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Araştırma Makalesi/Research Article

Adaptive Spiral Optimization Algorithm for Benchmark Problems

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Abstract- **In this study, Spiral Optimization Algorithm (SOA) that is one of the heuristic algorithms was improved by the self-adaptive concept. Adaptive Spiral Optimization Algorithm (ASOA) includes the self-adaptive structure to adjust the spiral radius and spiral angle values that are the parameters of SOA during the optimization. Three different ASOA versions were proposed in this paper. To evaluate the performance of the ASOA's versions, five benchmark optimization problems were taken from the literature. The proposed ASOA versions are more successful than classic SOA according to the mean best value and NFE indicators.**

***Keywords*- Spiral Optimization, Self-Adaptive, Benchmark.**

I. INTRODUCTION

In recent years, the heuristic algorithms have become popular optimization algorithms to overcome the nonlinear and complex problems [1-3]. These algorithms are inspired by biological or natural events. Some of the heuristic algorithms include firefly optimization algorithm, cuckoo optimization algorithm, bat algorithm, fruit fly optimization algorithm, crab mating optimization algorithm, flower pollination algorithm and spiral optimization algorithm [4]. The SOA is a heuristic algorithm inspired by spiral motion in nature. The basic characteristic of SOA is based on the dynamic step size in its spiral trajectory. At the beginning of the optimization, the step size is large value and it becomes less at near the optimum point that is always placed at the centre of the spiral form [5-6]. SOA has got two parameters whose names are spiral angle and spiral radius. In general, these parameters have fixed values during optimization [7].

In this study, self adaptive concept is proposed to adjust the SOA's parameters during optimization instead of the fixed spiral radius and spiral angle. Three different ASOA versions are presented by adaptive structure. Five benchmark optimization problems were taken from the literature to evaluate the performances of the ASOA versions and classic SOA. The effects of the spiral angle and spiral radius parameters are presented in the section of result. The diversity that is one of the important criterions in the heuristic algorithms is examined. As last, the comparison between classic SOA and ASOA versions is represented according to the mean best value and number of function evaluations (NFE) indicators.

II. SPIRAL OPTIMIZATION ALGORITHM (SOA)

Two dimensional SOA that was recently proposed by Tamura and Yasuda is a multipoint metaheuristics search method for two dimensional continuous optimization problems based on the analogy of spiral events in nature. Then Tamura and Yasuda proposed that n dimensional SOA using a design method of two dimensional optimization. The SOA has some advantages including its few control variables, local searching capacity, fast results, easy of using for optimization process, simple structure, etc [4-7].

The SOA structure is based on the dynamic step size in its spiral path trajectory. The step size is larger at the beginning of optimization process and then it becomes smaller at close to the optimum point that is located at the centre of the spiral form. The length of the step size from iteration to iteration is calculated by spiral radius parameter. On the one hand, the shape of spiral form is designed by the spiral angle parameter. Furthermore, this parameter influences the distance between two points on the spiral path [5-6]. The updating the individuals in the population for spiral model are formulated as below:

$$x_{(k+1)} = r.M(\theta).x_k - (r.M(\theta) - I_n).x_{best} \tag{1}$$

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where r denotes the spiral radius parameter, θ represents the spiral angle parameter, M is the rotation matrix, I_n denotes unit matrix. In the study by Tamura and Yasuda, a multipoint search model was proposed instead of a one-point search model [6]. In case of the one point search model, the updating mechanism based on Eq. (1) with x_{best} does not work completely. Because the initial point becomes the best solution and the center x_{best} as evaluating the initial point. As results of above considerations, a multipoint updating mechanism was proposed for the SOA as below:

$$x_{(k+1)_i} = r.M(\theta).x_{k_i} - (r.M(\theta) - I_n).x_{best}, \qquad i = 1,2,\dots m \tag{2}$$

where the common center x_{best} denotes a best solution obtained during the optimization process. The rotation angle around the origin at each iteration is $0 \leq \theta < 2\pi$ and a convergence rate of distance between a point and the origin at each iteration is $0 < r < 1$. $M(\theta)$ is the rotation matrix. The rotation matrix for two dimensional SOA is given below:

$$M_{2_{(1,2)}}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

Each two dimensional rotation matrix for n dimensional space is defined in Eq. (4). Whose blank elements mean zero. As this definition, many rotation matrices are formed by the way selecting two axes consist of each rotation plane during their permutations or combinations [5-6].

