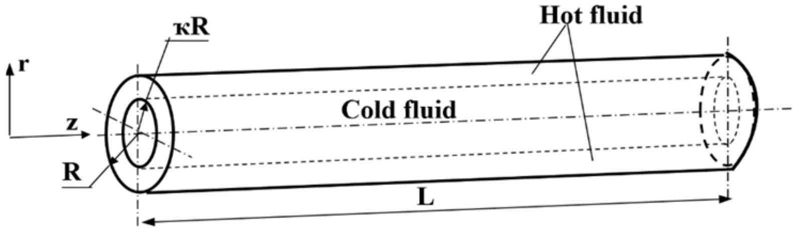

Figure 1. Pipe geometry

The energy equation used for temperature distribution was established by assuming that convection heat transfer was only in the axial direction, and that conduction heat transfer was only in the radial direction. Instead of the velocity change in the axial direction, the average axial velocity in the pipe was calculated and included in the energy equation. The energy equation for the above-mentioned assumptions is given below,

$$\frac{1}{\alpha} V_{z,ave} \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}, \tag{1}$$

where α is the thermal diffusivity of water, T is the water temperature and $V_{z,ave}$ is the average velocity of water in the axial direction. The dimensionless form of the energy equation is expressed by Eq. (2),

$$\beta \frac{\partial \theta(\eta,\xi)}{\partial \xi} = \frac{1}{\eta}\frac{\partial \theta(\eta,\xi)}{\partial \eta} + \frac{\partial^2 \theta(\eta,\xi)}{\partial^2 \eta} \tag{2}$$

In the Eq.2, θ is the dimensionless temperature, η and ξ are the dimensionless radial and axial distance, respectively. β is a coefficient that includes the pipe dimensions and the thermal properties of the fluid. Dimensionless expressions are given below,

$$\beta = \frac{(\kappa R)^2 V_{z,ave}}{\alpha L}; \quad \eta = \frac{r}{\kappa R}; \quad \theta(\eta,\xi) = \frac{T(\eta,\xi) - T_w}{T_i - T_w}; \quad \xi = \frac{z}{L} \tag{3}$$

In the equations, L is the pipe length, κR is the radius of the inner pipe, T_i is the inlet temperature of water and T_w is the wall temperature of inner pipe.

The boundary conditions established according to the reference coordinate axes by considering the pipe outer surface as adiabatic and centerline of inner pipe is symmetry axis are given below,

$$\left.\frac{\partial\theta(\eta,\xi)}{\partial\eta}\right|_{\eta=0}=0 \tag{4a}$$

$$\theta(\eta,0)=1,\ \ 0\le\eta\le 1 \tag{4b}$$

$$\theta(1,\xi)=0,\ \ 0\le\xi\le 1 \tag{4c}$$

3. ANALYTICAL SOLUTION

We apply the variables separation technique for Eq. (2) and (4). Let $\theta = X(\eta)Y(\xi)$ in Eq. (2). Then

$$\beta XY' = \frac{1}{\eta}X'Y + X''Y \tag{5}$$

or dividing by XY,

$$\beta\frac{Y'}{Y} = \frac{1}{\eta}\frac{X'}{X} + \frac{X''}{X} = -\lambda^2 \tag{6}$$

from which

$$X'' + \frac{1}{\eta}X' + \lambda^2 X = 0,\ \ \beta Y' + \lambda^2 Y = 0 \tag{7}$$

General solutions using boundary conditions in Eq. (4)

$$X(\eta) = A_1 J_0(\lambda_n\eta) + B_1 Y_0(\lambda_n\eta),\ \ Y(\xi) = C_1 e^{-\frac{\lambda_n^2}{\beta}\xi} \tag{8}$$

Since $\theta = X(\eta)Y(\xi)$ is bounded at $\eta = 0$, then $B_1 = 0$. Therefore, we take

$$\theta(\eta,\xi) = CJ_0(\lambda_n\eta)e^{-\frac{\lambda_n^2}{\beta}\xi}, \tag{9}$$

where $C = A_1C_1$.

