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Research Article

Students' Algebra Achievement, Algebraic Thinking and Views in the Case of Using Algebra Tiles in Groups

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Abstract – The study investigated how using algebra tiles in group work affected students' algebra achievement, algebraic thinking and views about using algebra tiles. 40 sixth grade students in the same school participated in a pretest-posttest control group design study. Students in the experimental group (EG) used algebra tiles in groups and those in the control group (CG) continued with their regular instruction. Prior Algebra Knowledge Test and Algebra Achievement Test were implemented to both groups as pretest and posttest. EG students expressed their views in the Views about Algebra Tiles Questionnaire. Although students' performances did not differ in the statistical analysis in both tests, qualitative analysis of the responses revealed that algebra tiles had positive effect on EG students' algebraic thinking. EG students indicated that using algebra tiles in group work supported their learning, they made them understand the concepts meaningfully, and the lessons were more enjoyable.

Key words: algebra achievement, algebra tiles, algebraic thinking, group work, middle school students

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Introduction

Algebra appears less concrete than arithmetic to the students because it requires thinking about sets of numbers rather than only a few numbers (Palabıyık & Akkuş, 2011). It is considered as remembering a set of rules to apply while operating with letters and numerals by students (Kaput, 1995) because algebra teaching in middle schools emphasizes procedures that involve rules and steps separated from other mathematical concepts and real life of students

(Kaput, 1999). Therefore, students have difficulties in understanding multiple meanings of letters and the underlying logic, and making transition from arithmetic to algebra while learning algebra and developing algebraic thinking (Jupri et al., 2015; MacGregor, 2004).

Algebraic thinking is one’s potential to reveal the relationship between the variables in one’s representations of quantitative situations (Driscoll, 1999). It is employing representations that illustrate quantitative relationships (Kieran, 1996). Improvement of algebraic thinking results in understanding algebra meaningfully rather than focusing only on procedures (Windsor, 2010). Presenting elementary and middle school students with situations that incorporate relationships in contexts improves their algebraic thinking (Lawrence & Hennessy, 2002). Manipulatives can be used for meaningful and effective algebra learning process to eliminate students’ difficulties and to develop students’ algebraic thinking (Akkuş, 2004; Chappell & Strutchens, 2001; Larbi & Okyere, 2016). Students have higher algebraic abilities, such as representing algebraic expressions and interpreting them, and making connections between algebraic concepts, when manipulatives are used (Chappell & Strutchens, 2001). Particularly, using manipulatives enables middle school students to make meaningful connections in algebraic thinking (Chappell & Strutchens, 2001).

Algebraic thinking is promoted when students are provided with the opportunities to convey their mathematical ideas in a classroom context that values and encourages collaborative learning (Windsor, 2010). Group work enhances students’ algebra learning and increases their self-efficacy in algebra (Fletcher, 2008). Moreover, group work has the potential to foster positive dispositions towards mathematics, students’ procedural fluency, and the development of their mathematical reasoning skills (Jansen, 2012). Students can notice what their peers struggle to capture, help them understand better, and also eliminate misunderstandings while learning a new concept in a group (Webb & Farivar, 1994). Furthermore, explaining to peers allows students to develop a deep understanding by identifying knowledge gaps and filling those gaps (Fuchs et al., 1997).

Algebra tiles, which are one of the manipulatives that are used in teaching algebra, help students see the relationship between algebraic and geometric concepts (Leitze & Kitt, 2000). Students can make the transition from concrete to symbolic representations of algebraic concepts with the help of algebra tiles (Fennema, 1972). Use of algebra tiles eliminates students’ mistakes and confusion between expressions such as 2x and 2+x (Picciotto & Wah, 1993). Furthermore, algebra tiles provide visual and hands-on approach for the newly-introduced concepts. Students can reach the algebraic rules from their own experiences with the help of algebra tiles (Okpube & Anugwo, 2016). One specific importance of algebra tiles is that they can be easily prepared by teachers by cutting the cardboards (Karakırık & Aydın, 2011). In this respect, it is important to investigate the effects of algebra tiles since they are easily created, replicated and used by the teachers cheaply when the resources are not sufficient. Examining the effects of manipulatives, specifically algebra tiles, is important in students' initial encounter with the algebraic concepts because students' learning can be supported by using manipulatives when students learn abstract concepts for the first time (Akkaya, 2006).

Research on the use of algebra tiles in teaching and learning of algebra have mostly investigated teaching solving linear equations in one variable to middle school students with algebra tiles (Magruder, 2012; Saraswati et al., 2016), a system of two linear equations to senior high school students by using algebra tiles (Akpalu et al., 2018), factoring algebraic expressions to high school students with algebra tiles (Larbi & Okyere, 2016; Schlosser, 2010; Sharp, 1995; Thornton, 1995), polynomial multiplication by using algebra tiles (Goins, 2001; Johnson, 1993; Ünlüer & Kurtuluş, 2021; Wingett, 2019), distributive property to expand algebraic expressions (Larbi & Okyere, 2016); solving quadratic equations by completing a square (Vinogradova, 2007) and algebraic expressions to students with learning disability through algebra tiles (Castro, 2017). These studies have found that middle school students reflected on their actions while solving linear equations with the help of algebra tiles (Magruder, 2012) and reached the solution of linear equation in one variable easily (Saraswati et al., 2016); high school students conceptually understood a system of two linear equations (Akpalu et al., 2018), learned factoring and distributive property meaningfully (Larbi & Okyere, 2016), and could make geometric connection to factoring polynomials when algebra tiles were used (Schlosser, 2010). In addition, high school students stated meaningful learning of factoring with ease (Sharp, 1995), had understood the concepts much better (Thornton, 1995) and could provide better explanation of polynomial multiplication process with the help of algebra tiles (Goins, 2001). Furthermore, students who have failed in algebra before performed better when they used algebra tiles (Wingett, 2019) and algebra tiles made a difference in the post-test scores of students with learning disabilities in a positive manner (Castro, 2017). Similarly, middle school students could easily model algebraic expressions (Ünlüer & Kurtuluş, 2021) and algebra tiles helped students build connections between algebraic and geometric concepts (Vinogradova, 2007). Using algebra tiles supported even teachers' understanding of polynomial multiplication (Johnson, 1993).