$$M_{n_{(1,2)}}(\theta_{(i,j)}) = \begin{bmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 1 & & & & & & \\ & & & \cos\theta_{(i,j)} & & & -\sin\theta_{(i,j)} & & \\ & & & & 1 & & & & \\ & & & & & \ddots & & & \\ & & & & & & 1 & & \\ & & & \sin\theta_{(i,j)} & & & \cos\theta_{(i,j)} & & \\ & & & & & & & 1 & \\ & & & & & & & & \ddots \\ & & & & & & & & & 1 \end{bmatrix} \tag{4}$$

The using of composition rotation matrix $M_n(\theta)$ that consists of rotation matrices according to Eq. (4) is based on all combination ($n.(n-1)/2$ combinations) of 2 axes. $M_n(\theta)$ is defined as Eq. (5):

$$M_n(\theta_{(i,j)}) = \prod_{i<i} M_{n_{(i,j)}}(\theta_{(i,j)}) \tag{5}$$

In Figure 1, graphical representations for two dimensional spiral models are shown with the different spiral angles and spiral radius values. The flow chart of n dimensional basic SOA is presented as below:

Step 0: Select to parameters:
Select to number of population size $Np \geq 2$, the parameters of SOA $0 \leq \theta < 2\pi$, $0.9 < r < 1$. Calculate $rM_n(\theta)$ and the maximum iteration number $iter_{max}$.

Step 1: Initialization:
Set random initials points $x_{i_{(0)}} \in IR_n$, $i = 1,2,\dots m$ in the feasible region of the optimization problem and calculate the fitness values for each initial points of the population center. Then x_{best} is determined as $x_{best} = x_{i_{g(0)}}$, $i_g = \arg min_i\, f x_{i_{(0)}}$, $i = 1,2,\dots m$.

Step 2: Updating the each individuals ($x_{i_{k+1}}$) according to Eq. (2).

Step 3: Calculate the fitness functions for the new individuals.

Step 4: Updating : x_{best} : $x_{best} = x_{i_{g(k+1)}}$, $i_g = \arg min_i\, f x_{i_{(k+1)}}$, $i = 1,2,\dots m$.

Step 5: New population is used instead of the current population.

Step 6: If $k = iter_{max}$ then terminate. Otherwise set $k = k + 1$ and return to step 2.