From the second boundary condition (4b)

$$\theta(1,\xi) = Ce^{-\frac{\lambda_n^2}{\beta}\xi}J_0(\lambda_n) = 0, \tag{10}$$

from which $J_0(\lambda_n) = 0$ and $\lambda_n = \lambda_1, \lambda_2, ...$ are the positive roots.

Thus a solution is

$$\theta(\eta,\xi) = Ce^{-\frac{\lambda_n^2}{\beta}\xi}J_0(\lambda_n\eta),\ \ n = 1,2,3,.... . \tag{11}$$

By superposition, a solution is

$$\theta(\eta,\xi)=\sum_{n=1}^{\infty}C_nJ_0(\lambda_n\eta)e^{-\frac{\lambda_n^2}{\beta}\xi} \tag{12}$$

To determine the unknown C_n, we constrain the above solution to satisfy the condition $(4c)$ as:

$$\Theta(\eta,0)=\sum_{n=1}^{\infty}C_nJ_0(\lambda_n\eta)=1\,. \tag{13}$$

Using the Fourier series, we obtain

$$C_n=\frac{2}{J_1^2(\lambda_n)}\int_0^1\eta J_0(\lambda_n\eta)d\eta\,. \tag{14}$$

Therefore, the temperature distribution for parallel and counter-flow in the inner and the outher pipe can be written as

$$\theta(\eta,\xi)=\sum_{n=1}^{\infty}\left\{\left[\frac{2}{J_1^2(\lambda_n)}\int_0^1\eta J_0(\lambda_n\eta)d\eta\right]J_0(\lambda_n\eta)e^{-\frac{\lambda_n^2}{\beta}\xi}\right\}. \tag{15}$$

Since

$$\int_0^1\eta J_0(\lambda_n\eta)d\eta=\frac{1}{\lambda_n}J_1(\lambda_n), \tag{16}$$

we obtain

$$\theta(\eta,\xi)=\sum_{n=1}^{\infty}\left\{\frac{2}{\lambda_nJ_1(\lambda_n)}J_0(\lambda_n\eta)e^{-\frac{\lambda_n^2}{\beta}\xi}\right\}. \tag{17}$$

Thus, as a solution of the Eq.2 , Eq.17 was obtained.

4. NUMERICAL SOLUTION

4.1. Grid structure

The flow in the pipes is axisymmetric for both parallel and counter flow conditions. For this reason, half of the double pipe geometry was modeled for the numerical analysis. The structured grid was used in the calculation area and the grid structure was intensified in areas near the pipe walls. The geometry, mesh structure and reference coordinate axes for parallel and counter flow conditions are shown in Fig. 2.