On the other hand, some research studies conducted in Turkey did not specifically examine the use of algebra tiles but the effects of various manipulatives including algebra tiles used together to teach algebra (e.g., Akyüz & Hangül, 2013; Gürbüz & Toprak, 2014; Işık & Çağdaşer, 2009; Koğ & Başer, 2012; Palabıyık & Akkuş, 2011; Türksever, 2019; Yıldız, 2012). While these studies generally reported the positive effects of manipulatives in students' learning of algebra, the studies in which algebra tiles were used did not focus on students' development of algebraic thinking when they met algebra concepts for the first time.

Gathering students' views has always been important in educational research because those views show us how they perceive teachers' efforts and learning the content. Students' positive and negative views about using manipulatives in learning mathematics help researchers have better interpretations of their views and opinions (Enki, 2014) and teachers to make sense of their learning with manipulatives (Yıldız, 2012). Students' negative views about using manipulatives that arise from seeing mathematics as a body of algorithms and rules that should be followed (Hinzman, 1997) and students' perceptions of the activities including manipulatives in which they engage might have an effect on the way teachers teach (Thompson & Lambdin, 1994).

Working in small groups in the classroom improved middle school students' algebra achievement and students expressed their satisfaction with being a part of the group work (Balt, 2017). Being engaged in group work enabled students improve their conceptual understanding in algebra (Jones, 2008) and promoted improvement of their algebra learning and their self-efficacy (Fletcher. 2008). Hinzman (1997) found that hands on manipulatives and group activities were useful in enhancement of middle school students' performance in algebraic concepts. Moreover, students stated that they enjoyed the use of manipulatives in activities while learning algebraic concepts.

Understanding the effects of using algebra tiles in group work on students' algebra achievement and algebraic thinking is important because algebra tiles have the potential to help students to internalize algebraic ideas when they first meet them. They are easy to produce for teachers, even for students. Therefore, they can provide a convenient material for teaching and learning algebra, especially when resources are scarce. Using algebra tiles in group work may provide students an idea-rich environment for learning the basic algebra concepts, which affects their future mathematics performance (Wang & Goldschmidt, 2003). In this sense, it is also important to explore students' views about using algebra tiles because their views can help teachers and researchers understand the useful features of these manipulatives for students' learning of algebra and development of algebraic thinking. Despite the above presented literature, there seems to be lack of studies on students' learning of algebra by using the algebra tiles in group work when they are introduced the algebra concepts for the first time. Thus, it is this study may contribute to the literature by providing knowledge on students' algebra achievement when they use algebra tiles in groups.

The present study investigated the effects of using algebra tiles in group work on sixth grade students' algebra achievement, algebraic thinking and views about using algebra tiles when they first met algebra concepts. For this purpose, following research questions were formulated:

1) Do 6th grade students who use algebra tiles in group work significantly outperform those who do not use algebra tiles in the algebra achievement test?
2) How does students' algebraic thinking differ for those who use algebra tiles in group work and who do not use in the algebra achievement test?
3) What views do 6th grade students develop about using algebra tiles in group work in the mathematics lessons?

In the study, the term "algebra achievement" was used to address achievement scores of the 6th grade students in the algebra achievement test that was developed by the researchers and that included questions about algebraic concepts based on the 6th grade objectives. "Algebraic thinking" includes recognizing and analyzing numerical and geometric patterns and expressing them mathematically in word or symbols, representing relationships, making generalizations about the mathematical relationships, and thinking with unknown quantities (NCTM, 2000). Students' algebraic thinking was investigated in this study by deeply analyzing their responses to all questions in the algebra achievement test. In this study, views refer to the 6th grade students' opinions, beliefs and feelings about using algebra tiles in group work in mathematics lessons and investigated by an open-ended questionnaire. Detailed information about mentioned tests is provided below.

Method

The study is a mixed-method study employing both quantitative and qualitative methods. A pretest-posttest control group design was chosen in order to investigate the effects of using algebra tiles in groups on 6th grade students' algebra achievement and algebraic thinking. Experimental group (EG) students learned initial algebra concepts by using algebra tiles in groups while control group (CG) students learned through regular instruction. Both EG and CG students' responses in the Prior Algebra Knowledge Test (PAKT) and Algebra Achievement Test (AAT) were examined in detail and experimental group students' views were explored through the Views about Algebra Tiles Questionnaire (VATQ) which was a qualitative survey.

The Curriculum Context in Relation to the Study

At the time of the study, there was a change in the middle school mathematics curriculum in Turkey. The new (2018) curriculum had just been initiated in the middle schools starting from the 5th grade. Students in the 6th grade were learning mathematics based on the previous (2013) curriculum. Algebra topics were introduced in the 6th grade in both curricula (MEB, 2013; 2018). Table 1 shows the objectives related to algebraic expressions at the 6th grade level in 2013 curriculum (MEB, 2013) and the corresponding grade level of the same objective in 2018 curriculum (MEB, 2018).