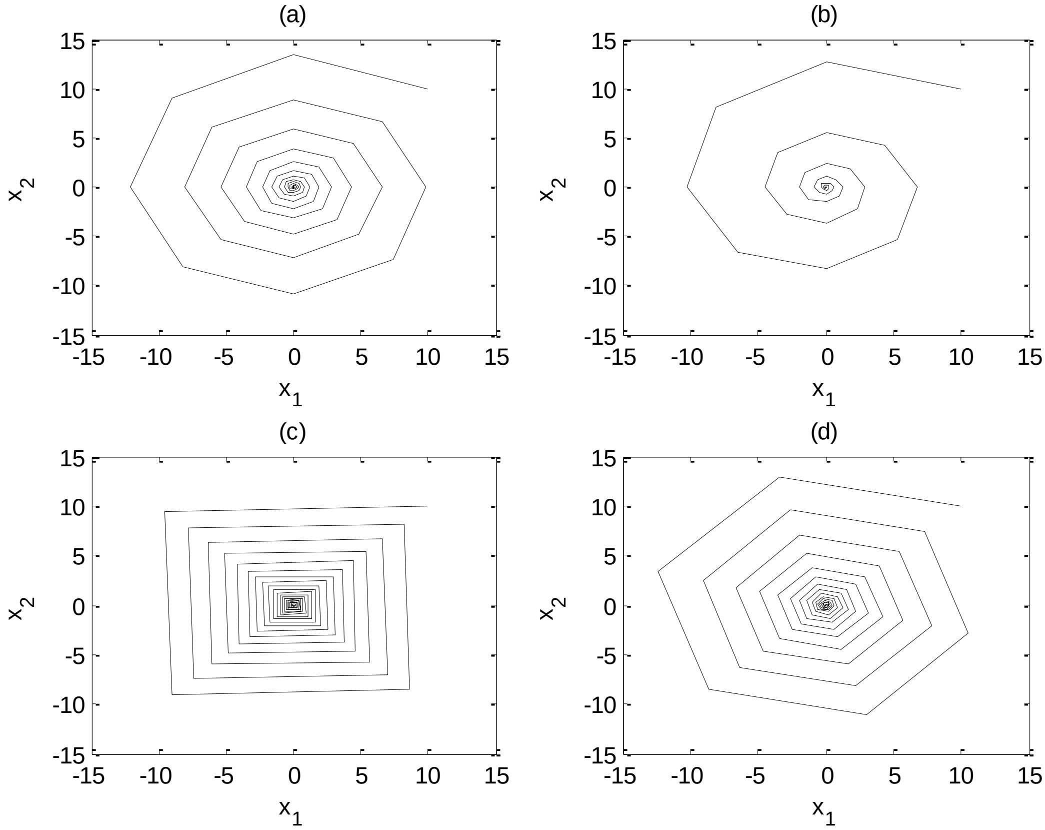


Figure 1. Graphical representation of spiral form with different spiral angel/radius parameters.
(a : ϴ=0.95, r=π/4), (b : ϴ=0.90, r=π/4), (c : ϴ=0.95, r=π/2), (d : ϴ=0.95, r=π/3).

III. ADAPTIVE SPIRAL OPTIMIZATION ALGORITHM (ASOA)

In the optimization problems, selecting the parameter values of the heuristic algorithms are very important. These parameters are depended on problem's structure. In general, the trial-and-error method is used for adjusting these parameters before the final optimization run. In this section, a self-adaptive method for the tuning the parameters of SOA is proposed. In ASOA, there are two adjustable parameters as spiral radius and spiral angle. The advantage of this approach is that all search points in ASOA are updated by tuning the parameter values randomly in each iteration. The spiral radius parameter is updated by means of the adaptive approach in below:

$$r_{i,g+1} = \begin{cases} r_{low} + rnd_1 \times (r_{up} - r_{low}), & if(rnd_2 < \tau) \\ r_{i,g}, & otherwise \end{cases} \tag{5}$$

where $r_{i,g}$ denotes the spiral radius of i_{th} individual and g_{th} generation, r_{low} and r_{up} are the lower and the upper value of the spiral radius, $rnd_{1,2}$ stand for the random values in range [0 1], τ represents the probability to adjust the spiral radius. In this study, the spiral radius limit values were determined as $r_{low} = 0.9$ and $r_{up} = 1.0$. During the optimization, the other parameter whose name is spiral angle is calculated in Eq. (6):

$$\theta_{i,g+1} = \begin{cases} \theta_{low} + rnd_3 \times (\theta_{up} - \theta_{low}), & if(rnd_4 < \tau) \\ \theta_{i,g}, & otherwise \end{cases} \tag{6}$$

where $\theta_{i,g}$ denotes the spiral angle of i_{th} individual and g_{th} generation, θ_{low} and θ_{up} are the lower and the upper value of the spiral angle, $rnd_{3,4}$ stand for the random values in range [0 1]. In this equation, τ represents the probability to adjust the spiral angle. In this study, the limit values of the spiral angle were used as $\theta_{low} = 0$ and

$\theta_{up} = 2\pi$. In our experiments, τ coefficient was used as 0.1. In Figure 2, the results of one from the test benchmark functions are presented for different values of the probability to tune the spiral radius and angle. As can be seen this figure, the best probability coefficient that is denoted as k, was obtained as 0.1. Figure 3 shows the variations of the spiral angle and spiral radius during the optimization process of Rosenbrock function.

