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Quantum Reservoir Parameter Estimation via Fisher Information

Ufuk KORKMAZ*[1], Deniz TÜRKPENÇE[1]

Abstract

In this study, we show that as a result of weak interaction of different information environments structured with a single probe qubit, these environments can perform binary classification of the information they contain. In this way, we refer to these environments as quantum information baths because they consist of sequences of identical qubits in certain pure quantum states. A micro-maser like master equation has been developed to clearly describe the system dynamics analytically and the quantum states of different information reservoirs. The model can also be treated as a quantum neuron, due to the single-qubit probe that makes a binary decision depending on the reservoir parameters in its steady state. The numerical results of the repeated interaction process based on the divisibility and additivity of the quantum dynamic maps are compared with the analytical results. Besides being a single quantum classifier, the model we present can also serve as a basic unit of a quantum neural network within the framework of the dissipative model of quantum computing.

Keywords: Binary classification, information reservoir, collision model, quantum Fisher information

1. INTRODUCTION

Classification of data plays an important role in extremely important applications such as machine learning, medical diagnostics and pattern recognition. In recent studies, machine learning algorithms made within the scope of quantum framework present the advantages of quantum computing [1-7]. The standard circuit model of quantum computation relies on quantum registers in pure states. That is, the quantum register is a closed quantum system.

However, we propose a quantum processing task based on open quantum dynamics. Recent reports show that quantum baths can be evaluated as communication channels through which information is transmitted [8, 9]. These studies encourage us to examine the existence of quantum classifiers where their dynamics are not unitary.

In our study, we consider a two-level quantum system (probe qubit) weakly interacting with different quantum reservoirs carrying information content. We refer to these reservoirs information reservoirs as we assume they are identical qubit strings in pure states with specific information parameters. [10, 11]. The probe qubit is subject a dissipation process in the presence of information reservoirs [12], such as quantum reservoir engineering, where the steady state is a non-trivial

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quantum state. We show that the steady state of the probe qubit acts as a quantum binary classifier as a result of interacting with various information environments. Using the mixing properties of quantum dynamic maps, we show that under some conditions the mixture of quantum dynamical maps can be natural data classifiers. Though it seems useless to use mixed states for quantum computation processing, it is shown that the dissipative model of quantum computing provides an alternative route for quantum computation. [14, 15].

For quantum thermodynamics or materials sciences, it is important to take advantage of quantum sources that exhibit non-linear response with respect to linear variation of dynamical parameters [16-18]. We construct our model by considering a single spin weakly connected to information reservoirs where reservoir induced non-linearity could be encountered. We then trace out the environmental states to calculate the reduced dynamics. Single spin magnetization as a steady-state response of reduced dynamics is an indication of merit. In the model, the input data represents information reservoirs connected to a single spin. The physics we use in this model is based on the complete positivity, additivity [19] and divisibility [20, 21] of quantum dynamical maps. As has been done in previous studies [7], [22], superconducting circuits can be shown as a physical model to apply the theoretical model with possible defects. We leave the activation and training tasks out of the scope of this study for our proposed classifier.

2. FRAMEWORK AND MODEL DYNAMICS

Perceptron is the simplest developed mathematical model for binary data classification. More precisely, the perceptron predicts binary decision output corresponding to a weighted sum of data instances as input. In general, the output is modulated by a rule, so-called activation function. For instance, a step function $f(y)$ where $y=\sum_i w_i x_i$ with x_i data instances and w_i corresponding weights [23] can be defined as output. By the behaviour of the step function, the output node returns $f(y)=+1$ for $y\geq 0$ and returns $f(y)=0$ else. Activation functions can be selected from either linear or non-linear functions. However, nonlinear functions are attractive in multilayer neural network applications as they support back propagation. There have been studies based on the advantages of quantum computing using quantum perceptrons or neural network models [2, 3, 6, 24].