Table 1. Objectives related to algebraic expressions in the 6th grade in 2013 curriculum and 2018 curriculum

Objectives Students should be able to…	2013 Curriculum	2018 Curriculum
O1. Write a phrase as an algebraic expression and write a phrase for a given algebraic expression.	6th grade	6th grade
O2. Evaluate an algebraic expression for different values of variable.	6th grade	6th grade
O3. Express the meaning of simple algebraic expressions.	6th grade	6th grade
O4. Make addition and subtraction in algebraic expressions.	6th grade	7th grade
O5. Multiply an algebraic expression with a natural number.	6th grade	7th grade

As can be observed in Table 1, O4 and O5 moved to the 7th grade level in the recent curriculum change although they were covered in the 6th grade at the time of the study. This presented an obstacle for the study in terms of its significance and the implications of the results for teaching mathematics and curriculum. However, the study addressed both the curriculum objectives and students' algebraic thinking, which was an overarching construct based on students' existing knowledge and skills both in other concepts and in algebra as explained below in AAT. Consequently, objectives O1 and O2 were considered as the prerequisite algebra knowledge in this study for both EG and CG. Algebra tiles were introduced and used in the EG while covering O3, O4, and O5, while CG continued with the teacher's regular instruction. This provided a research base to compare both curricula and inform the algebra teaching practices in the 2018 curriculum.

Participants and the School Context

Participants of the study consisted of 40 sixth grade students (age 11-12) in two 6th grade classrooms taught by the same teacher in a public school in one of the cities in Turkey, selected based on the convenience of the first author (hereafter, the researcher). One class was randomly assigned as the EG and the other was assigned as the CG. Students in both classes had similar mathematics achievement according to the school-based mathematics examinations. Students in both CG and EG did not learn any algebraic concept until the 6th grade. Table 2 shows the number of students who took pretest and posttest in EG and CG.

Table 2. The number of students who took pretest and posttest in EG and CG

Groups	Pretest	Posttest	Pretest ∩ Posttest
Experimental	23	23	22
Control	21	20	18
Total	44	43	40

The school was an inner-city school where students from middle socioeconomic status families attended, as indicated by the classroom teacher. Students in both groups have used counters and fraction tiles as materials but they have never used algebra tiles before. In addition, students have not worked in a group in the mathematics lessons.

Data Collection Tools

Prior Algebra Knowledge Test (PAKT), Algebra Achievement Test (AAT) and Views about Algebra Tiles Questionnaire (VATQ) were used to collect data for the study. PAKT, AAT, and lesson plans were prepared according to the 2013 curriculum. O1 and O2 (see Table 1) were the objectives where algebra tiles were not used in teaching. Therefore, these two objectives were used as a base for students' existing algebra knowledge to see if EG and CG groups were at the same achievement level before the implementation started. Then, algebra tiles were used by the students in groups in EG while the regular instruction continued in CG.

Prior Algebra Knowledge Test (PAKT): PAKT consisted of 4 essay type questions with subquestions (15 questions in total) and developed by the researchers to see if EG and CG students had differences in their existing algebra knowledge before the treatment. Both EG and CG students were introduced with algebra topics for the first time in the 6th grade. The treatment in the EG in this study started after the students learned to (i) write a phrase as an algebraic expression and write an algebraic expression as a phrase (O1) and (ii) evaluate an algebraic expression for different values of variables (O2). Therefore, these two objectives were covered in the pretest to see if students in both groups had similar achievement in basic algebraic concepts. PAKT was implemented to both EG and CG students as a pretest in 40 minutes. The 6th grade objectives covered in the PAKT and sample questions for each objective are presented in Table 3.

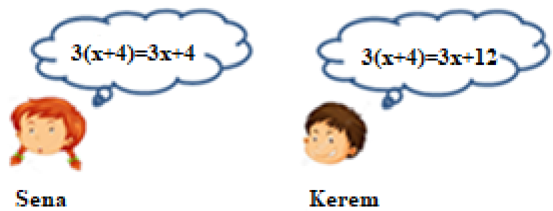
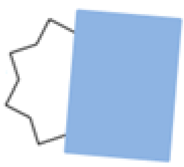
Table 3. The 6th grade objectives (MEB, 2013) covered in the PAKT and sample questions for each objective

Objectives	Sample Questions
Write a phrase as an algebraic expression and write a phrase for a given algebraic expression.	Write each phrase as an algebraic expression. • 3 less than twice a number of candies in the jar • 2 less than a number plus twice the same number • The amount of remaining time of the exam when 15 minutes of the time completed • If the sum of two numbers is 80 and one of the numbers is m, the other number is Write a phrase for each algebraic expression given below. 5(c-2) 7k-6 (m+1)/2 x/2+5
Evaluate an algebraic expression for different values of variable.	Evaluate each algebraic expression given below for a given value of variables. $\frac{2(n-3)}{5} \quad for\ n = 13$ $\frac{3x+4}{2} \quad for\ x = 6$

Algebra Achievement Test (AAT): AAT included 11 essay type questions with subquestions (35 questions in total) and developed to examine students' algebra achievement and algebraic thinking. Two questions in the test were taken from "Chelsea Mathematics Diagnostic Tests-Algebra" developed by Hart et al. (1985) and adapted to Turkish by Altun (2005). These questions were modified by the researchers. Other questions were developed by the researchers according to the literature and objectives in the curriculum. The test was used to reveal possible differences in EG and CG students' algebra achievement and algebraic thinking based on the use of algebra tiles. It was administered to EG and CG students as a posttest after the treatment allowing 40 minutes. The objectives covered in the AAT, sample questions for each objective, and sample questions about algebraic thinking are given in Table 4.

In addition to questions related to objectives O3, O4, and O5 in the curriculum (see Table 1), there were questions in the AAT that were developed to address students' algebraic thinking in a different way based on their existing mathematics knowledge and skills including the 6th grade algebra objectives. They were related to finding the perimeter of a geometric figure in terms of algebraic expressions, writing given algebraic expressions as multiplication of a natural number and an algebraic expression, comparing algebraic expressions, and finding the length of one side of a geometric figure in terms of algebraic expressions. These questions were based on a combination of students' existing knowledge and skills in geometry, multiplication, comparison of quantities, and recently learned algebraic concepts.

