The Comparison of Velocity Formulas Used in Stormwater and Sewage Design

Tulin Cetin[1]*, Mehmet Ali Yurdusev[2]

[1]Celal Bayar Üniversitesi, Mühendislik Fakültesi, İnşaat Mühendisliği Bölümü, Muradiye, Manisa, +90 236 2012337, tulin.cetin@cbu.edu.tr
[2]Celal Bayar Üniversitesi, Mühendislik Fakültesi, İnşaat Mühendisliği Bölümü, Muradiye, Manisa, +90 236 2012308, yurdusev@cbu.edu.tr
*Corresponding author / İletişimden sorumlu yazar

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Abstract

The diameter and the slope of the pipe for the desired discharge value are calculated in stormwater / sewage systems design by using different velocity formulas. In Turkey, governmental offices that prepare contracts for stormwater and sewage collection systems prefer and use different velocity formulas. In this study, the effects of using such formulas on system characteristics are investigated in an attempt to provide guidance for system designers.

Keywords — Stormwater, sewage, velocity formulas

1 Introduction

Urban stormwater and sewer collection systems are one of the most important areas of civil engineering. With developing technology and changing understanding of issues, human health and safety have become the most significant issues for all countries in the world as it is all agreed that people deserve to live in a healthy environment [1].

Waste water should be collected and taken directly to a treatment plant with gravity and / or reverse flow in urban areas. Then the treated water is discharged to the natural environment. The waste water influences directly the human health because of its polluted nature. Beyond this direct hazardous affect, the waste water also leads to groundwater pollution if mixed into it.

The storm water should also be collected and discharged to an available discharge point not to affect lives negatively. With the absence or inappropriate construction of storm water systems, living conditions become hard, damages to properties and even loss of lives occur.

The most important issue of designing storm–sewer systems is to define pipe diameters and burying depths that means to determine slopes. While finding the pipe diameter and slope that could allow debris determined to flow, there are some issues that should be taken into account. The most important one is the velocity of the discharge which should be within the permissible limits. To make sure that the velocity is within the limits, changes in pipe diameters and slopes could be performed.

There are several formulas available to calculate the velocity of the discharge. The well-known ones of these formulas include Manning, Kutter, Prandtl-Colebrook, Hazen-Williams and Darcy-Weisbach [2]. In Turkey, governmental offices that prepare contracts for stormwater and sewage systems prefer using different velocity formulas. For instance, IllerBank [3] chooses Kutter formula while ISKI [4] uses Prandtl-Colebrook formula.

Kavvas conducted a research into velocity changes in a pipe designed to flow with the recommended minimum velocity [5]. But in his study, he tried only a few diameter and slope values without considering the velocity formula is used. Authors of this study were presented a paper in the same topic with only limited comparisons in VII. National Hydrology Congress [2]. In our study, an attempt has been made to investigate the effects of using such formula on system characteristics to provide guidance for system designers.

2 Methods

Hydraulic calculations of stormwater and sewage systems are undertaken using the formulas for open channel hydraulic. At the beginning, the velocity calculation assuming the pipe is full is performed and then the full flow is calculated [6]. The discharge is defined as follows :

$$Q_D = V_D \times A \tag{1}$$

where QD is the full of flow, VD is the velocity of full flow and A is the cross sectional area of the pipe.

To find the velocity of the full flow, mostly used velocity formulas are given below :

Manning : $V = \frac{1}{n} R^{2/3} J^{1/2}$ (2)

Kutter : $V = \frac{100R\sqrt{J}}{m+\sqrt{R}}$ (3)

Prandtl-Colebrook :

$$V = -2\mathrm{Log}\left(\frac{2{,}51v}{\mathrm{D}\times\sqrt{2\mathrm{gDJ}}} + \frac{k/D}{3{,}71}\right) \times \sqrt{2\mathrm{gDJ}} \tag{4}$$

Hazen-Williams : $V = 0.849 C_{hw} R^{0.63} J^{0.54}$ (5)

Darcy-Weisbach : $J = \frac{fQ^2}{8gRA^2}$ (6)

Wetted perimeter : $P = D\cos^{-1}\left(1 - 2\,{}^{d}/_{D}\right)$ (7)

Parameters used in formulas above are as follows :

V : Velocity
R : Hydraulic Radius
D : Diameter
d : Water height of partially full section
J : Slope
n : Manning roughness coefficient
m : Kutter roughness
ν : Kinematic viscosity
k : Prandtl-Colebrook roughness coefficient
g : Gravitational acceleration
C_{hw} : Hazen-Williams friction coefficient
ε: : Darcy-Weisbach roughness height
f : Friction factor
P : Wetted perimeter