Our model is an open quantum system and we take advantage of dissipative processes for data classification. Such a system ρ is defined by

$$\frac{\partial\rho}{\partial t}=P_1\mathcal{L}_t^{(1)}+\cdots+P_N\mathcal{L}_t^{(N)} \tag{1}$$

a weighted combination of generators representing the non-unitary evolutions induced by N distinct reservoirs. Here, the generators $\mathcal{L}_t^{(i)}$ are, in general, time-dependent and P_i are non-negative coefficients representing the probabilities of the probe qubit experiencing from the ith information reservoir. Note that, Eq. (1) represents the quantum equivalent of a classical perceptron model defined and represents a physical system as long as the weak coupling condition is satisfied, which ensures the additivity of the generators [19, 21, 25].

To this end, each generator will be represented by a completely positive trace-preserving (CPTP) quantum dynamical map

$$\Phi_t^{(i)}=Tr_{\mathcal{R}_i}\left[U_t\left(\rho_0\otimes\rho_{\mathcal{R}_i}\right)U_t^{\dagger}\right] \tag{2}$$

where U_t is a unitary propagator acting both on the system and the environment and $\mathcal{R}_i$ represents the quantum state of the ith environment. In addition, if a dynamical map satisfying $\Phi_{t+s}=\Phi_t(\Phi_s[\rho])$ is completely positive (CP) for t and $s\geq 0$, this is referred to as a CP divisible map. It has been shown that CP divisibility ensures the additivity of quantum dynamical maps in the weak coupling condition where cross-correlations between distinct reservoirs are avoided [21, 26].

Therefore, we represent the open quantum dynamics of our model by using a weighted combination of quantum dynamical maps satisfying CP divisibility as

$$\Phi_t[\rho_0] = \sum_i P_i\, \Phi_t^{(i)}. \tag{3}$$

Note that, Eq. (3) can only be cast in place of Eq. (1) only if all the dynamical maps $\Phi_t^{(i)}$ in the summation is CP divisible.

Single qubit quantum information is parametrized by polar and azimuthal angles as $|\Psi(\theta,\emptyset)\rangle = \cos\frac{\theta}{2}|e\rangle + e^{i\emptyset}\sin\frac{\theta}{2}|g\rangle$ in the well-known Bloch sphere representation, here $|e\rangle \equiv |0\rangle$ and $|g\rangle \equiv |1\rangle$ stand for excited and ground states, respectively. Throughout of our study we take $\emptyset = 0$ fixed and parametrize 'quantum features' by θ. In our model, we treat the classifier as a schema that carries informational content to different reservoirs where the probe qubit is weakly coupled. We model the open quantum dynamics using a repeated interaction process [27]. The ancilla units in these models are identical. Ancilla units do not interact with each other and the dynamics of the repeated interactions are unitary with very small interaction time τ. Thanks to the standard formulation of the collision models, memoryless open quantum dynamics equivalent to the master equations is obtained. At the beginning, the ancillas $\mathcal{R}_i$, which are prepared identically, sequentially collide with system S in equal duration τ. The system plus reservoir $S\mathcal{R}$ state is initially assumed to be in a product state $\rho(0) = \rho_S(0) \otimes_i^N \rho_{\mathcal{R}_i}$ where $\rho(0) = |+\rangle\langle+|$ and $\rho_{\mathcal{R}} = |\Psi_\theta\rangle\langle\Psi_\theta|$. By choosing the initial system states as $|+\rangle = \frac{|e\rangle+|g\rangle}{\sqrt{2}}$, we initially provide a null magnetization. These identical qubit states $\mathcal{R}_i$ read as

$$\rho_{\mathcal{R}_i} = \otimes_{k=1}^{N} \rho_{i_k}(\theta,\emptyset). \tag{4}$$