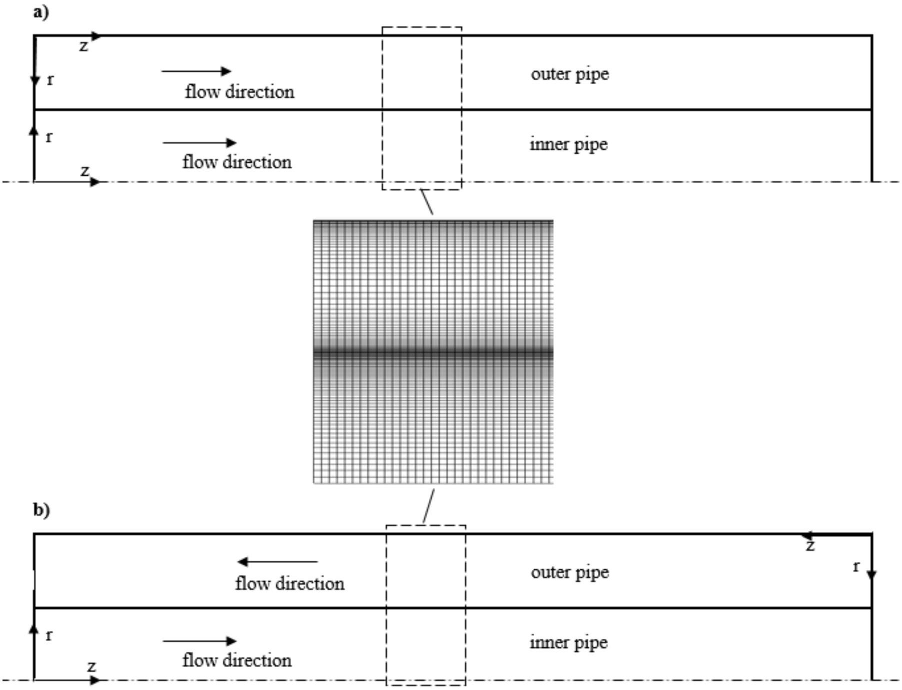

Figure 2. Geometry of pipes and reference coordinate axes for inner and outer pipe a) parallel flow b) counter flow

4.2. Solution method

In the numerical solution of the problem in which the mathematical model is formed, the finite volume method is used. Numerical analyses were performed using the ANSYS-Fluent program. The SIMPLE algorithm was used to solve the pressure-velocity coupling in discretized conservation equations and Second Order Upwind scheme was used to find the intermediate point values in the grid structure created for the numerical analysis. The convergence criterion for the mass, momentum and energy conservation equations is expressed by the equation given below.

$$\left|\frac{\delta^{n+1}(i,j)-\delta^{n}(i,j)}{\delta^{n+1}(i,j)}\right|<10^{-6}, \tag{18}$$

where n and $n+1$ represent two consecutive iterations and δ is any variable, and i and j are the grid locations at the direction of radial and axial, respectively.

4.3. Verification of numerical results

For the fully developed laminar flow condition, the following analytical equation is used in obtaining the velocity profile [11],

$$V(r)=2V_{ave}\left(1-\frac{r^{2}}{R^{2}}\right), \tag{19}$$

where V_{ave} is the average velocity in the pipe and R is the outer radius of pipe. Value of $r = 0$ is the point on the centerline of the pipe.

The comparison of the numerical results for different grid sizes with the velocity values obtained using Eq. 19 is shown in Fig. 3. As seen in Fig. 3, the analytical results are compared for four different grid sizes in the radial and axial direction. In areas near the pipe wall, the velocity values remain about the same for all grid sizes. However, the velocity values along the pipe axis are influenced by the variation of grid size. After the value of 1x2mm grid size, the results are not affected by variation of grid size. Thus, 0.5x1mm grid size is preferred with the aim of reducing the analysis time. The highest error rate between analytical and numerical model is 0.5% for 0.5x1mm grid size. This indicates that the numerical model is in agreement with the analytical model.

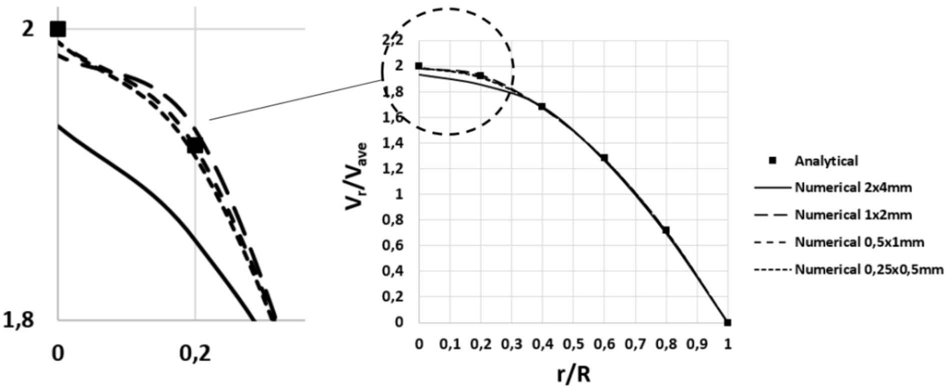

Figure 3. Grid independent test and validation of numerical results

5. RESULTS AND DISCUSSION

Analytical and numerical analyses were carried out for parallel and counter flow conditions where the cold fluid was in the inner pipe and the hot fluid was in the outer pipe. The thermophysical properties of the fluid were determined based on the film temperature (T_f) . The values of pipe measurements and fluid properties used in the analyses are given in Table 1.