ASCE Task Force recommends the following formula for the friction factor of Darcy-Weisbach formula to ease the calculation [7-9].

$$f = 1.325\left[ln\left(\frac{\varepsilon}{12R} + \frac{0.625v}{VR\sqrt{f}}\right)\right]^{-2} \tag{8}$$

Swamee [10] redefined the formula in Equation.8 as Equation.9,

$$f = \frac{4}{3}\left[ln\left(\frac{\varepsilon}{12R} + 1.63\left(\frac{vP}{Q}\right)^{0.9}\right)\right]^{-2} \tag{9}$$

Substituting the friction factor defined in Equation.9 into Equation.6, the Darcy-Weisbach formula becomes,

$$J = \frac{Q^2}{6gA^2R}\left[ln\left(\frac{\varepsilon}{12R} + 1.63\left(\frac{vP}{Q}\right)^{0.9}\right)\right]^{-2} \tag{10}$$

In this study, for the calculations of Darcy-Weisbach formula obtained given in the Equation.10 is used.

The discharge rate of the pipe (Q/Q_D) is calculated using the full flow and the real flow [6].

$$Q/Q_D \tag{11}$$

where Q defines the flow inside the pipe.

To ease the calculations pre-prepared tables like the one given in Table 1 are used. Using the Table 1, Q/Q_D is found and then the pipe discharge rate (d/D) and velocity rate (V/V_D) are calculated with interpolation.

The velocity value of the full flow (V_D) and velocity rate (V/V_D) is multiplied, and the real velocity of the flow (V) inside the pipe is found. With these values and the accepted value of the pipe diameter, the pipe is checked if it fits for the flow [6]. The real velocity formula is given as follows :

$$V = V_D \times {}^{V}/_{V_D} \tag{11}$$

Table 1. Discharge, discharge rate and velocity rates of partially full flow

d/D	V/V_D	Q/Q_D		d/D	V/V_D	Q/Q_D
0,01	0,089	0,000		0,44	0,944	0,400
0,02	0,141	0,001		0,45	0,954	0,417
0,03	0,184	0,002		0,46	0,964	0,433
0,04	0,222	0,003		0,47	0,973	0,450
0,05	0,257	0,005		0,48	0,983	0,466
0,06	0,289	0,007		0,49	0,991	0,483
0,07	0,319	0,010		0,50	1,000	0,500
0,08	0,348	0,013		0,51	1,007	0,516
0,09	0,375	0,017		0,52	1,013	0,532
0,10	0,401	0,021		0,53	1,019	0,548
0,11	0,426	0,025		0,54	1,025	0,564
0,12	0,45	0,031		0,55	1,030	0,581
0,13	0,473	0,036		0,56	1,035	0,597
0,14	0,495	0,042		0,57	1,040	0,613
0,15	0,517	0,049		0,58	1,045	0,628
0,16	0,538	0,056		0,59	1,049	0,644
0,17	0,558	0,063		0,60	1,053	0,660
0,18	0,577	0,071		0,61	1,057	0,675
0,19	0,597	0,079		0,62	1,060	0,691
0,20	0,615	0,088		0,63	1,063	0,706
0,21	0,633	0,097		0,64	1,066	0,721
0,22	0,651	0,106		0,65	1,068	0,735
0,23	0,668	0,116		0,66	1,070	0,749
0,24	0,684	0,126		0,67	1,072	0,764
0,25	0,701	0,137		0,68	1,073	0,777
0,26	0,717	0,148		0,69	1,074	0,791
0,27	0,732	0,159		0,70	1,075	0,804
0,28	0,747	0,171		0,71	1,075	0,816
0,29	0,762	0,183		0,72	1,075	0,829
0,30	0,776	0,196		0,73	1,075	0,841
0,31	0,790	0,209		0,74	1,074	0,852
0,32	0,804	0,222		0,75	1,073	0,864
0,33	0,814	0,235		0,76	1,072	0,874
0,34	0,830	0,249		0,77	1,071	0,885
0,35	0,843	0,263		0,78	1,069	0,894
0,36	0,855	0,277		0,79	1,067	0,904
0,37	0,868	0,292		0,80	1,064	0,913
0,38	0,879	0,307		0,81	1,062	0,921
0,39	0,891	0,322		0,82	1,059	0,929
0,40	0,902	0,337		0,83	1,056	0,937
0,41	0,913	0,353		0,84	1,053	0,944
0,42	0,924	0,368		0,85	1,050	0,951
0,43	0,934	0,384		0,86	1,046	0,957
d/D	V/V_D	Q/Q_D		d/D	V/V_D	Q/Q_D
0,87	1,043	0,963		0,93	1,020	0,989
0,88	1,039	0,968		0,94	1,016	0,991
0,89	1,035	0,973		0,95	1,013	0,994
0,90	1,031	0,978		0,96	1,009	0,996
0,91	1,028	0,982		0,97	1,006	0,997
0,92	1,024	0,985		0,98	1,004	0,999