In order to represent the additivity of quantum dynamical maps in terms of quantum collisional model, each dynamical map $\Phi_t^{(i)}$ in Eq. (3) can be rephrased as

$$\Phi_{n\tau}^{(i)}[\rho_0] = Tr_n\left[U_{0i_n} \cdots Tr_1\left[U_{0i_1}\left(\rho_0 \otimes \rho_{\mathcal{R}_{i_1}}\right)U_{0i_1}^{\dagger}\right] \otimes \cdots \rho_{\mathcal{R}_{i_n}} U_{0i_n}^{\dagger}\right] \tag{5}$$

by using the reduced dynamics where $n\tau$ is the time elapsed for n collisions. When the scheme above was repeated a sufficient number of times, the system state becomes identical to that of the reservoir state. Thus, the system reaches steady state where the process is referred to as 'quantum homogenization' [28]. From now on, we drop the index k for convenience.

We use the standard collision model where the bath of N qubits do not interact each other, so the open system evolution is Markovian analougus to the central spin model [29]. As shown in Figure 1, a cluster of Nqubits interacts randomly with the probe qubit. The total Hamiltonian reads $H = H_0 + H_{\mathcal{R}_i} + H_{int}$, where

$$H_0 + H_{\mathcal{R}_i} = \frac{\hbar w_0}{2}\sigma_0^z + \frac{\hbar w_b}{2}\sum_{i=1}^{N}\sigma_i^z. \tag{6}$$

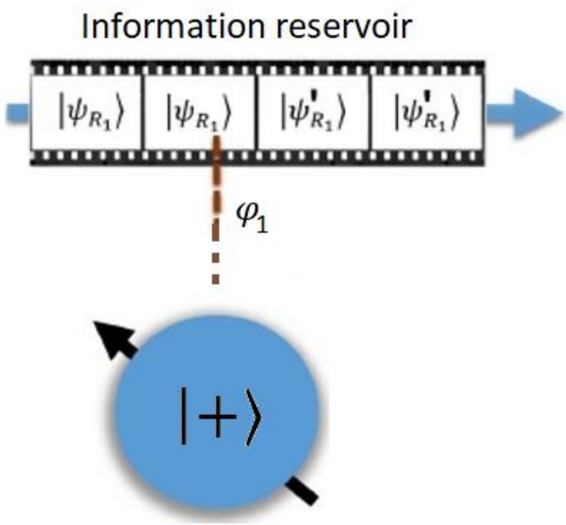

Figure 1 Statistically identical prepared cluster qubits interacts randomly with the probe qubit

Eq. (6) is the free Hamiltonian of the probe qubit and the N reservoir qubits, respectively. $\hbar$ is the reduced Planck constant. $\sigma_{0,i}^z = |e_{0,i}\rangle\langle e_{0,i}| - |g_{0,i}\rangle\langle g_{0,i}|$ are the Pauli z matrices for the probe and ith qubit, respectively. For simplicity, we assume that the system reservoir resonance is $w_0 = w_b$. We define interaction Hamiltonian as

$$H_{int} = \hbar\sum_{i=1}^{N}\varphi_i(\sigma_i^+\sigma_0^- + H.c.), \tag{7}$$

where φ_i is the coupling constant between the probe qubit and the ith reservoir qubits, $\sigma_i^+ = |e_i\rangle\langle g_i|$, $\sigma_i^- = |g_i\rangle\langle e_i|$ are the individual Pauli -

raising and -lowering operators. $\sigma_0^{\pm}$ are the Pauli matrices act on the probe qubit. The coupling strength is directly related to the $\varphi_i \propto P_i$ probability expressing the ith reservoir encounters. We derive a micromaser-like master equation based on repeated, random interactions. In this way, we establish the connection between the dynamic model of the study and real physical systems [17, 29-31]. In this article, open system evolution is Markovian. Because we used the standard collision model where the ancillas do not interact with each other. $U(\tau) = exp[-iH_{int}\tau/\hbar]$ are unitary propagators and describe the collisions between the system qubit and each ancilla.

