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Longevity Risk and Modelling in The Life and Pension Insurance Company: Mortality Forecasting with the Lee–Carter Method

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Abstract

Longevity risk is exactly the opposite of mortality risk and indicates that live longer than life expectancy has a cost for insurance companies. Longevity risk is one of the important topics which take part in actuary literature. One of the most widely used model is Lee-Carter (LC) model which allow to be expressed as a stochastic process of mortality models.

The study was carried out in order to model male and female mortality rates in Turkey by means of Lee-Carter (LC) method and in order to make predictions for the future. Thus, male and female death rates associated with age between 1950-2020 years of Turkish statistical institute in Turkey was used as data. At the end of study, it was found that death rate of men may be more than those females for the future.

Keywords: *Insurance, Longevity Risk, Mortality Risk, Lee Carter Method.*

Introduction

Life and pension insurance companies face with two important actuarial risks when conducting insurance operations. First one is the longevity risk life and pension companies are exposed to with regards to their insurance products based on the longevity of the insured. The second, on the other hand, is the mortality risk which may arise due to reduced mortality rate of the insured.

Longevity Risk may arise when individuals in a specific group live longer than they were expected to. It is the result of different outcomes obtained for longevity and mortality risk when compared to the assumptions insurance companies use when they assign prices for their products. In other words, longevity risk is the case when more than expected number of insured survives in terms of annuity products (Black and Skipper, 2000: 161). Private insurance companies, particularly life and pension insurance companies along with social security institutions, take heed of longevity risk when developing their long-term plans and programs.

Longevity risk arises from the ambiguity of the survival of individuals which can be calculated using the mortality rate of the society in question reflecting a somewhat acceptable estimate for each individual (Hanewald; Piggott; Sherris, 2013: 88). On the other hand, more individuals may live longer than expected or the mortality rate may be lower than expected. Especially as the developments in health and medicine technologies may lead to short-term (i.e., one or two years) or long-term (10+ years) increase in the human life expectancy, this may lead to longevity risk (Coxa; Lin; Pedersen, 2010: 243). However, longevity risk is a long-term risk, it is mostly necessary to be evaluated for time periods as short as one year (Richards; Currie; Ritchie, 2012: 1).

Longevity risk is an important risk factor for life and pension insurance companies and it may have a significant impact on the risk status (Gatzert and Wesker, 2012: 1). It is known that mortality rate is related with socioeconomic and behavioral risk factors and this fact is taken into consideration in actuarial calculations for life insurance policies. It is of utmost importance to understand mortality fully for product development in life insurance, pricing, evaluation and profitability analysis (Kwon and Jones, 2008: 394). Life expectancy of insured is a source of costs for the insurance company in terms of health expenses, medical treatment expenses, etc. during the pension (Brouhns, Denuit and Vermunt, 2002:373). Increased life expectancy is a serious problem for life insurance companies and pension funds which are oblige to make monthly pension payments. In the light of these issues, it is important to take measures against this kind of a risk.

Lee and Carter (1992) offered a model which uses age and time factors in combination in order to model and estimate the age-specific mortality rates. Developed in 1992, this method is recognized as one of the most commonly used methods for the mortality rate estimations. Following the research conducted by Lee and Carter, Lee and Miller (2001), Booth et al. (2002), De Jong and Tickle (2006), and Hyndman and Ullah (2007) have further developed this method with several additions.

Among the research which estimates mortality rates in Turkey are Gençtürk and Genç (2012), and Demircioğlu and Büyükyazıcı (2013). In their study, Gençtürk and Genç (2012) estimated the mortality rates based on the mortality statistics available in Turkey using Trend method and Lee-Carter method and they have compared the results obtained from these methods. It was found that mortality rates obtained from Lee-Carter and Trend methods were similar while these two methods were more compatible for the data obtained from women. Demircioğlu and Büyükyazıcı (2013) estimated the age-specific mortality rates in Turkey using the Lee-Carter method and Poisson Log-Bilineer approach which was developed as an alternative to Lee-Carter method. Researchers have compared the suggestions obtained from both methods and found that these two methods gave somewhat different results.