Hydraulic definitions used for the partially flow calculation is shown in the Figure.1. As can be seen from the Figure.1, d is the water depth, D is the diameter and d/D is pipe discharge rate [6].

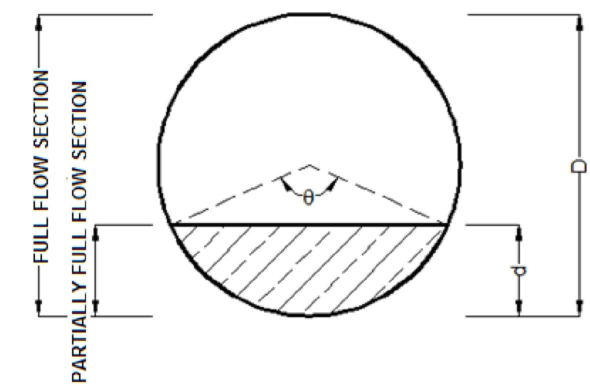

Figure 1 Hydraulic definitions of partially full flow [6]

Hydraulic definitions used for the partially flow calculation is shown in the Figure.1. As can be seen from the Figure.1, d is the water depth, D is the diameter and d/D is pipe discharge rate [6].

2.1 Evaluation of Velocity Formulas - The Software Used

The software is prepared to investigate the effects of velocity formulas for the hydraulic calculations of storm water and sewage systems. The general structure of the program is given in Figure.2.

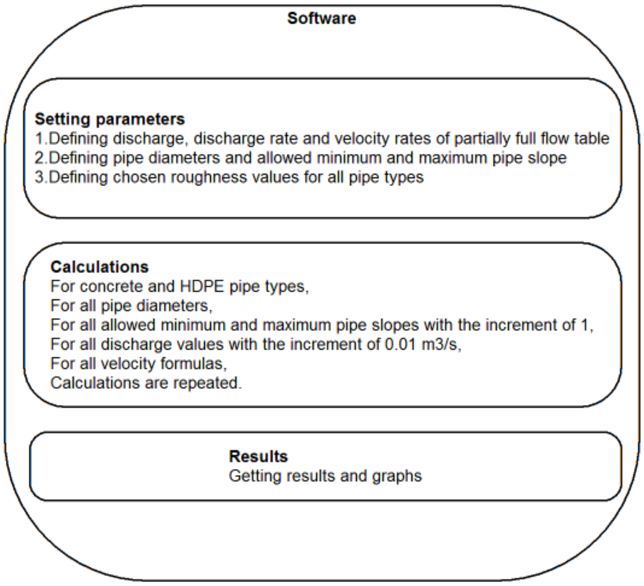

Figure 2 The general structure of the program

The five velocity formulas mentioned previously have been included in the software. Roughness values according to pipes are given in the Table.2. These values are taken from catalogues of various concrete and HDPE pipe fabricators.

Table 2. Chosen roughness values according to pipe types

Roughness values	Concrete pipe	HDPE pipe
n	0.014	0.008
m	0.35	0.1
k (mm)	1.5	0.02
C_{hw}	130	150
ε	0.36	0.02

Pipe diameters and their slope limits used in the software is given in the Table.3

Table 3. Pipe diameters and slope limits used in the software

Diameter (mm)	Min. Slope (1/)	Max. Slope (1/)
200	300	7
300	300	7
400	500	15
500	500	15
600	1000	50
800	1000	50
1000	2000	75

For the each value of diameters given in Table.3, each integer values between minimum and maximum permitted slope values are used in calculations. At each diameter the value of the discharge inside the pipe are taken as minimum value of 0.01 m³/s. With the extend of the pipe capacity in each calculation, accepted flow is increased by 0.01 m³/s.

Minimum and maximum velocity values are not taken into consideration for the purpose of not limiting the results. Therefore some velocity values are greater than the maximum permitted velocity values in some results. It is aimed to see the general trend. Therefore exceeding velocity limits is not considered.