$$U(\tau) = \mathbb{1} - i\tau\left(\sigma_0^+ J_{\varphi_i}^- + \sigma_0^- J_{\varphi_i}^+\right) - \frac{\tau^2}{2}\left(\sigma_0^+\sigma_0^- J_{\varphi_i}^- J_{\varphi_i}^+ + \sigma_0^-\sigma_0^+ J_{\varphi_i}^+ J_{\varphi_i}^-\right) \quad (8)$$

up to second order in τ. Here, $J_{\varphi_i}^{\pm} = \sum_{i=1}^{N} \varphi_i \sigma_i^{\pm}$ are the collective operators weighted by φ_i. The initial system is assumed to be factored $\rho(t) = \rho_0(t) \otimes \rho_{\mathcal{R}_i}$, assuming the reservoir states are reset to their initial state after each interaction. In the light of micro-maser theory, we explain random interactions with a Poisson process. The dynamics of the system is

$$\rho(t+\delta t) = r\delta t U(\tau)\rho(t)U^{\dagger}(\tau) + (1 - r\delta t)\rho(t) \quad (9)$$

in a time interval δt, where where $r\delta t$ is the probability of an interaction event at a rate r and $(1 - r\delta t)$ is the probability of occurring a non-interaction state. For the reduced dynamics of the probe qubit, the master equation is obtained as

$$\dot{\rho}_0(t) = Tr_{\mathcal{R}_i}[rU(\tau)\rho(t)U^{\dagger}(\tau) - r\rho(t)] \quad (10)$$

in the time limit $\delta t \to 0$ for $\dot{\rho}_0(t) = [\rho_0(t+\delta t) - \rho_0(t)]/\delta t$. After some adjustments, the master equation obtained for our model is

$$\dot{\rho}_0 = -i\left[H_{eff}, \rho\right] + \sum_{i=1}^{N} \varphi_i^2 (\zeta^+ \mathcal{L}[\sigma_0^+] + \zeta^- \mathcal{L}[\sigma_0^-]) + \sum_{i<j}^{N'} \varphi_i \varphi_j (\zeta_s^+ \mathcal{L}_s[\sigma_0^-] + \zeta_s^- \mathcal{L}_s[\sigma_0^+]) \quad (11)$$

where $H_{eff} = r\tau \sum_i^N \varphi_i (\langle\sigma_i^-\rangle \sigma_0^+ + \langle\sigma_i^+\rangle \sigma_0^-)$ denotes the effective Hamiltonian describing a coherent drive on the probe qubit. $\langle \mathcal{O}_i \rangle = Tr\left[\mathcal{O}\rho_{\mathcal{R}_i}\right]$ are averages calculated over identical reservoir units. The superoperators here, respectively, $\mathcal{L}[o] \equiv 2o\rho o^{\dagger} - o^{\dagger}\rho o - \rho o o^{\dagger}$ is the standard Lindblad term and $\mathcal{L}_s[o] \equiv 2o\rho o$ describes a squeezing effect by the reservoir. Since the standard Linbladian coefficients $\zeta^{\pm} = \frac{r\tau^2}{2}\langle \sigma_i^{\pm}\sigma_i^{\mp}\rangle$ contain diagonal inputs and $\zeta_s^{\pm} = 2r\tau^2 \langle\sigma_i^{\pm}\rangle\langle\sigma_j^{\pm}\rangle$ contain off-diagonal inputs, the Linbladian coefficients carry information corresponding to the different inputs of the density matrices of the reservoir units. $N' = N(N-1)/2$ are terms in the summation.