Wiśniowski et al., (2015) estimated the population changes in England until the year 2024 using the Lee-Carter method. This study modeled the population movements such as age-specific birth, mortality and immigration. Danesi, Haberman and Millossovich (2015) have estimated the mortality improvement rates using the Lee-Carter method based on the mortality data obtained from Italy between 1974 and 2008. In their study, Richards and Currie (2009) estimated the mortality rates for England and Whales using the Lee-Carter method with the data collected from a period between 1961 and 2006. The study showed that the Lee-Carter method should also take heed of liabilities such as pension payments. Antolin (2007) has conducted a mortality modeling study for some of the OECD countries (Canada, France, Germany, Italy, Mexico, United Kingdom, United States of America).

This study conducts a mortality modeling for Turkey using the Lee-Carter model, one of the most important mortality models for life insurance products and social security insurance policies priced with estimated longevity of individuals and masses. Data on the mortality rates of Turkey based on Turkish Statistical Institute (TUIK) for a period between 1950 and 2010 were taken as a reference point. Future estimations were then addressed calculating the longevity and mortality rates for the periods after 2010. This study includes longer-term estimations compared to other studies conducted in Turkey. By using historical data, life and death times in Turkey until 2060 were calculated. These results will guide insurance companies operating in Turkey in presenting their retirement plans.

Mortality Models and Lee-Carter (LS) Method

However, there is no universal method for mortality rate estimations, the method to be used is determined considering criteria such as accuracy, reliability, and simplicity. Mortality modeling most commonly uses variables such as age and gender while it is obvious that there are other factors affecting the mortality when health and mortality statistics are investigated. The effects of these descriptive variables on mortality model are disregarded as data is not available or insufficient for that variable. With this variables disregarded, an assumption of homogeneity of variance is not possible for mortality models commonly used in the literature. Some unexpected results may be obtained when the assumption of homogeneity of variance is not established (Gençtürk and Genç, 2012: 64)

Several methods are developed in order to estimate the mortality rates and pattern and also to anticipate the future mortality. The efforts to interpret mortality in the form of a curve go back to the 19th century, starting from the work of Gompertz, the "Law of Mortality". These first trials on mortality curve tried to divide mortality in early age, middle age and advanced age, and merely considered the age as a factor in interpreting the mortality (Hári et al., 2007). In the recent years, accurate and reliable calculations in mortality models have gained importance for actuaries and policy-makers. Many models are developed for mortality calculations in time and it was found that stochastic models are most commonly preferred as they give more accurate and reliable outcomes as a result of the studies conducted in this respect (Koissi and Shapiro, 2008).

In 1992, Ronald Lee and Lawrance Carter have offered a model which includes age and time factors for age-specific mortality rate modeling and estimations (Lee and Carter, 1992). This model utilizes a time series model which is able to reflect the changes in the mortality rates of the past to a model. The estimations of mortality rate and life expectancy are in correlation with the estimation of the time-dependent mortality levels (Lee and Carter, 1992).

Lee-Carter model is a mortality projection approach which explains the multiplication of components namely a parameter which varies in time and reflects the general mortality rate as the log of age-specific death rate and a parameter which defines the speed of change in general mortality level for each age group and the addition of time-independent age-specific component values. This method suggests a linear approach with variables such as x (age) and t (time) and builds on the future estimations of mortality rates based on the recorded past mortality rates. (Haberman and Russolillo, 2005: 2-3).

Lee-Carter model has found a widespread use in this field in the last 18 years and it can be said that it is the golden standard of our time in anticipating the mortality using models (Li and Chan, 2007: 68).