3. The comparisons of the results according to velocity formulas

Several illustrative graphs are prepared from results obtained by running the software. As there are so many results obtained from the software, the graphs for specific diameter, slope and discharge (flow) values are shown only.

Firstly, concrete and HDPE pipes are compared. In each formula and under the same conditions, HDPE pipes leads to have faster flow and therefore have more flow in case of having low roughness. So there are similar results at each formulas, for the sake of illustration the Manning results are shown in Figure.3 for velocity versus discharge and in Figure.4 for velocity versus slope.

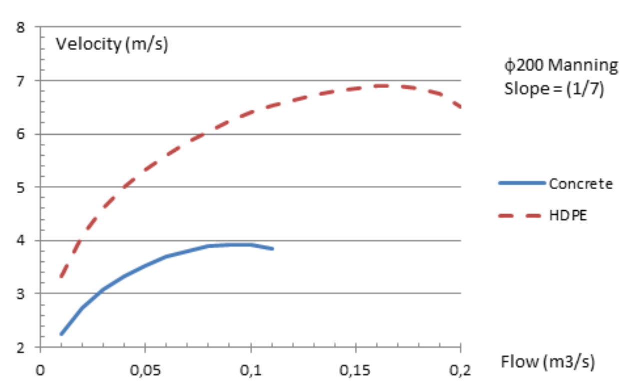

Figure 3 Velocity calculated with Manning formula versus discharge of the pipe of ϕ200 with slope 1/7

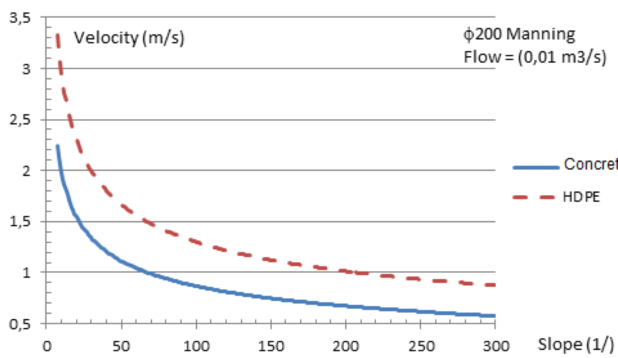

Figure 4 Velocity calculated with Manning formula versus slope of the pipe of ϕ200 having discharge of 0,01 m^3/s

In Figure 4, it is shown that having the same diameter value, HDPE pipes have faster flow according to concrete pipes. As the slope is getting steeper, the velocity of the flow inside the pipe increases geometrically while the slope becomes flat, velocity decreases and the rate of the decrease is lower.

Discharge rate (flow rate) versus slope graphic is shown in Figure 5. For all formulas, all diameters and all discharge values, concrete pipes have more discharge rate while HDPE pipes have less at the same slope. As the slope is getting more flat, the discharge rate of the discharge increases with a lower increasing rate.

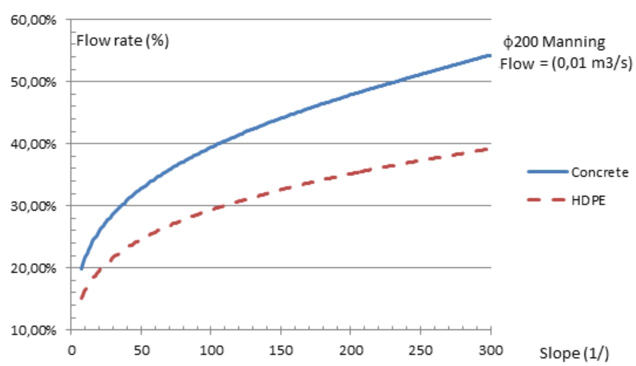

Figure 5 Discharge rate (velocity calculated with Manning formula) versus slope of the pipe of ϕ200 having discharge of 0,01 m^3/s

For ϕ200 concrete pipe with a slope of 1/7 which is the maximum permitted slope value for this diameter, velocity versus discharge graphs for all formulas are shown in Figure 6. For all formulas after a certain discharge value, the velocity, inside the pipe decreases. For ϕ200 concrete pipe, Hazen-Williams formula gives the maximum velocity while the Kutter gives the minimum value.