Let us express the density matrix of the probe qubit as $\rho(t) = p_e(t)|e\rangle\langle e| + p_g(t)|g\rangle\langle g| + (c(t)|e\rangle\langle g| + H.c.)$ in the standard basis. When we substitute these expressions in Eq. (11) as $\dot{\rho}_0$=0 and $c = c^* = 0$, the steady state equation is obtained as follows

$$\rho_0^{ss} = \frac{1}{\sum_i^N \varphi_i^2} \sum_{i=1}^{N} \varphi_i^2 (\langle \sigma_i^+ \sigma_i^- \rangle |e\rangle\langle e| + \langle \sigma_i^- \sigma_i^+ \rangle |g\rangle\langle g| + [i\gamma_1^- (\langle \sigma_i^+ \sigma_i^- \rangle - \langle \sigma_i^- \sigma_i^+ \rangle)|e\rangle\langle g| + H.c.]) \quad (12)$$

where $\gamma_1^- = r\tau \sum_i^N \varphi_i \langle \sigma_i^- \rangle$. Eq. (12) reduces to

$$\rho_0^{ss} = (\langle \sigma_1^+ \sigma_1^- \rangle |e\rangle\langle e| + \langle \sigma_1^- \sigma_1^+ \rangle |g\rangle\langle g| + [i\gamma_1^- (\langle \sigma_1^+ \sigma_1^- \rangle - \langle \sigma_1^- \sigma_1^+ \rangle)|e\rangle\langle g| + H.c.]) \quad (13)$$

in the presence of a single (only $\varphi_1 \neq 0$) information reservoir. As clear above, the steady state density matrix involves the same diagonal terms as that of the reservoir density matrices of the reservoir units.

This result is typical for the steady state response of small quantum systems for dissipative interactions. However, the off-diagonal elements above depend on γ_1^-, which is a function of r. In other words, a complete implementation of a single information reservoir depends on the interaction statistics. That is, for Poisson interaction statistics $r\delta t \to 0$, off-diagonals will not appear in the steady state, while for regular statistics $r\delta t \to 1$, one obtains more data about the reservoir in the steady state with the inclusion of off-diagonal terms.

2.1. Fisher Information

As mentioned above, the probe qubit encodes the binary classification result in the steady state. A qubit can be represented as

$$\rho = \frac{1}{2}(\mathbb{1} + \boldsymbol{\omega} \cdot \boldsymbol{\sigma}) \tag{14}$$

in the Bloch representation where, $\mathbb{1}$ is the unit matrix, $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)^T$ is the Bloch vector and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes a vector whose components are the Pauli matrices. When the qubit undergoes a dissipation process with Pauli channels, this can be defined by a CPTP map

$$\varepsilon(\rho) = \frac{1}{2}\mathbb{1} + \frac{1}{2}(A\boldsymbol{\omega} + \boldsymbol{c}) \cdot \boldsymbol{\sigma} \tag{15}$$

where A is a 3×3 transformation matrix with $A_{i,j} = \frac{1}{2}Tr[\sigma_i \varepsilon(\sigma_j)]$ real elements and $\boldsymbol{c} \in R^3$ is a translation vector with $c_i = \frac{1}{2}Tr[\sigma_i \varepsilon(\mathbb{1})]$. Here, the transformation of the Bloch vector $\boldsymbol{\omega}$ is defined by an affine map $\varepsilon(\boldsymbol{\omega}) \coloneqq A\boldsymbol{\omega} + \boldsymbol{c}$.

An unknown parameter λ of a random variable can be estimated by Fisher information. The classical Fisher information for a discrete random variable reads

$$\mathcal{F}_\lambda = \sum_r p_r(\lambda) \left[\frac{\partial ln p_r(\lambda)}{\partial \lambda}\right]^2 \tag{16}$$

where $p_r(\lambda)$ is the probability of obtaining the result r conditioned on the parameter λ. Quantum Fisher information (QFI) can be quantified by the generalization of Eq. (16) as

$$\mathcal{F}_\lambda = Tr[\rho_\lambda L_\lambda^2] = Tr[(\partial_\lambda \rho_\lambda) L_\lambda] \tag{17}$$

where L_λ is the symmetric logarithmic derivative defined through $\partial_\lambda \rho_\lambda = \frac{1}{2}\{\rho_\lambda, L_\lambda\}$ [32].