Mortality Modeling

Let $\mu_x(t)$, represent the instantaneous death rate of an individual at the age of *x* and time of *t*. Then the probability of the deaths in that year is calculated (Kogure and Kurachi, 2010:162):

$$q_x(t) = 1 - \exp\left\{-\int_0^1 \mu_{x+s}(t+s)\mathrm{ds}\right\}.$$

$$\mu_{x+s}(t+u) = \mu_x(t) \qquad (1)$$

For x and t integers $0 \leq s, u < 1$. Exponential of death is constant in age and time.

$$q_x(t) = 1 - \exp\{-\mu_x(t)\}.$$

Lee-Carter method models the mortality rates.

$$q_x\,(t) = 1 - \exp\,\{a_x + \beta_x k_t\},$$

First, it is necessary to estimate the parameters.

$$q_x\,(t) = 1 - \exp\,\{-\exp\{a_x + \beta_x k_t\}\}$$

It is assumed that *x* number of individuals will be alive for *t* years.

$$_tp_x(t_0) = \left(1 - q_x(t_0)\right) \mathrm{x} \left(1 - q_{x+1}(t_0 + 1)\right) \qquad (2)$$

$$\mathrm{x} \ldots \mathrm{x} \left(1 - q_{x+t-1}(t_0 + t - 1)\right)$$

$$= \exp\left\{-\sum_{j=0}^{t-1} \exp\left\{a_{x+j} + \beta_{x+j} k_{t_{0+j}}\right\}\right\}$$

Model Development

Lee-Carter Model is described as follows;

$$\text{In}\left(m_{x,t}\right) = a_x + b_x k_t + \epsilon_{x,t},$$

$m_{x,t}$: represents the approximate mortality rate for the age of *x* at the time of *t* (Chan, 2013: 19-20).
a_x : Age-specific parameter; this set {a_x , *x* =0, 1 ,…………..….} represents the general outlook of the death chart.

k_t : Time-dependent parameter; k_t represents the trend of the improvement in mortality in time.

b_x : Age-specific parameter; characterizes the k_t sensitivity at the age of *x*.

$\epsilon_{x,t}$: represents the error term.

Time-dependent parameter, k_t, represents the change in the logs of death rates in time. However, one should not expect to have similar changes in mortality values for every age group. It is not possible for any effect which may occur at the general level of mortality to be observed in the same pattern at any age. Therefore, any decrease in the general level of mortality will affect any age in a different manner. It is the b_x parameter which makes it possible to separately define the effects of these changes for each age group. a_x parameter, on the other hand, is the age-specific mortality level which is found using the averages over the years for each age group and therefore is separated from the t index (Brouhns, Denuit, and Vermunt (2002).

For the estimation of the parameters b_x, which is the age-specific pattern of mortality that changes according to years, and k_t, which shows the variation of mortality by years, the $\epsilon_{x,t}$ matrix is obtained by subtracting the a_x vector from the logarithmic mortality matrix. In the application, the TDA method used by Lee and Carter in their original work and the two-stage estimation method were used to find the bx and k_t vectors. As a first step, b_x and kt values were obtained by applying the TDA method to the $\epsilon_{x,t}$ matrix. In this decomposition process, the Biplot software, which was prepared as an add-on to the Microsoft Excel program, and the Matlab program were used.

Application

Data Collection

Population related mortality rates are regularly announced by the Turkish Statistics Institution (TUIK). Data collection involved the censuses held by TUIK in Turkey in 1960, 1965, 1970, 1975, 1980, 1985, 1990, 1995, 2000, 2005, 2010 and 2020, then the mortality rates were calculated and the results were modeled using the Lee-Carter model. Data on the age-specific mortality rates of Turkey was not made available by Turkish Statistical Institute (TUIK) between 1960 and 1965. Data for this period was obtained from the research conducted by Yıldırım (2010). Data for the remaining years were obtained from the official website of TUIK.