Table 1. Parameter values

$T_{inlet, i}$ (°C)	$T_{inlet, o}$ (°C)	T_f (°C)	α (m^2/s)	L (mm)	κR (mm)	R (mm)	Re
20	80	50	$1{,}56 \times 10^{-7}$	1000	12,5	25	1500

Fig. 4 includes the axial temperature change for different points in the radial direction under parallel flow conditions in the inner and outer pipe. As shown in Fig. 4a, the analytical and numerical results are closer to each other due to the reduced effect of the velocity boundary layer towards the inner tube axis (η=0,4). Also, the close proximity of the temperature values can be explained by the fact that the average velocity value in the axial direction is close to the velocity value and the temperature distribution is uniform at the measuring points for η=0,4 (Fig. 6a). In areas near the inner pipe wall (η=0,6-0,9), especially near the pipe inlet, deviations between the numerical and analytical results have occurred. Since the velocity boundary layer is not fully developed in the regions close to the pipe inlet, the velocity values in the axial direction are more variable and the validity of the assumption of the mean axial velocity decreases (Fig.6a). Thus, deviations between temperature values occur. On the other hand, in the energy equation, the assumption that convection and conduction heat transfer only occurs in the axial and radial direction leads to the difference between the analytical and numerical results. Evaluations of the temperature distribution at the different axial locations for the inner pipe also apply to the outer pipe (Fig.4b). The temperature distribution is uniform because the fluid temperature at the points near the pipe outer wall is less influenced by the cold flow. In the regions close to the inner pipe wall, however, heat transfer is more effective and velocity boundary layer is emerged. As in the case of the inner pipe, fully developed flow conditions are approached in the regions close to the pipe outlet. Differences arise between the analytical and numerical results, depending on the validity of the mean velocity assumption in the axial direction. Despite the deviations between the results, the highest error rate is 11% for the inner and outer pipe. This result indicates that the analytical model for estimating the axial temperature distribution in parallel flow conditions is appropriate. For the temperature distribution in the radial direction (Fig. 4c and 4d), both the temperature distributions of the inner and outer pipes are similar. Analytical and parametric results are better aligned in regions close to the pipe exit. This situation shows that the use of the analytical model for temperature estimation is more appropriate in regions approaching fully developed flow conditions. In the pipe inlet area, the differences between the results are increasing (Fig.6a). The highest difference between analytical and numerical results is 9% at Ƹ=0,1.

In Fig. 5, the axial and radial temperature distributions for the counter-flow condition are given. The analytical and numerical results of axial temperature values for the inner and outer pipe for η=0.4 are in agreement (Figs.5a and 5b). The uniformity of the temperature distribution near the pipe outer wall and its axis ensured this result (Fig.6b). In Fig. 5a, the difference between the analytical and numerical results begins to increase in the regions of the pipe exit for the inner pipe. As seen from the temperature contour in the counter-flow condition (Fig.6b), the high temperature of the fluid in both pipes in the outlet region of the inner pipe causes a sudden increase in the inner pipe wall temperature. This causes the numerical results to differ from the results obtained by the exponential curve in the analytical solution. The same situation for the axial temperature distribution is more pronounced in the outlet regions of outer pipe (Fig. 5b). For the counter-flow condition, the outlet region of the outer pipe and the inlet region of the inner pipe are in the same position. At this position, both fluids are at low temperature and this leads to a further decrease in the internal pipe wall temperature, resulting in an increase in the difference between analytical and numerical results. When this difference is considered in terms of the velocity profile of the flow, it is seen that the velocity profiles at a certain point are different since the inlet and outlet regions of the inner and outer pipes are opposite to each other. Thus, the difference between the temperature gradients in the inner and outer pipes increases depending on the velocity profile between the viscous and inviscid regions. The analytical and numerical results of the temperature distributions in the radial direction for the inner and outer pipes coincide with each other as they are in the parallel flow condition (Fig.5c and 5d). Analytical and numerical results are in agreement in the other regions while the difference between the results in the regions close to the pipe outlet increases. When the inner and outer pipes are considered together, the highest difference between the temperature values is about 7%.