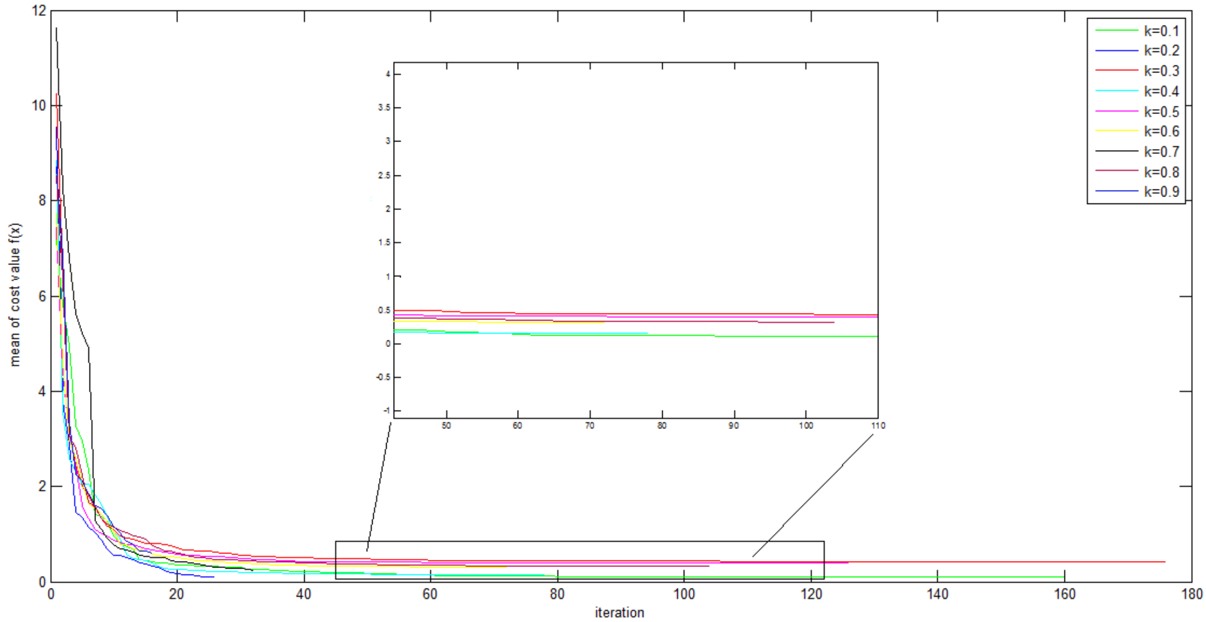
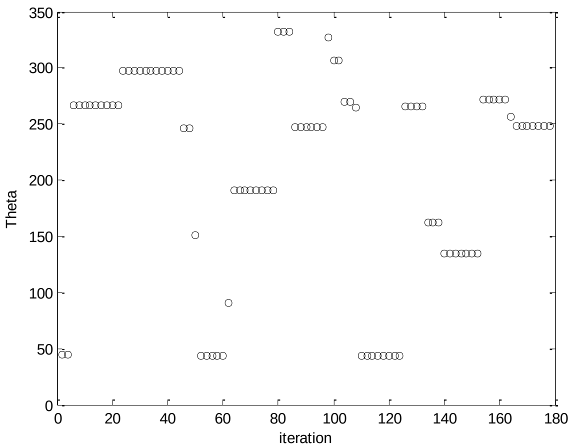


Figure 2. Results of Rosenbrock function for different spiral angle/radius probability coefficients.