Mortality Modeling Using the Lee-Carter Method

Mortality rate in Turkey was adapted to the TUIK data using the Lee-Carter method. Gender-specific mortality data of Turkey for the period between 1950 and 2020 was used. A data matrix was established for year and age values following the calculation of the conversions of the mortality rates in Turkey for ages between 1 and 85, both Women and Men.

The results of the model parameters are shown below in Figures based on the data collected between 1950 and 2020.

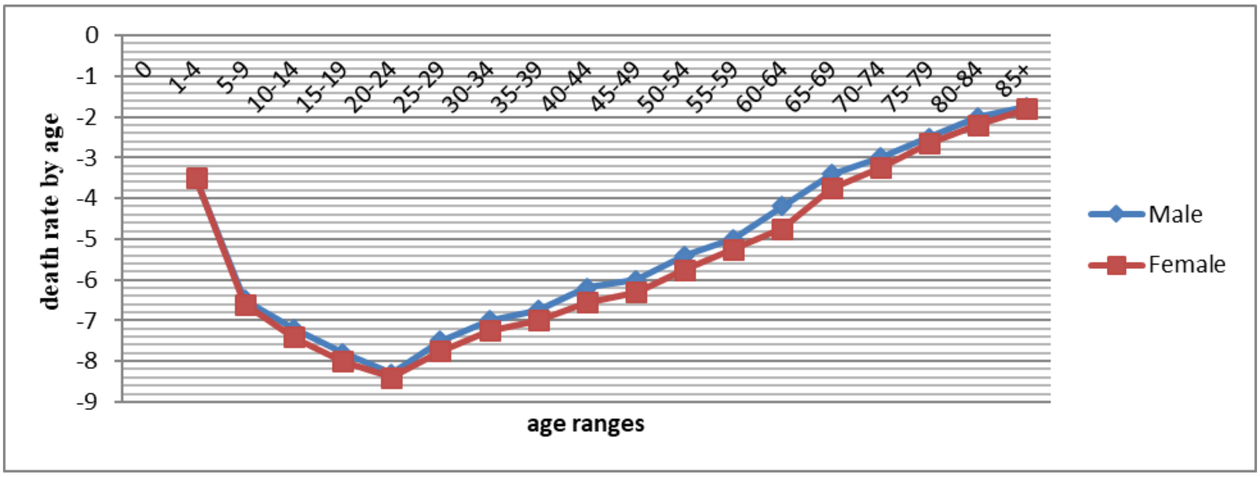

Figure 1: a_x parameter value

When the age-specific parameter of mortality, ax, is investigated (Figure 1), it was found that the mortality rate decreases between the age groups of 0 and 20-24 for both Women and Men. It is observed that mortality rates of early ages are lower than advanced ages for both genders and it turns into an increasing trend especially after the age group of 20-24. Nevertheless, it was found that mortality of men is higher than mortality of women for all age groups (0 to 85) according to the time-independent and age-specific parameters.