Table 4. The 6th grade objectives (MEB, 2013) covered in the AAT, sample questions for each objective, and sample questions about algebraic thinking

Objectives	Sample Questions
Express the meaning of simple algebraic expressions.	Determine whether given representations are correct or incorrect and rewrite incorrect representations as correct representations. ….. $y + y + 1 = 3y$ ….. $x + x - 1 = - 1 + 2x$ ….. $\frac{a}{2} + \frac{a}{2} = 2a$ ….. $5 - c - c + c = 5 - 3c$
Make addition and subtraction in algebraic expressions.	Perform operations for the algebraic expressions given below. (4x–5) + (–2x+3) (x+3) – (–2x – 1)
Multiply an algebraic expression with a natural number.	 Sena and Kerem think the equivalent algebraic expression of 3(x+4) as shown in the picture. Explain which representation is correct.
Find the perimeter of a geometric figure in terms of algebraic expressions.	Assume that one part of the regular polygon, whose the length of one side is 4 unit and number of the side is unknown, is covered by paper. 

	Find the perimeter of the polygon in terms of algebraic expression.
Compare algebraic expressions.	When you compare 3n and (n+3) algebraic expressions for different values of n, which algebraic expression is greater? Explain.

Both PAKT and AAT, and the rubrics which were prepared by the researchers and used to evaluate and score students' responses in the tests (explained below) were reviewed for validity by two mathematics education researchers, one of whom has an experience in teaching middle school mathematics more than 10 years. They claimed that the questions in these instruments reflected the objectives and the goals of the instruments, as well as the rubrics. Then, PAKT was piloted with 55 7th grade students and AAT was piloted with 52 7th grade students in a non-participant middle school in the same city that the study was conducted. One question was removed from the PAKT because none of the students could write the general rule of sequence algebraically. After the question was removed, the last version of PAKT was presented to the same mathematics education researchers for the final review. The test was ready to administer to the students after their final approval. PAKT had 0.73 and AAT had 0.62 reliability coefficients calculated by Kuder-Richardson (KR-21) formula, which indicated that both tests were reliable (Hinton et al., 2014).

Views about Algebra Tiles Questionnaire (VATQ): VATQ consisted of 5 open-ended questions and developed by the researchers in order to gather students' views about using algebra tiles in group work. Questions were reviewed by mathematics teachers and two mathematics education researchers for validity of the questionnaire. There was no pilot study for VTQA but two 6th grade students who were not in the EG and CG were asked to read the questions and comment on their clarity. VTQA was implemented only to EG students after the treatment and was completed in 20-30 minutes.

Procedure and Treatment

The study was conducted in the two 6th grade classes of one mathematics teacher after the necessary ethical and formal procedures were completed. The teacher was trained by the researcher before the implementation about how to use algebra tiles. Then, the lesson plans of the EG were prepared by the researchers and reviewed by the teacher and the researcher together. The teacher used her own lesson plans for the CG, thereby continuing her regular instruction. She taught the algebra topics in both EG and CG classrooms.

Lessons plans of the EG were prepared according to 5E instructional model including engagement, exploration, explanation, elaboration, and evaluation phases. This model enables students to reconstruct initial concepts through self-reflection and continuous interaction with social and physical environment and they build a more grounded conceptual understanding based on their interpretations of the concepts (Bybee, 1997). The 5E model suited the purposes of the study because it provided a structure for students' use of algebra tiles while collaborating with their peers. Throughout the treatment, three objectives (O3, O4, and O5), were covered in both EG and CG in 7 class hours by the mathematics teacher. PAKT was implemented to both EG and CG before the treatment. When the treatment ended, AAT was implemented to both EG and CG. Additionally, views of EG students were explored by VATQ after the treatment.

Experimental group (EG) treatment: EG lessons employed questioning, discussion, group work, and individual work including algebra tiles. For all of the three objectives, the sequence of engagement, exploration, explanation, elaboration and evaluation phases were followed. In the engagement phase, at the beginning of the lessons, for the first objective, the teacher introduced the algebra tiles to the students. For the second objective, she asked the concept of zero pair in algebra tiles in order to enable students to connect their existing knowledge to the new concepts. For the third objective, the teacher presented students with real life problems in order to puzzle and motivate them for the lesson. Other phases were similar to each other for the three objectives as described below.

In the exploration phase, after the students modelled algebraic expressions and operations with algebraic expressions with the help of algebra tiles in groups of 2-4 students, they showed the same operations by drawing pictures that represent algebra tiles and wrote their work with algebraic notations. In the explanation phase, the teacher went through the process that students involved in the exploration part and made necessary explanations. In the elaboration phase, she got the algebra tiles back from the students, presented activity sheets to the students and asked them to complete the items individually without using algebra tiles. Thus, the students could apply what they have learned. In the evaluation phase, at the end of the lessons, the teacher presented exit cards to the students and asked them to give back to her while leaving the classroom after completing the tasks in exit cards. During the instructions in EG, the researcher did not participate and only observed the class to make sure that treatment was being implemented as in the lesson plans. Table 5 summarizes the treatment in EG.