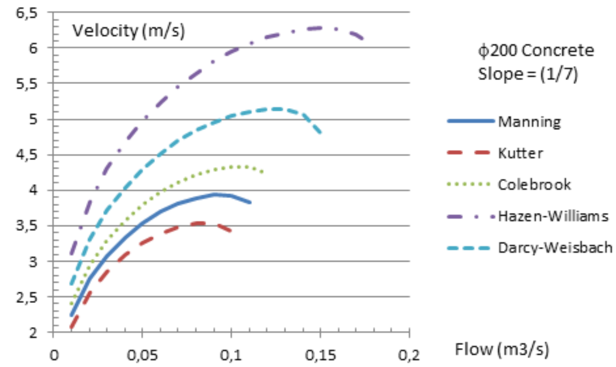

Figure 6 Velocity calculated with different formulas versus discharge for ϕ200 concrete pipe having slope 1/7

The case for concrete pipes of all diameters between ϕ200 and ϕ1000 with maximum permitted slope is investigated. The results are presented in Figure.7 and Figure.8. Smaller diameters including ϕ400 Kutter formula give less velocity values, at the diameter ϕ500 Manning and Kutter formulas give similar velocity values as shown in Figure 7, while for ϕ600 and bigger diameters Manning formula give the minimum velocity value as shown in Figure 8. For all diameters of concrete pipes, Hazen-Williams formula gives the maximum velocity value.

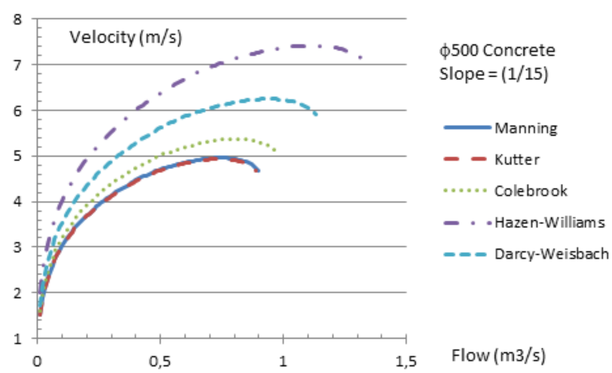

Figure 7 Velocity calculated with different formulas versus discharge for ϕ500 concrete pipe having slope 1/15

For the same conditions at other slopes, similar results are obtained.

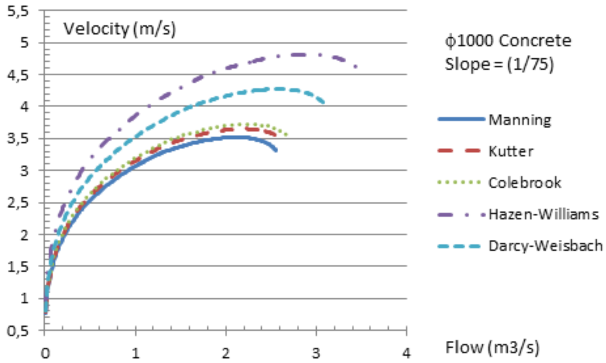

Figure 8 Velocity calculated with different formulas versus discharge for ϕ1000 concrete pipe having slope 1/75

For the purpose of comparing velocity versus discharge rate for concrete pipes, Figure 9 is prepared. For all diameters, all slope values and all velocity formulas of concrete pipes, velocity increases till discharge rate of approximately 70% while discharge rate increases. After this value of discharge rate, the velocity inside the pipe starts decreasing.

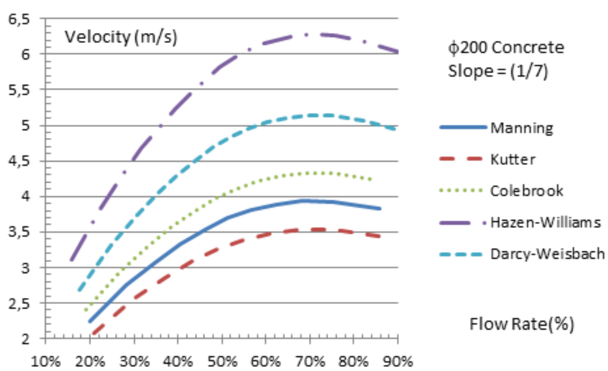

Figure 9 Velocity calculated with different formulas versus discharge rate for ϕ200 concrete pipe having slope 1/7

Under the same conditions for HDPE pipes, Kutter formula gives less velocity values for all diameters. For the diameter of ϕ200 graph as shown in Figure 10 is given as an example. For diameters between ϕ200 - ϕ400, Hazen-Williams formula gives the maximum velocity values as can be seen from Figure 10 and 11. Manning formula gives more velocity values while diameter is increasing. For HDPE pipe with the diameter of ϕ400 Manning and Hazen-Williams formulas give approximately same velocities.