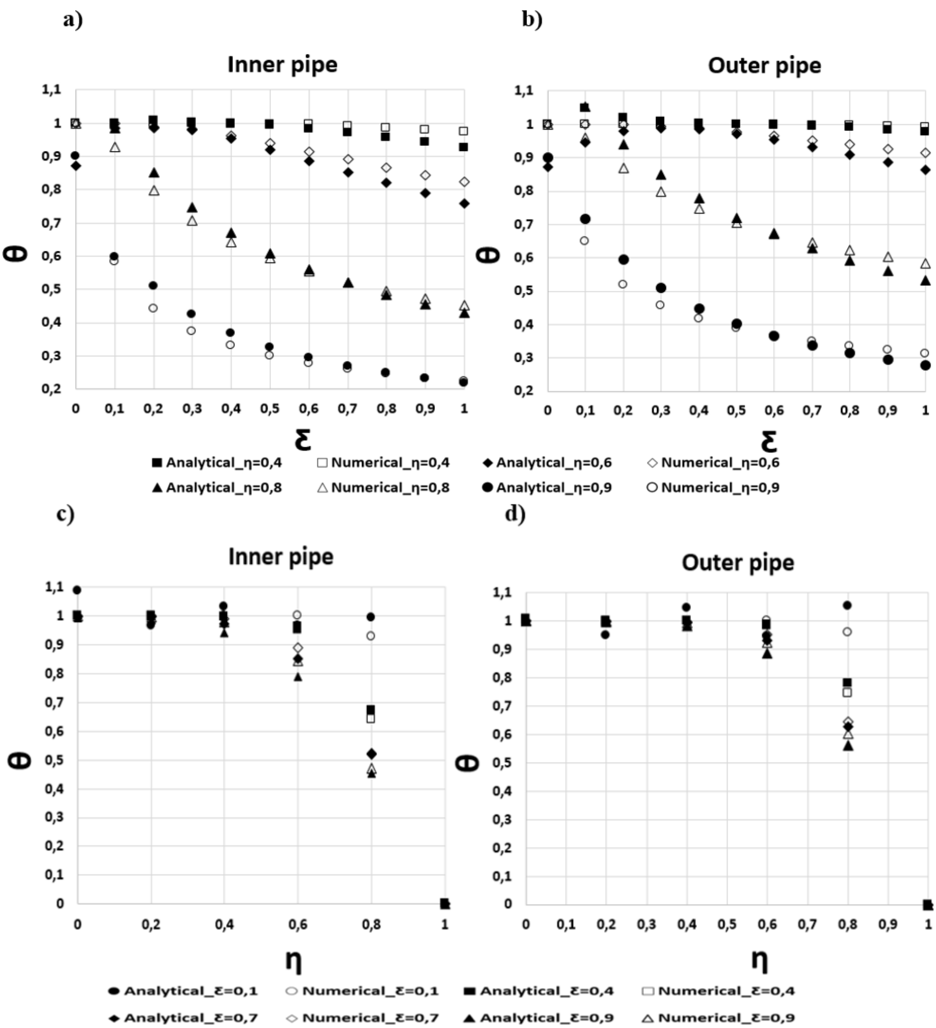

Figure 4. Variation of temperature for parallel flow a) Axial direction for inner pipe, b) Axial direction for outer pipe, c) Radial direction for inner pipe, and d) Radial direction for outer pipe