Table 5. The summary of the treatment in EG

Phase	Explanation
Engagement Phase	Introducing algebra tiles

	Asking the concept of zero pair Presenting real life problems
Exploration Phase	Discovering the rules in operations with algebraic expressions by using algebra tiles as concrete manipulative in groups of 2-4 students
Explanation Phase	Teacher's reviews and explanations about the process
Elaboration Phase	Completing the items in the activity sheet individually without using algebra tiles
Evaluation Phase	Completing the tasks in exit cards

Control group (CG) treatment: Algebra tiles were not used in CG and students had regular instruction where questioning, drill and practice, and individual work took place. The teacher explained the concepts on the board during which she asked some questions to the students. For instance, while teaching subtraction in algebraic expressions, she asked "How do you make subtraction in integers?" to connect subtraction in algebraic expressions and subtraction in integers. After she explained the concepts, she solved some related problems on the board. She then asked the students to solve the problems that she wrote on the board individually. For each problem, one student came to the board to show his/her solution. At the end of the lessons, homework related to the concepts was given to the students. The researcher also observed the CG class to document how the regular instruction was implemented. Students in CG did not work in groups, and they did not work with manipulatives. However, after the treatment ended, activities in the EG were also conducted in CG by using algebra tiles.

Data Analysis

In order to analyze students' answers in PAKT and AAT a rubric for each of the instruments were prepared by the researcher and reviewed by the mathematics teachers and the mathematics education researchers who were involved in the instrument construction process. Students' correct answers were coded as 1 and their incorrect answers were coded as 0 according to the rubrics, and total score from the tests were calculated for all students. To answer the first research question, means and standards deviations of PAKT and AAT of EG and CG were calculated. Table 6 shows the result of Shapiro-Wilk Test conducted to check normality assumption for PAKT. Since the significance values for both groups violated the normal distribution assumption, Mann-Whitney U test was conducted to determine if there was a statistically significant difference between the EG and CG in terms of prerequisite knowledge before the treatment in PAKT.

Table 6. Result of Shapiro-Wilk Test for PAKT

	Statistic	df	Sig.
Experimental Group	0.888	23	0.014
Control Group	0.881	21	0.015

Table 7 presents the result of Shapiro-Wilk test for AAT. Since normality assumption was ensured, independent samples t-test was conducted to compare the scores of EG and CG in AAT after the treatment.

Table 7. Result of Shapiro-Wilk Test for AAT

	Statistic	df	Sig.
Experimental Group	0.977	23	0.858
Control Group	0.930	23	0.154

In order to investigate the second research question, students' responses in AAT were analyzed in-depth and their mistakes, misconceptions and alternative solutions were determined. For the third research question, students' responses in VATQ were read several times carefully and codes representing meaning units and possible upper-level categories that encompassed these codes and represented the data set were identified by the researcher, with examples from data for the codes. Then, both authors discussed these codes, upper-level codes, and code-coded data pairs and finalized the names of the codes. The researcher used these codes to label the responses in VATQ, brought them under the upper-level categories, and discussed the findings with the second author. The upper-level categories were finalized after this discussion. Intercoder reliability measure suggested by Miles and Huberman (1994) was used to calculate the agreement and it was found 92.3%.

Results

In this section, the results of the descriptive and inferential statistics analysis for PAKT and AAT and findings of VATQ will be presented respectively.

Students' Prior Algebra Knowledge

PAKT was administered to 23 students in EG and 21 students in CG as a pretest before the treatment. Descriptive statistics of PAKT scores for both groups is given in Table 6.

Table 6. Descriptive statistics of PAKT scores for EG and CG

	Experimental Group	Control Group
N	23	21

Minimum	0	0
Maximum	15	15
Mean	8.61	6.95
Standard Deviation	5.42	5.56

Table 6 shows that EG students' mean score (M=8.61, SD=5.42) was higher than CG students' mean score in PAKT (M=6.95, SD=5.56). In order to determine whether there was a significant mean difference between the groups before the treatment, Mann-Whitney U Test was performed since normality assumption for independent t-test could not be ensured. The result of Mann Whitney U test for PAKT is presented in Table 7.

Table 7. Result of Mann-Whitney U Test for PAKT

	Mann-Whitney U	Sig.
PAKT	205.500	0.395

$p>0.05$

Table 7 shows that there was no statistically significant mean difference between the groups before the treatment (U=205.500, $p>0.05$). Thus, based on the statistical results, it can be said that EG and CG students had equal prior algebra knowledge.

Students' Algebra Achievement After the Treatment

AAT was implemented to 23 students in EG and 20 students in CG as a posttest after the treatment. Descriptive statistics of AAT scores for both groups is given in Table 8.

Table 8. Descriptive statistics of AAT scores for EG and CG

	Experimental Group	Control Group
N	23	20
Minimum	1	0
Maximum	34	32
Mean	19.65	14.85
Standard Deviation	8.51	10.32

As seen from Table 8, EG students' mean score (M=19.65, SD=8.51) was higher than CG students' mean score in AAT (M=14.85, SD=10.32). In order to investigate the first research question, independent samples t-test was conducted as the data set was suitable for the analysis. Table 9 shows independent samples t-test results of AAT.

Table 9. Results of independent samples t-test for AAT

	Experimental Group		Control Group		t Value	df	p
AAT	Mean	SD	Mean	SD	1.67	41	.10
	19.65	8.51	14.85	10.32			

$p>0.05$

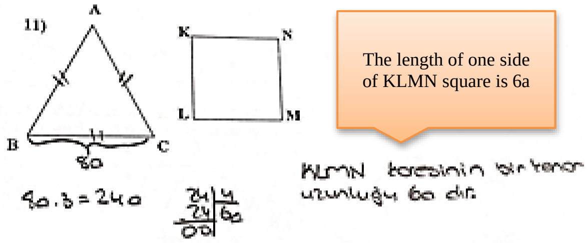
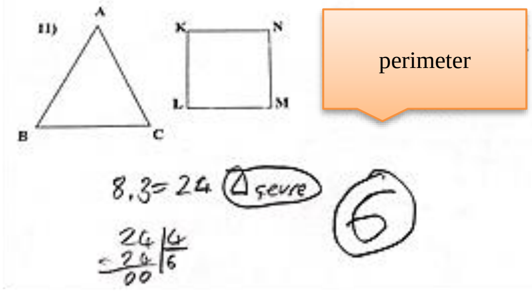
There was no statistically significant mean difference between the groups after the treatment ($t(41)=1.67$, $p= .10$). These results indicated that using algebra tiles in group work did not lead to significantly better results in terms of students’ algebra knowledge and algebraic thinking than regular instruction.