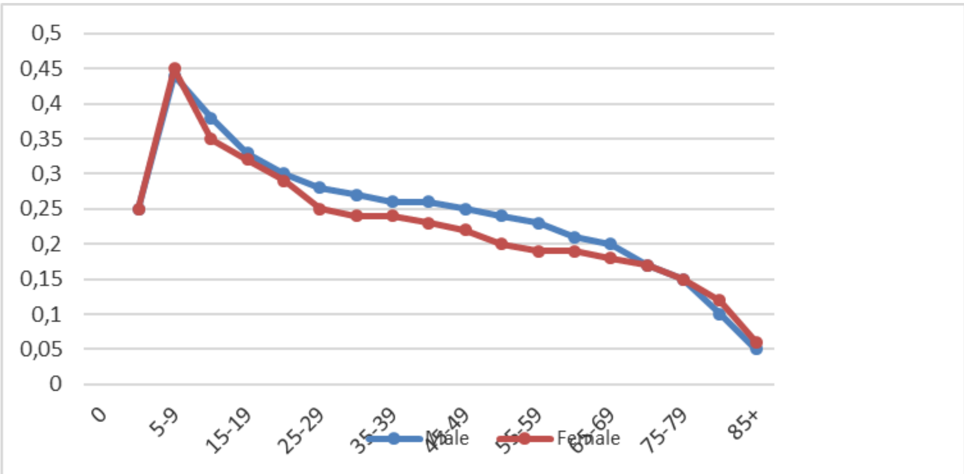

Figure 2: b_x parameter value

Figure 2 shows the results obtained for the bx parameter which gives the mortality rates for each age group. A review of the parameters shows that parameters have positive values for each age group. These results reveal that mortality rates in Turkey are decreasing for all age groups. They especially show that mortality rate will not increase for any age group based on a change in the mortality structure. According to these results, mortality rate is decreasing particularly for the age group of 1-4 and the mortality rate for this age group will be reduced in the future. This result confirms the reduced number of infant deaths in

Turkey reported in the recent years. bx values follow a decreasing pattern for the advanced age groups between 1-4 and 85. These results indicate that the effect of the change in the mortality rate will be reduced between age groups. Mortality rate increases in a dramatic manner especially with the age group of 50.

kt represents the general trend of mortality improvement in time. Calculated for mortality rates of both men and women, kt values are given in Figures 3 and 4.

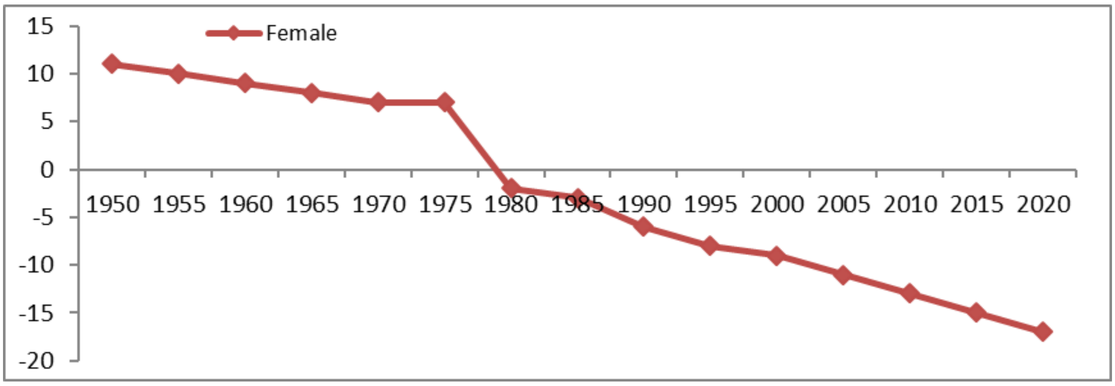

Figure 3: Change of k_t parameter according to years (female)

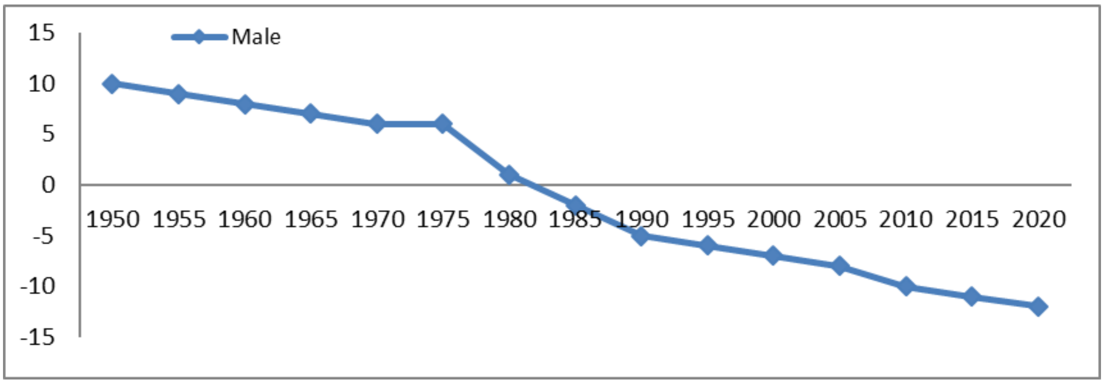

Figure 4: Change of k_t parameter according to years (male)