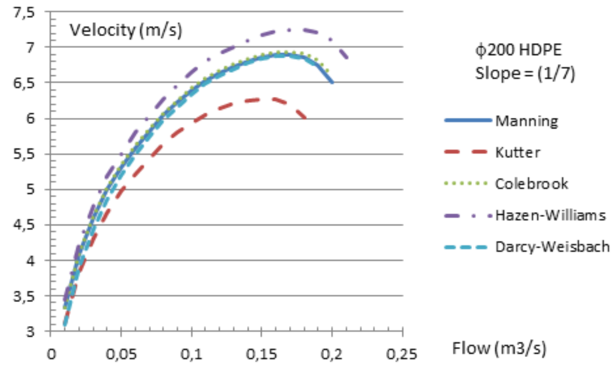

Figure 10 Velocity calculated with different formulas versus discharge for ϕ200 HDPE pipe having slope 1/7

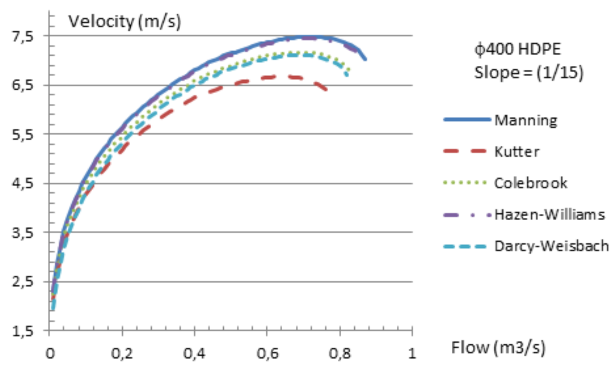

Figure 11 Velocity calculated with different formulas versus discharge for ϕ400 HDPE pipe having slope 1/15

Manning formula gives the maximum velocity value for the diameters of ϕ500 and greater as shown in Figure 12.

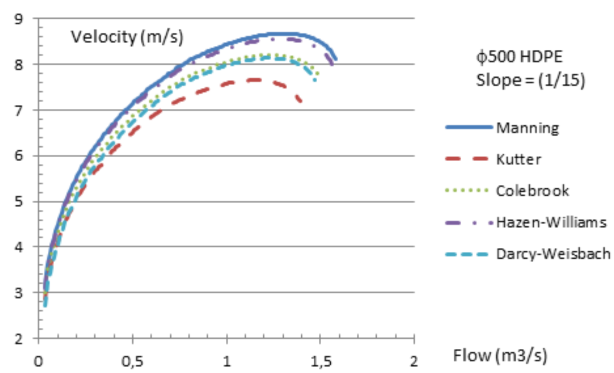

Figure 12 Velocity calculated with different formulas versus discharge for ϕ500 HDPE pipe having slope 1/15

For HDPE pipe of ϕ1000, the difference between Manning and Hazen-Williams velocity values increases as shown in Figure 13. Prandtl-Colebrook and Darcy-Weisbach velocity formulas give similar results to each other for all diameters of HDPE pipes.

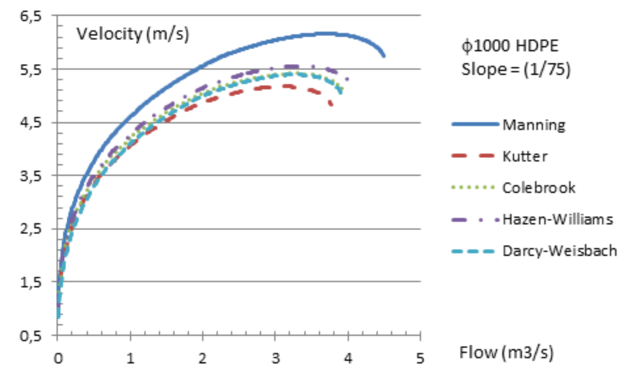

Figure 13 Velocity calculated with different formulas versus discharge for ϕ1000 HDPE pipe having slope 1/75

To compare velocity and discharge rate for HDPE pipes, Figure 14 is prepared. As for the concrete pipes, for all diameters and all formulas of HDPE pipes, the velocity inside the pipe decreases after having discharge rate of approximately 70%.