Differences in Algebraic Thinking

In order to investigate the second research question, EG and CG students’ responses to each question in AAT were analyzed in detail. The detailed analysis of the responses revealed that EG students provided more correct answers than CG students for the questions related to the following objectives: determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions; performing addition and subtraction with the given algebraic expressions; writing given algebraic expressions as multiplication of a natural number and an algebraic expression; and finding the length of one side of a geometric figure in terms of algebraic expressions. Representative responses of EG and CG students to the related questions are given in Table 10.

Table 10. Representative responses of EG and CG students to the questions in which EG students performed better

One EG student’s response	One CG student’s response
Perform operations for the algebraic expressions given below.	
i) $(4x-5)+(-2x+3)$ $= 2x-2$	i) $(4x-5)+(-2x+3) = 4x-2$
ii) $(x+3)-(-2x-1)$ $(x+3)+(+2x+1) = 3x+4$	ii) $(x+3)-(-2x-1) = -1x+2$
Write each algebraic expression given below as multiplication of a natural number and an algebraic expression.	
• $6x+8$ $2(3x+4)=6x+8$	• $6x+8$ $6(x+8)$
• $9-3x$ $3(3-x)=9-3x$	• $9-3x$ $3(9-x)$
• $-2x-10$ $2(-x-5)=-2x-10$	• $-2x-10$ $2(-x-10)$

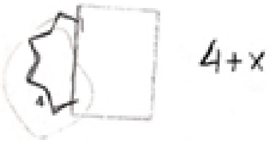
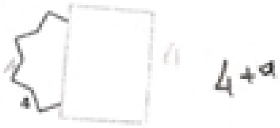
ABC equilateral triangle and KLMN square have equal perimeters. If the length of one side of the triangle is 8a, find the length of one side of the square.	
	

The examples in Table 10 revealed that exploring how to perform operations with algebra tiles in group work helped EG students make addition and subtraction with the given algebraic expressions meaningfully. CG students had difficulties in adding and subtracting variables and in subtracting the negative variable as in the representative response. Moreover, some CG students made operations between unlike terms. When students were asked to write given algebraic expressions as multiplication of a natural number and an algebraic expression, EG students performed considerably better than CG students. Most of the CG students multiplied the number outside the parenthesis by only x inside the parenthesis as in the representative response. These representative responses illustrated that EG students were able to connect their new learning of the algebra concepts with their existing knowledge and skills of arithmetic better than CG students. In the question that asked students to find the length of one side of a geometric figure in terms of algebraic expressions, most of the EG students were able to give the correct answer. However, most of the CG students ignored the variable and wrote only 6 as the answer instead of 6a as in the representative response.

On the other hand, both EG and CG students did not perform well in answering some of the questions, those especially related to finding the perimeter of a geometric figure in terms of algebraic expressions and explaining which algebraic expression is greater. Representative responses of EG and CG students to the related questions are given in Table 11.

Table 11. Representative responses of EG and CG students to the question in which they did not perform well

One EG student's response	One CG student's response
Assume that one part of the regular polygon, whose the length of one side is 4 unit and number of the side is unknown, is covered by paper. Find the perimeter of the polygon in terms of algebraic expression.	

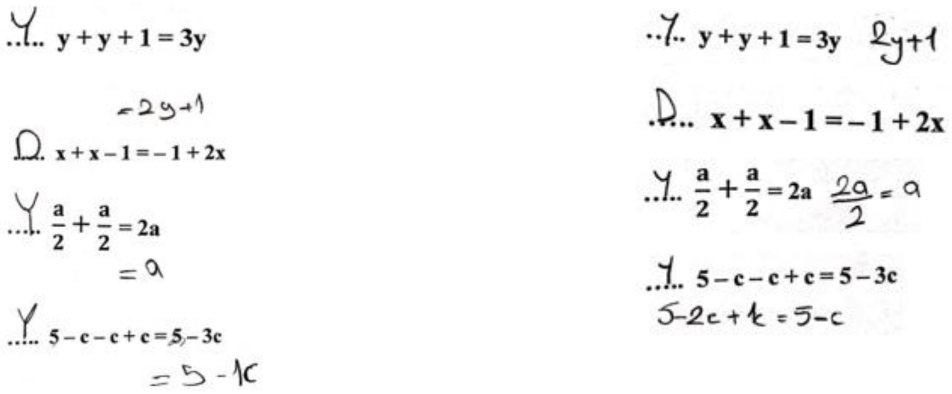
	
When you compare 3n and (n+3) algebraic expressions for different values of n, which algebraic expression is greater? Explain.	
$3.9=27$ $9+3=12$ 3n daha büyük olur. 3n is greater	$3n > (n+3)$ 3n 'de çarpıyoruz (n+3) 'de topluyoruz. We multiply in 3n. We add in (n+3).

As it is seen from the representative responses, while finding the perimeter of a geometric figure whose number of the side is unknown, some students in both EG and CG added 4 and variable rather than multiplying. Also, some students wrote 32 by adding the visible eight sides of the polygon. In the question that asked students to explain which algebraic expression was greater, students generally tended to evaluate algebraic expressions for only one value and decide that one was greater than other according to this evaluation. Also, some students thought that 3n was greater (because of including multiplication operation in it) than n+ 3 which includes addition operation.