(kt) values show a similar pattern for both men and women. Ignoring the small changes, kt values obtained from men and women does not show a significant deviation. While having a similar pattern for both men and women, (kt) values are also irregular. In general, kt values have a decreasing trend for both men and women. It can be observed that these values have been decreasing for women as of 1975 and for men as of 1980.

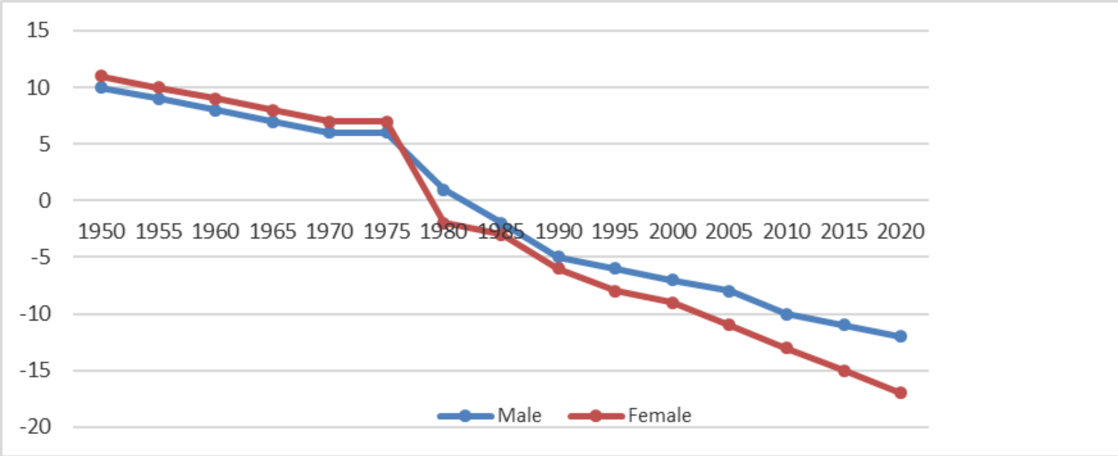

Figure 5: Comparison of k_t parameters by gender

kt, which is the death rate indicator in the Lee-Carter Model, shows the change in mortality rates over the years for all ages. The first stage kt estimation values obtained as a result of TKA are shown in Figure 5. The first-stage cut-off values of the kt parameter, which expresses the course of mortality over the years, show that the mortality of the Turkish population has decreased over the years for both men and women.

Mortality Estimation Modeling for the Future

Projections were created for the period between 2010 and 2060 according to the life expectancy, and age groups obtained from Lee-Carter method using the data recorded between 1950 and 2020. Figures 5 and 6 show the men and women mortality rates until 2060 based on the approximate death rates recorded between 1960 and 2020 in Turkey.