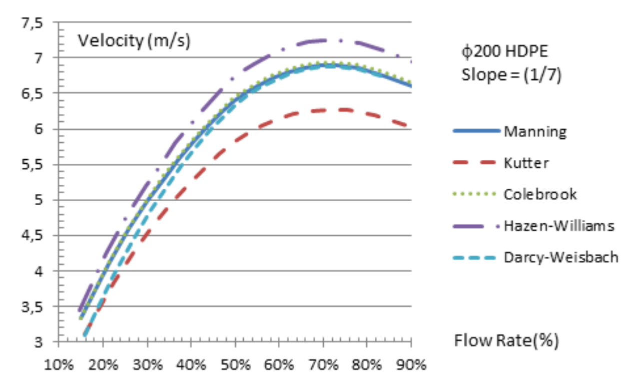

Figure 14 Velocity calculated with different formulas versus discharge rate for ϕ200 HDPE pipe having slope 1/7

For the discharge of 0.01 m³/s that is the minimum discharge value exercised in this study for the concrete pipe of ϕ200, discharge rate versus slope is shown in Figure 15. As can be seen from this figure, at the steeper slopes, the increase in the discharge rate value is higher. When the slope becomes more flat than the slope of 1/50, the increase in the discharge rate becomes stable.

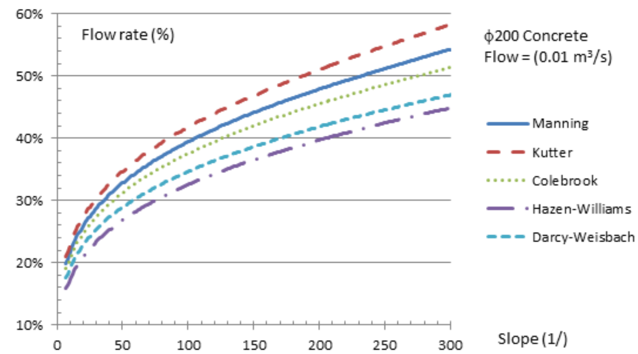

Figure 15 Discharge rate calculated with different formulas versus slope for ϕ200 concrete pipe having flow 0.01 m³/s

Same conditions for the discharge value of flow of value 0.07 m³/s are shown in Figure 16. For higher flow, discharge rate increases geometrically with respect to slope. For the same flow value, the diameter could be insufficient for the flow inside when the slope is decreasing.

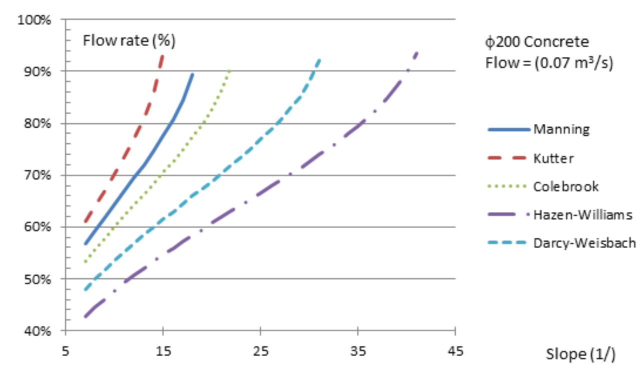

Figure 16 Discharge rate calculated with different formulas versus slope for ϕ200 concrete pipe having flow 0.07 m³/s

For ϕ200 HDPE pipe with discharge value of 0.01 m³/s which is the minimum flow value considered in this study, discharge rate versus slope graph is shown in the Figure 17. It can be seen from this Figure that HDPE pipes act similar to concrete pipes at steeper slopes. At steeper slopes, the increase in the discharge rate is higher, while the increase in the discharge rate becomes stable for more flat slopes than 1/50.

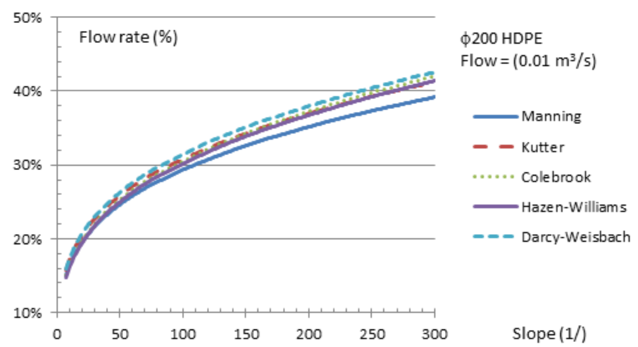

Figure 17 Discharge rate calculated with different formulas versus slope for ϕ200 HDPE pipe having flow 0.01 m^3/s

Same conditions are observed for discharge value of 0.07 m^3/s and associated results are shown in Figure 18 as at steeper slope velocity is higher while in more flat slope velocity is lower.