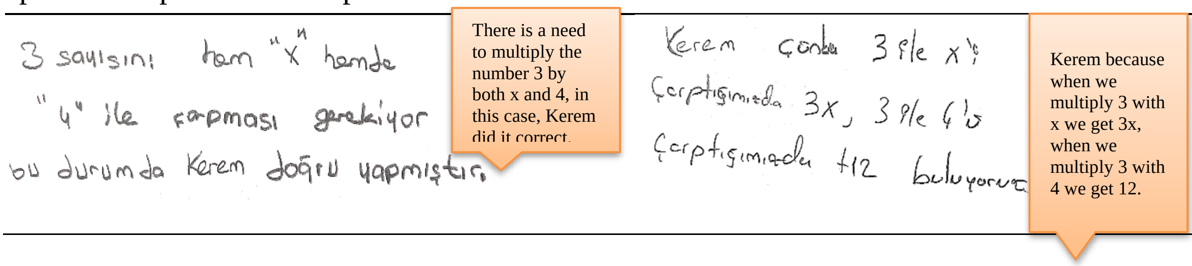
In addition, in two questions related to the representation of algebraic expressions, most of the both EG and CG students could determine whether given representations were correct or incorrect and could explain which representation was correct by using multiplication of a natural number and an algebraic expression. As a different solution than other students, it was seen that one EG student showed the correct representation by assigning an arbitrary value to the x and comparing Sena's and Kerem's responses. Representative responses of EG and CG students to the related questions are given in Table 12.

Table 12. Representative responses of EG and CG students to the question in which they performed well

One EG student's response	One CG student's response
Determine whether given representations are correct (D) or incorrect (Y) and rewrite incorrect representations as correct representations.	



Sena and Kerem think the equivalent algebraic expression of 3(x+4) as shown in the picture. Explain which representation is correct.



Students' Views about Using Algebra Tiles in Group Work

In order to investigate the third research question, EG students' responses to the questions in VATQ were examined and three major categories were identified as using algebra tiles, group work, and enjoyment. Sub-categories and examples related to these categories are presented in Table 13.

Table 13. Categories, sub-categories and examples obtained from VATQ

Category	Sub-category	Example
Using algebra tiles	Effective understanding of algebra	Algebra tiles clarified the concept better and helped me learn the concept better.
		When I saw the operation on the board, I did not understand what to do. With the help of algebra tiles, I understood and comprehended the concept better.
		Algebra tiles enabled me to remember the concept easily.
		Algebra tiles enabled me to perform complicated operation easily.
Group work	Learning easier	With my group mates, we concentrated more and [completed the task] easier altogether. Additionally, I helped my group mates for their understandings.
		Group work helped me understand things that I did not understand before.
		Group work made understanding easier for all of us.

Enjoyment	Enjoying the lesson	By using algebra tiles, we both made activity and had fun. I learned with fun, I did not get bored.

Findings showed that both algebra tiles and group work aspects of the EG courses were enjoyable for the students. There were responses combining both aspects of the EG lessons. Some students remarked on using algebra tiles in group work in the way that using them together eased their understanding. The following excerpt illustrates this:

"In group work, students who understood explained to those who did not understand. Before the group work, we were confused about what to do. We had some questions such as "How will we do?" in our minds. However, when we used algebra tiles, we understood immediately."

Enjoyment appeared in data also in relation to using algebra tiles and working in groups. Some students made positive comments on the lessons where algebra tiles were used as illustrated below:

"While using algebra tiles, we did not only have fun, but we also comprehended topic. I have already liked mathematics, now, I began to like much more."

Discussion and Conclusion

The analyses documented above indicated that although there was no statistically significant mean difference between the groups, EG had a higher mean score than CG in AAT. It can be said that using algebra tiles in group work might have some effect on difference in scores. This result is consistent with the result of previous studies which reported similar findings for different grade levels and algebraic concepts (Castro, 2017; Schlosser, 2010; Sharp, 1995; Wingett, 2019). The difference could be the result of students' increased motivation to respond to the questions due to their use of algebra tiles. The non-significant result could be related to allocating only 7 class hours according to the curriculum to teach algebra concepts with algebra tiles. It was not possible to extend the time for the implementation because of the curriculum requirements. Otherwise, when students are engaged with algebra tiles longer, their learning is significantly positively affected (Larbi & Okyere, 2016). In addition, EG students stated that they have only used counters and fraction tiles as manipulatives in the mathematics lesson before. They used algebra tiles for the first time in this study. This non-significant result might be due to students' limited exposure to manipulatives. If these students had used more manipulatives in mathematics lessons, it would be possible that they would benefit more from the treatment. It is argued that students perform better when they use multiple materials while learning algebra (Koğ & Başer, 2012). Therefore, for meaningful learning and better algebraic thinking, instruction should involve more manipulatives.

In terms of algebraic thinking, there were qualitative differences in students' responses in approximately half of the questions in AAT favouring EG. Use of algebra tiles in group work in lessons could lead to this qualitative difference between EG and CG students. EG students performed better in analysing the given algebraic expressions and determining their parts; performing operations in algebraic expressions meaningfully; writing given algebraic expressions as multiplication of a natural number and an algebraic expression and finding the length of one side of a geometric figure in terms of algebraic expressions. It can be said that interaction with algebra tiles in group work in the form of concrete manipulatives has some positive effect on students' algebraic thinking (as indicated by the mean scores) in the present study confirming the findings where middle school students established meaningful connections in algebraic thinking by using manipulatives (Chappell & Strutchens, 2001).

The use of algebra tiles did not make a difference in EG students' responses to the questions that included both algebra and geometry concepts. Students' low performance in these questions can be due to the lack of their prior knowledge in geometry concepts. In addition, both EG and CG students might not be familiar with these kinds of questions, which could result in students' low performance. This might, in a limited sense, also reveal that when the students were asked to combine their knowledge and skills of different concepts to solve a question, they might not always perform well. However, this is rather an inference and should be investigated in further studies. As a result, it can be concluded that using algebra tiles in group work had limited but qualitatively positive effect on the students' algebraic thinking.