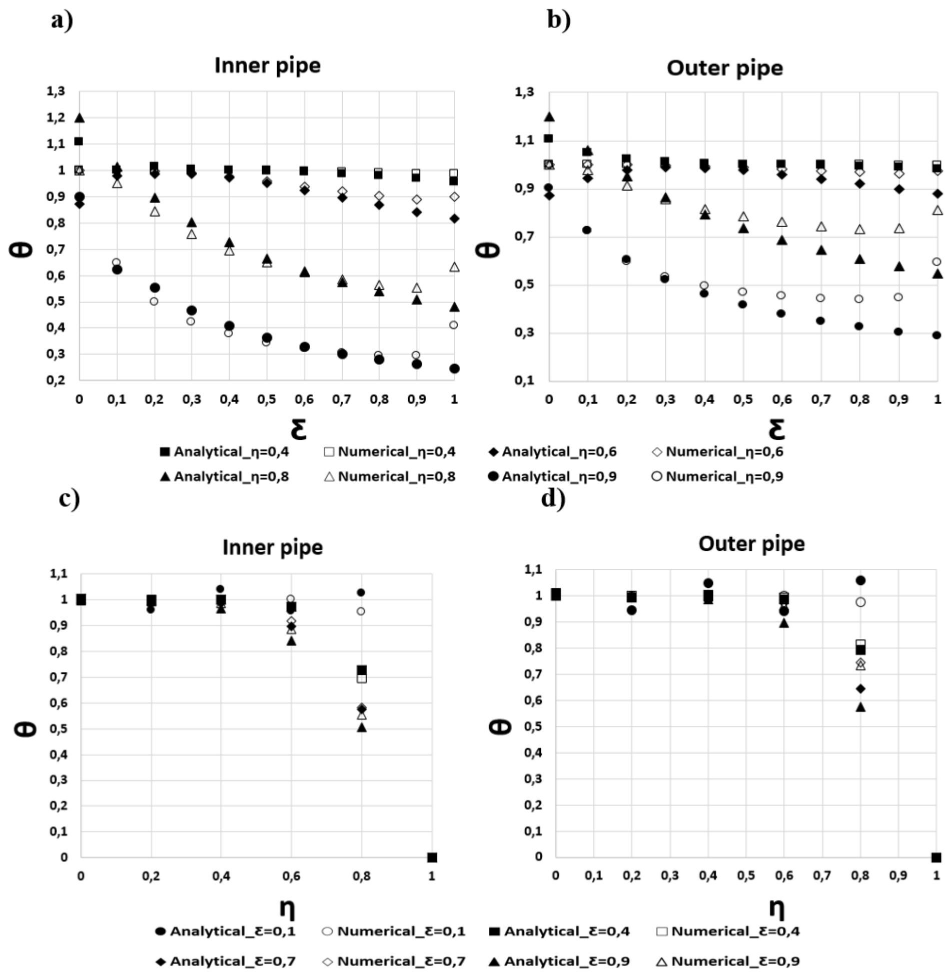

Figure 5. Variation of temperature for counter flow a) Axial direction for inner pipe, b) Axial direction for outer pipe, c) Radial direction for inner pipe, and d) Radial direction for outer pipe