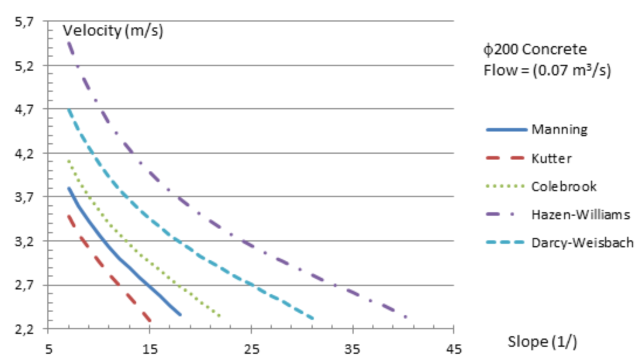

Figure 18 Velocity calculated with different formulas versus slope for ϕ200 concrete pipe having flow of 0.07 m^3/s

All these conditions mentioned in this study are also tried and have yielded similar results for other diameters and pipe material. For example velocity-flow graph, velocity-slope graph and discharge rate-slope graph for Manning formula, only the diameter value of ϕ200, the slope value of 1/7 and the discharge value of 0.01 m^3/s are only illustrated in this study while Kutter, Prandtl-Colebrook, Hazen-Williams and Darcy-Weisbach formulas, diameter values other than ϕ200, the slope value other than 1/7 and the discharge value other than 0.01 m^3/s are not illustrated. The velocity-flow graph for concrete pipes are illustrated only for diameter values of ϕ200, ϕ500 and ϕ1000 and for the minimum allowed slope, while graphs for diameter values of ϕ200, ϕ500 and ϕ1000 other slope values than the minimum allowed slope and the diameter values of ϕ300, ϕ400, ϕ600 and ϕ800 are not presented in this study. The same situation is also valid for HDPE pipes. Velocity - discharge rate graphs for concrete and HDPE pipes are illustrated only for the diameter value of ϕ200 and for the minimum allowed slope, while the graphs for the diameter value of ϕ200 slope values other than the minimum allowed slope and diameter values of ϕ300, ϕ400, ϕ500, ϕ600, ϕ800 and ϕ1000 are not presented in this study. Discharge rate - slope graphs for concrete pipes are illustrated only for the diameter value of ϕ200 and only for discharge values of 0.01 and 0.07 m^3/s, while the graphs for the diameter value of ϕ200 discharge values other than 0.01 and 0.07 m^3/s and diameter values of ϕ300, ϕ400, ϕ500, ϕ600, ϕ800 and ϕ1000 are not illustrated. For the HDPE pipes, discharge rate - slope graphs are illustrated only for the diameter value of ϕ200 and only for the discharge value of 0.01 m^3/s. For velocity - slope graphs, only for concrete pipes, the diameter value of ϕ200 and the discharge value of 0.07 m^3/s is illustrated.

4 Results

To investigate the effects of Manning, Kutter, Prandtl-Colebrook, Hazen-Williams and Darcy-Weisbach formulas on the sewage and stormwater hydraulic calculations a software is developed. Using the necessary roughness values for concrete and HDPE pipes, for several diameters, slopes and discharge values, velocity and discharge rate values are calculated.

Firstly concrete and HDPE pipes are compared. It is seen that, HDPE pipes have more flow and cause faster flow as their roughness values are better.

At steeper pipe slopes, the velocity inside the pipe increases geometrically. While the slope becomes more flat, velocity decreases and the decrease rate is also lower. The discharge rate at steeper slopes is higher. While slope becomes flat, the increase rate of discharge rate becomes stable.

Hazen-Williams formula gives maximum velocity value for concrete pipes. For smaller diameters Kutter formula, for larger diameters Manning formula give the minimum velocity value. For HDPE pipes, Prandtl-Colebrook and Darcy-Weisbach formulas give similar velocity values, Kutter formula gives the minimum for small diameters Hazen–Williams formula for larger diameters Manning formula give the maximum velocity values.

It is true for all velocity formulas that the velocity inside the pipe, whether it is concrete or HDPE type, decreases for all diameters after the discharge rate of 70% and more. The findings of this study have shown that the type of the formula used affect the results depending on the pipe material and the diameter used. It is hoped that these suggestions will help designers use the formulas wisely and produce better designs.

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