Students are reported to have positive views and state meaningful learning outcomes when they use algebra tiles (Schlosser, 2010; Sharp, 1995; Thornton, 1995). Similarly, in this study, although statistically significant difference could not be found between the groups, EG students stated that they learned and understood algebraic expressions better and faster with the help of algebra tiles in their responses in VATQ. Findings showed that some students were aware that algebra tiles helped them build conceptual connections, which helped them understand the concepts better.

EG students were exposed to group work while exploring the concepts with algebra tiles. However, this was the first time for the students working in a group in the mathematics lesson as the teacher indicated. Although group work has positive effects on students' learning (Koblitz & Wison, 2014), these effects might not be seen because of the limited exposure to group work in this study. Students in EG asserted that group work made their understanding easier confirming previous studies (Balt, 2017). Group work facilitates students' noticing of other group members' understanding, and enables students to help their friends and deepen their own knowledge by explaining the concept (Webb & Farivar, 1994). This was the case in the present study and some students mentioned that they helped their group mates for their understanding in the group work. EG students also expressed enjoyment for working with algebra tiles in group work, which has been reported elsewhere (Jansen, 2012; Koçak et al., 2009; Mulryan, 1994; Sofroniou & Konstantinos, 2016).

We would like to make cautious speculations about the relationship between the nature of the implementation and the findings here. The major input in teaching algebra concepts in this study was the algebra tiles whereas the group work provided an idea-rich context for students to make sense of the algebraic concepts. This might reveal questions about what really affected EG students' responses in AAT. The findings seemed to be due to the combined effect of the algebra tiles and group work. However, the group work was not designed to be structured during the implementation. Therefore, while we indicate that the findings could be attributed to the combined effect, we also address that algebra tiles might have more influence on the findings in this combination. Yet, there is a need to investigate both the combined and individual effects of the group work and algebra tiles.

There was a change in the curriculum at the time of the study and the 6th grade students in the following year did not learn about addition and subtraction of algebraic expression, and multiplication of an algebraic explanation with a natural number. A follow-up study could be to conduct a similar study in the 7th grade and see if there will be differences, which would provide curriculum developers with a feedback for the changes in the grade level of related objectives. Curriculum changes take place quite frequently in Turkey, which brings difficulties for the researchers who investigate teaching at schools in terms of justifying the significance of their ongoing studies and recent findings. This study emphasized both the objectives in the curriculum and a combination of students' existing knowledge and skills with their recent algebra knowledge and skills, and addressed them as algebraic thinking. Working with such focus is valuable because it provides an evaluation of and feedback for the current curriculum to the teachers and curriculum developers; and justify the findings of the study regardless of the changes. This study provided some findings for to what extent students could combine their existing knowledge with a new knowledge for the teachers. We suggest that further studies based on teaching in classrooms in frequently changing education systems could focus on more overarching constructs.

It should be kept in mind that the findings were limited to the conveniently selected schools and the teacher, and students who participated in the study. The implementation was limited by the designated lesson hour for the topic in the national curriculum. Further studies should consider these limitations. It can be recommended that future studies would include one group using algebra tiles in group work, one group using algebra tiles without group work, and one group without using algebra tiles and group work to increase the generalizability and to determine the effectiveness of algebra tiles with or without group work. Furthermore, interviews can be conducted with the students to examine their algebraic thinking and views in-depth.

In this study, we tried to address that the initial algebra topics could be taught by the algebra tiles, which are easy to produce within a limited time frame available for these topics in the curriculum. Our findings indicated that using algebra tiles in group work even for a short time has some promising effects on students who learn algebra for the first time. The students also mentioned that they liked using algebra tiles while learning algebraic expressions and were happy to work in groups while using the algebra tiles. Therefore, we recommend that teachers integrate algebra tiles to the mathematics lessons in the beginning of the algebraic concepts to enhance students' algebraic thinking and achievement. Familiarity of the students with the group work might increase the effectiveness of using the algebra tiles.

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Grup Çalışmasında Cebir Karoları Kullanılması Durumu: Öğrencilerin Cebir Başarısı, Cebirsel Düşünmeleri ve Görüşleri

Özet:

Bu çalışmada grup çalışmalarında cebir karosu kullanımının öğrencilerin cebir başarısını, cebirsel düşünmelerini ve cebir karosu kullanımına ilişkin görüşlerini nasıl etkilediği incelenmiştir. Çalışma 40 altıncı sınıf öğrencisinin katıldığı öntest-sontest kontrol gruplu deneysel desen ile gerçekleştirilmiştir. Deney grubu öğrencileri gruplar halinde cebir karolarını somut materyal olarak kullanırken, kontrol grubu öğrencileri herhangi bir somut materyal kullanmamış, öğretmen olağan dersini yapmıştır. Her iki gruba da Cebir Ön Bilgi Testi öntest olarak ve Cebir Başarı Testi sontest olarak uygulanmıştır. Deney grubu öğrencileri Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu ile görüşlerini belirtmişlerdir. İstatistiksel analizler her iki testte de öğrencilerin performansları arasında anlamlı bir fark göstermemesine rağmen, nitel analizler cebir karoları kullanımının deney grubu öğrencilerinin cebirsel düşünmeleri üzerinde olumlu bir etkisi olduğunu ortaya koymuştur. Ayrıca, deney grubu öğrencileri grup çalışmalarında cebir karoları kullanımının öğrenmelerini desteklediğini, kavramları anlamlı bir şekilde anlamalarını sağladığını ve dersleri daha eğlenceli hale getirdiğini ifade etmişlerdir.

Anahtar kelimeler: cebir başarısı, cebir karoları, cebirsel düşünme, grup çalışması, orta okul öğrencileri, öğrenci görüşleri