Based on these expressions, for a general mixed state $\rho_\lambda = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$, QFI is defined by

$$\mathcal{F}_\lambda = \sum_i \frac{(\partial_\lambda p_i)^2}{p_i} + \sum_i p_i \mathcal{F}_{\lambda,i} - \sum_{i \neq j} \frac{8 p_i p_j}{p_i + p_j} \left|\langle\Psi_i|\partial_\lambda \Psi_j\rangle\right|^2 \tag{18}$$

where $\{p_i\}$ are the eigenvalues of ρ and $\mathcal{F}_{\lambda,i}$ is the QFI for a pure state with

$$\mathcal{F}_{\lambda,i} = 4[\langle\partial_\lambda \Psi_i|\partial_\lambda \Psi_i\rangle - |\langle\Psi_i|\partial_\lambda \Psi_i\rangle|^2]. \tag{19}$$

In addition, a convenient formula specific to the two level system (TLS) was developed as follows [33]

$$\mathcal{F}_\lambda = Tr[(\partial_\lambda \rho)^2] + \frac{1}{det \rho_\lambda} Tr[(\rho_\lambda \partial_\lambda \rho_\lambda)^2]. \tag{20}$$

QFI can also be expressed in Bloch sphere representation under Pauli channels as [34]

$$\mathcal{F}_\lambda = |\partial_\lambda\, \varepsilon(\boldsymbol{\omega})|^2 + \frac{[\,\varepsilon(\boldsymbol{\omega}) \cdot \partial_\lambda\, \varepsilon(\boldsymbol{\omega})]^2}{1 - |\,\varepsilon(\boldsymbol{\omega})|^2}. \tag{21}$$

2.2. Numerical results for Fisher Information

The QuTIP package is used for numerical calculations [35]. It can define the density matrix for the information reservoir as follows

$$\rho_{\mathcal{R}_i} = \begin{bmatrix} \frac{1+\cos\theta_i}{2} & \frac{e^{-i\emptyset_i}}{2}\sin\theta_i \\ \frac{e^{i\emptyset_i}}{2}\sin\theta_i & \frac{1-\cos\theta_i}{2} \end{bmatrix} = \begin{bmatrix} \langle\sigma_i^+\sigma_i^-\rangle & \langle\sigma_i^-\rangle \\ \langle\sigma_i^+\rangle & \langle\sigma_i^-\sigma_i^+\rangle \end{bmatrix} \tag{22}$$

When we substitute the values in Eq. (22) in Eq. (12), $\varepsilon(\rho_0^{ss})$ is obtained according to the variables θ and $\emptyset$ as

$$\begin{aligned}\varepsilon(\rho_0^{ss}) = {} & \frac{1}{2\sum_i^N \varphi_i^2}\sum_{i=1}^N(\varphi_i^2 + \varphi_i^2\cos\theta_i)|e\rangle\langle e| + \\ & \left(+\frac{ir\tau}{2\sum_i^N \varphi_i^2}\sum_{i,j=1}^N \varphi_i\, \varphi_j^2 \sin\theta_i \cos\emptyset_i \cos\theta_j + \frac{r\tau}{2\sum_i^N \varphi_i^2}\sum_{i,j=1}^N \varphi_i\, \varphi_j^2 \sin\theta_i \sin\emptyset_i \cos\theta_j\right)|e\rangle\langle g| + \\ & \left(-\frac{ir\tau}{2\sum_i^N \varphi_i^2}\sum_{i,j=1}^N \varphi_i\, \varphi_j^2 \sin\theta_i \cos\emptyset_i \cos\theta_j + \frac{r\tau}{2\sum_i^N \varphi_i^2}\sum_{i,j=1}^N \varphi_i\, \varphi_j^2 \sin\theta_i \sin\emptyset_i \cos\theta_j\right)|g\rangle\langle e| + \\ & \frac{1}{2\sum_i^N \varphi_i^2}\sum_{i=1}^N(\varphi_i^2 - \varphi_i^2\cos\theta_i)|g\rangle\langle g|\end{aligned} \tag{23}$$