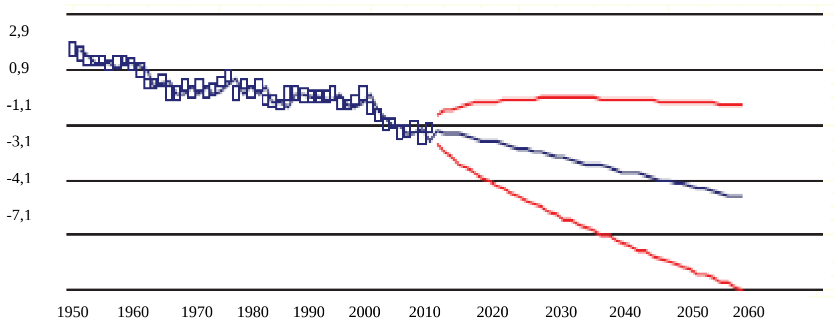

Figure 6: Projection Results for Women Until 2060

The projection results for women for the period between 2010 and 2060 show that these results comply with the data obtained from the period between 1950 and 2010 with a decreasing trend and the results are in a 95% confidence interval. Women mortality rates have been decreasing starting from 1950 and the trend is continued until 2060. Mortality rates have been rapidly decreasing especially after 2000s with negative values.

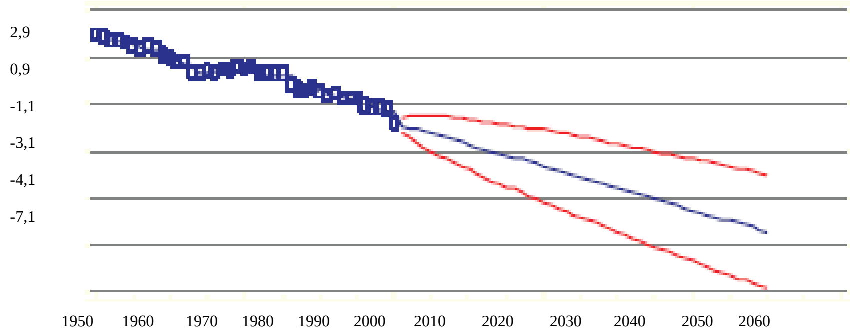

Figure 7: Projection Results for Men Until 2060

A review of the projection results for men until 2060 showed that these results are similar with the ones obtained for women. The confidence level of the projection results for men between 2010 and 2060 was 95% and the mortality rate follows a decreasing pattern.

Figure 8 shows the life expectancy estimations for both men and women in a time-depenndent manner using the Lee-Carter method.

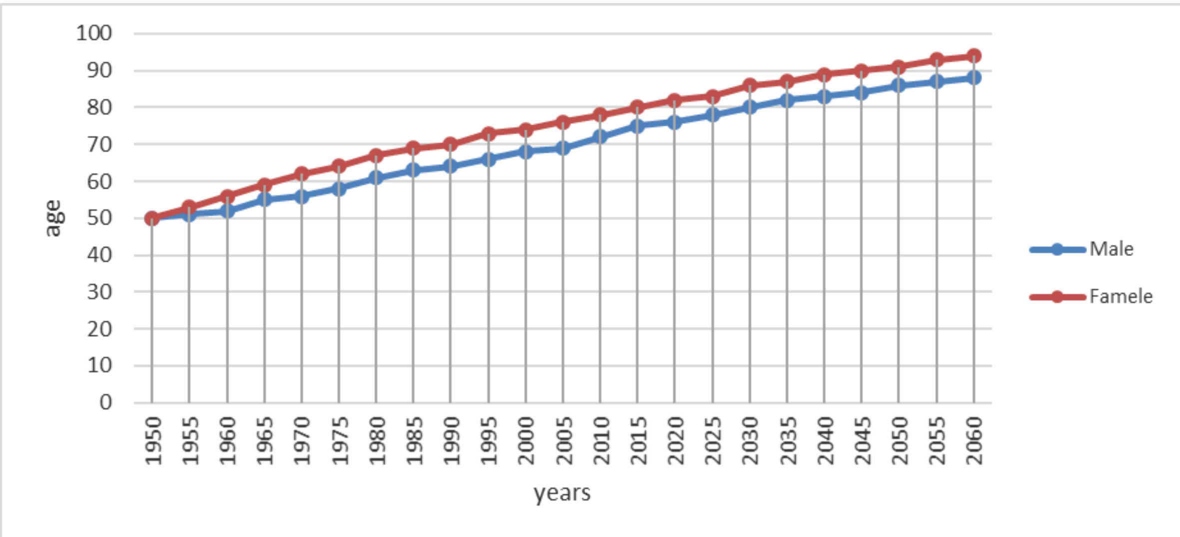

Figure 8: Life Expectancy Using Lee-Carter method.