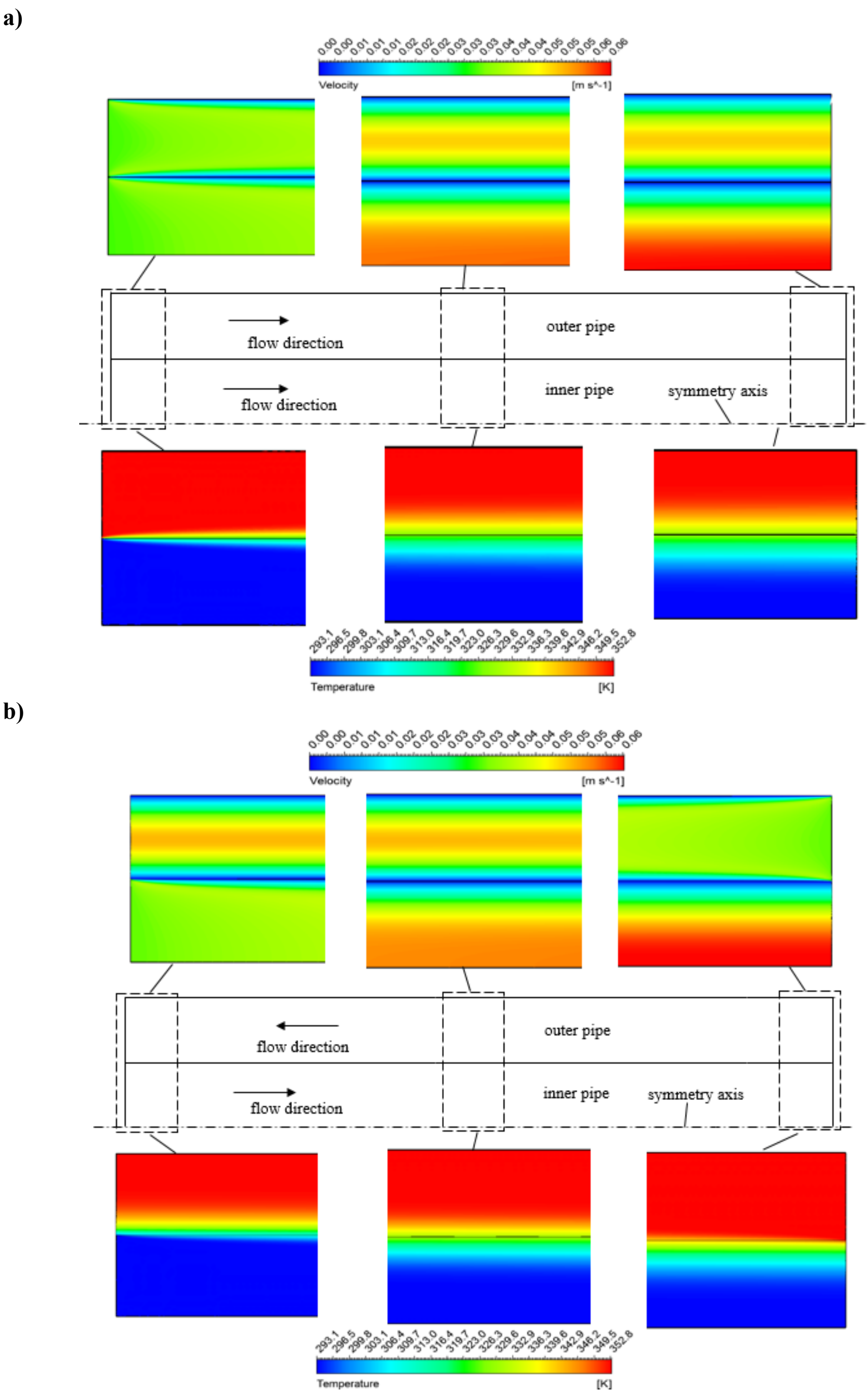

Figure 6. Temperature and velocity contours a) parallel flow b) counter flow

6. CONCLUSIONS

In the present study, an analytical model was developed to estimate the temperature distribution in a parallel and counter flow double-tube heat exchanger and the analytical results were compared with the numerical analysis results. The results for the axial and radial temperature distribution in laminar and steady-state flow conditions are given below.

The use of the average axial velocity assumption significantly affects the agreement between analytical and numerical results for parallel and counter flow conditions. The velocity boundary layer is not fully developed in the regions near to the pipe inlet. This causes an increase in the difference between the axial velocity value and the average velocity value in the inlet zone. Thus, the compatibility between analytical and numerical results is diminishing. In the regions close to the pipe axis and pipe outer surface, the uniform fluid velocity and temperature improve the compatibility of the analytical and numerical model. The maximum difference between the analytical and numerical results for the temperature distribution in the axial direction is 11%. Except for the factors mentioned above for the counter-flow condition, the analytical and numerical solutions differ for the temperature distribution in the axial direction due to the variation of the temperature gradients in the inner and outer pipes at different ratios. In particular, the sudden change of the inner pipe wall temperature in the outlet regions of the inner and outer pipes significantly reduce the adaptation of analytical and numerical results.

The values calculated by the analytical and numerical method for the temperature distribution in the radial direction comply with each other for both parallel and counter flow conditions. Results obtained from the analyses shows that the use of the analytical model for temperature estimation is more appropriate in the regions approaching fully developed flow conditions. The highest difference between the temperature values is about 9% and 7% for parallel and counter flow condition, respectively.

The results obtained for all conditions indicate that the analytical model is more suitable for parallel flow double pipe heat exchangers. Analyzing by including the velocity in the axial direction as a variable in the energy equation can provide the analytical and numerical results more compatible for parallel and counter flow conditions.

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