Substituting $\varepsilon(\rho_0^{ss})$ and Pauli matrices into Eq. (15),

$$\varepsilon(\rho_0^{ss}) = \frac{1}{2}\mathbb{1} + \frac{1}{2}\big(\varepsilon(\omega_x)\sigma_x + \varepsilon(\omega_y)\sigma_y + \varepsilon(\omega_z)\sigma_z\big) \quad (24)$$

we get

$$\varepsilon(\omega) = \begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \frac{1}{\sum_i^N \varphi_i^2}\begin{pmatrix} r\tau\sum_{i,j=1}^{N}\varphi_i\,\varphi_j^2\sin\theta_i\sin\emptyset_i\cos\theta_j \\ -r\tau\sum_{i,j=1}^{N}\varphi_i\,\varphi_j^2\sin\theta_i\cos\emptyset_i\cos\theta_j \\ \sum_{i=1}^{N}\varphi_i^2\cos\theta_i \end{pmatrix}$$
(25)

Finally, when we substitute Eq. (25) in Eq. (21), the QFI ($\mathcal{F}_\theta$) is obtained in Bloch sphere notation.

Figure 2 shows the couplings are equal and fixed QFI is investigated for different reservoir states defined by the geometrical qubit parameters. In this case, the preferred parameter is the Bloch ball azimuthal angle θ to define to information reservoir states $|\Psi_\theta\rangle$.

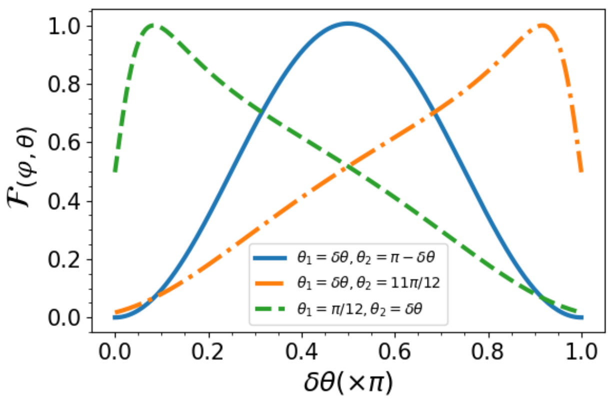

Figure 2 The variation of the QFI of the system qubit coupled to the two environments carrying different information contents parametrized by θ, where $\delta\theta$ is a fraction of θ with $\delta\theta = 1.0$. Coupling of the system to the reservoirs are fixed, equal and $\varphi_1 = \varphi_2 = 0.05$

While $t_r \sim 1 - 10ns$ is a single qubit reset time, current state-of-the art technology allows an energy dissipation on time scale $T_1 \sim 50 - 100\mu s$ [36]–[38]. We take $w_0 = w_b = 1\ GHz$ and is $k = 1 \times 10^3$ is number of successful. The interaction time is $\tau = 4ns$ corresponding to $\tau = 4$ when scaled w_0. The rate is $r = \frac{k}{T_1 w_0} = 0.2$.

Blue curve presents the QFI depicted against $\delta\theta$ which is a factor governs the variation of the Bloch ball azimuthal angle such as $\theta_1 = \delta\theta, \theta_2 = \pi - \delta\theta$ to the $|\Psi_{\theta_1}\rangle$ and $|\Psi_{\theta_2}\rangle$ reservoirs, respectively. For instance, when $\delta\theta = 0$; $\theta_1 = 0$ and $\theta_2 = \pi$. $\mathcal{F}_\theta$ is the maximum likelihood estimate when $\delta\theta = \pi/2$. This means that $\mathcal{F}_\theta$ has maximum information at $\delta\theta = \pi/2$.