Scientific research showed that life expectancy of women is longer than the life expectancy of men. Especially in Turkey, the life expectancy of women is longer than the life expectancy of man according to the data collected from TUIK. Based on the results of the estimations using Lee-Carter method, it was found that the life expectancy of women is consistently longer then the life expectancy of men between the years 1950 and 2060.

As of 2010, life expectancy of women is 70+ and it increases consistently. It is expected that the average life expectancy of women in Turkey to become 80+. Life expectancy of men, on the other hand, has also been increasing yet it was shorter than the life expectancy of women.

Results and Discussion

Life and pension insurance companies face a number of insurance risks due to their operations. The subject of the insurance policies life and pension companies create is humans. Based on the human life, these kinds of insurances are exposed to several risks. The most important of these risks are longevity risk and mortality risk. Insurance companies must estimate mortality rates and life expectancies in order to be able to fulfill their obligations towards the insured. The mortality rate and life expectancy play an important role in the determination of the amount of insurance premiums to be paid for a life insurance. It is vital that life insurance companies and social security institutions make successful predictions about future death rates so that they can meet their financial obligations to their commitments. Because wrong estimations may cause the insured to pay more premiums unnecessarily by increasing premiums, or may cause institutions or companies to be insufficient in meeting their financial obligations with less premiums than they should be.

The estimation of mortality rates is an important subject of the actuarial sciences. Estimations of mortality rate are commonly used in many fields. The most important feature of mortality rates and life tables is that they are developed as a result of reliable calculations. It is of utmost importance to estimate mortality rates of a country based on its specific population and mortality statistics.

Lee-Carter method is one of the most commonly used methods in the actuary calculations related with the mortality rates. Lee-Carter method has been reported to be a successful resource since 1992 and has been used by several countries in order to identify their mortality patterns.

This study estimates the mortality rates based on the population and death statistics of Turkey using the Lee-Carter method, a stochastic method used to model mortality rates.

Mortality rates exhibit a decreasing trend for the age groups between 0 and 20-24 for both men and women between the ages of 0 to 85 and living in Turkey. It is observed that mortality rates of early ages are lower than advanced ages for both men and women and it turns into an increasing trend especially after the age group of 20-24. However, mortality rates of men and women are similar in their trends, it was found that mortality rate of men is higher than the mortality rate of women for every age group.

The results obtained for all parameters were positive for each age group when mortality rates are investigated per age group. It can be observed that mortality rates in Turkey are decreasing for all age groups. They especially show that mortality rate will not increase for any age group based on a change in the mortality structure. It was shown that the mortality rate is decreasing in Turkey for the age group of 1-4 while the mortality rate increases for the age group of 50+.

When approximate mortality rate of both men and women is considered, it can be observed that these values have been decreasing for women as of 1975 and for men as of 1980. The projection results for men and women between 2010 and 2060 shows that the mortality rate will be decreasing as it was the case in the previous years. An increase in the life expectancy of both men and women is estimated while the life expectancy of women is anticipated to be higher than the men.

Several issues may arise in the estimation of mortality rate and life expectancy in Turkey, as it was the case also in this study. Without a regulated recording system, numerically insufficient mortality data and unreliable results are inevitable. Application results and the deviations in the findings of further research will show the significant data insufficiency for the mortality statistics available in Turkey. Consistency for the future in modeling and foresight studies Finding predictions depends on the goodness of fit of the model obtained with the method used to the data, as well as how accurate and explanatory the data used in the study is. Therefore, for a more accurate estimation of the general population mortality in Turkey, obtaining data on mortality indicators is of great importance.

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