The green curve represents the fixed state of the first environment represented by the azimuth angle $\theta_1 = 15^\circ$ and the variation of the state of the second environment parameterized by $\theta_2 = 1^\circ, 2^\circ, \cdots, 180^\circ$. Likewise, the orange curve represents the fixed state of the second environment represented by the azimuth angle $\theta_2 = 165^\circ$ and the variation of the state of the first environment parameterized by $\theta_1 = 1^\circ, 2^\circ, \cdots, 180^\circ$ (Figure 2). The shifts in the maximum information value of the $\mathcal{F}_\theta$ relative to the initial state of the information reservoirs are clearly visible. In this way, classification can be made as $\theta \leq \pi/2$ class1 and $\theta > \pi/2$ class2.

3. CONCLUSION

First, we derive a master equation, considering quantum reservoirs as information sources. Through this equation we have derived, we propose a classifier that makes a binary decision in its steady state depending on the coupling ratios and the parameters of the distinct, multiple, quantum information-carrying reservoirs. By specifying the classification according to the weighted sum of the amplitude parameters, the steady-state magnetization of the probe qubit is used as the classification tool. As a result, we analytically verify that a single probe qubit can classify it by reading the quantum information encoded in the reservoir qubits.

Second, the QFI is generalized for N reservoir states defined by the geometric qubit parameters. We then numerically obtain the QFI for $N = 2$ reservoir states. We also verify that quantum Fisher information can be used as a classification tool. In cases where quantum information tasks are based on dissipation, our results may contribute.

4. APPENDICES

Let's express Eq. (8) as follows

$$U(\tau) = \mathbb{1} - U_1 - U_2 \quad \text{(A.1)}$$

where $U_1 = i\tau\left(\sigma_0^+ J_{\varphi_i}^- + \sigma_0^- J_{\varphi_i}^+\right)$ and $U_2 = \frac{\tau^2}{2}\left(\sigma_0^+\sigma_0^- J_{\varphi_i}^- J_{\varphi_i}^+ + \sigma_0^-\sigma_0^+ J_{\varphi_i}^+ J_{\varphi_i}^-\right)$, respectively. If we substitute Eq. (A.1) in Eq. (10) and neglect the 3rd ($U_1(\tau)\rho(t)U_2^\dagger(\tau)$) and 4th ($U_2(\tau)\rho(t)U_2^\dagger(\tau)$) order terms, the following expression is obtained.

$$\dot{\rho}_0(t) = rTr_{\mathcal{R}_i}\big[U_1(\tau)\rho(t)U_1^\dagger(\tau) - U_1(\tau)\rho(t) - U_2(\tau)\rho(t) - \rho(t)U_1^\dagger(\tau) - \rho(t)U_2^\dagger(\tau)\big]. \quad \text{(A.2)}$$

Considering the cyclic and linearity properties of the trace operation, the explicit expression of the main equation (Eq. (A.2)) for the probe qubit is as follows when we trace the degrees of freedom of the information environments.

$$\dot{\rho}_0 = -ir\tau\left[\sum_i^N \varphi_i(\langle\sigma_i^-\rangle\sigma_0^+ + \langle\sigma_i^+\rangle\sigma_0^-)\,,\rho\right] + \frac{r\tau^2}{2}\sum_{i=1}^N \varphi_i^2(\langle\sigma_i^+\sigma_i^-\rangle\mathcal{L}[\sigma_0^+] + \langle\sigma_i^-\sigma_i^+\rangle\mathcal{L}[\sigma_0^-]) + 2r\tau^2\sum_{i<j}^{N'}\varphi_i\varphi_j\left(\langle\sigma_i^+\rangle\langle\sigma_j^-\rangle\,\mathcal{L}_s[\sigma_0^-] + \langle\sigma_i^-\rangle\langle\sigma_j^+\rangle\mathcal{L}_s[\sigma_0^+]\right) \quad \text{(A.3)}$$

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The Declaration of Conflict of Interest/ Common Interest

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Authors' Contribution

The authors contributed equally to the study.

The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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