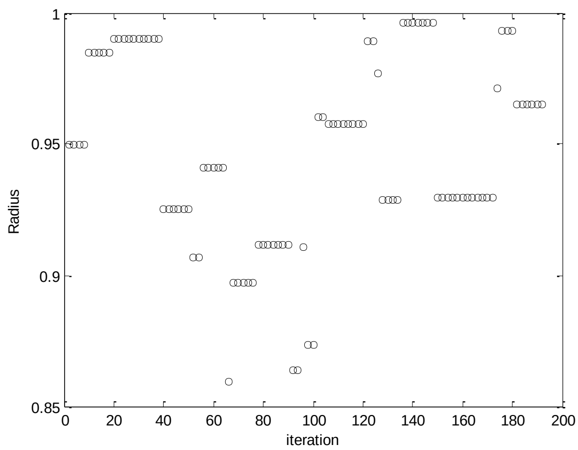


Figure 3. Spiral angle and spiral radius variations for Rosenbrock function.

In this study, three different adaptive concepts of spiral optimization algorithm are proposed. The details regarding these adaptive concepts are given below:

- $ASOA_1$: Adaptive spiral radius and fixed spiral angle ($\theta = \pi/4$).
- $ASOA_2$: Fixed spiral radius and adaptive spiral angle ($r = 0.95$).
- $ASOA_3$: Adaptive spiral radius and spiral angle.

IV. EXPERIMENTAL RESULTS

To evaluate the performances of the ASOA versions, five benchmark test functions taken from the literature were used. The characteristics of the benchmark functions that used in this study are given in Table 1. The proposed ASOA concepts were compared with the classic SOA. In the experimental works, the population size (NP) was used as 20, the maximum number of iterations was selected as 200 and the number of independent runs was used as 50. All spiral optimization algorithms were coded on PC with Intel(R) Core(TM) i5-3230M

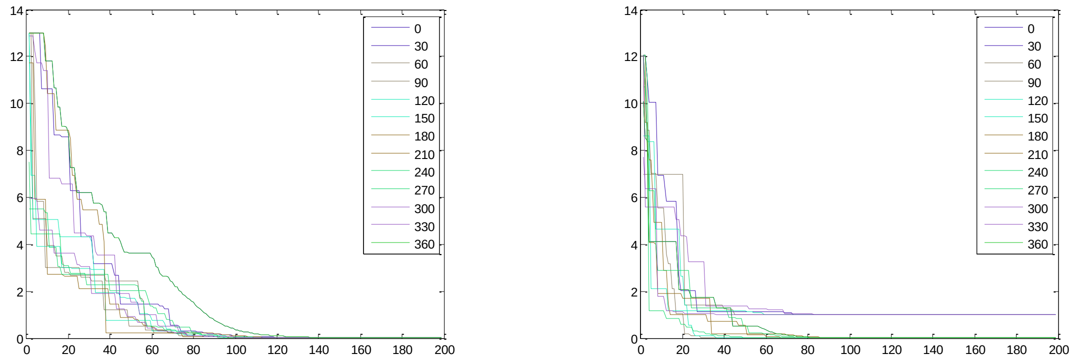
CPU 2.60GHz/8GB RAM. The termination criterion was determined as iteration reaches the maximum number of iteration and $|fitness(best\ individual) - fitness(worst\ individual)| = VTR$. *VTR* represents the value to reach and it was used as 1×10^{-6}. This section consists of four sub-sections. The effects of the spiral angle and spiral radius were presented in the first two sub-sections and then the diversity of the ASOA concepts and SOA structure were examined. Finally, SOA and the ASOA versions were compared with each other.

Table 1. The characteristics of the benchmark functions [8]

Name	Function	*S*	x_1, x_2	f_{min}
Ackley *(F1)*	$f(x) = -a \exp\left(-b\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d} \cos(cx_i)\right) + a + \exp(1)$	[-35 35]	(0, 0)	0
Himmelblau *(F2)*	$f(x) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$	[-5 5]	(3, 2) (-3.78, -3.28) (-2.81, 3.13) (3.58, -1.85)	0
Penholder *(F3)*	$f(x) = -\exp\left\|\exp\left(\left\|-\frac{\sqrt{x_1^2 + x_2^2}}{\pi} + 1\right\|\right)\cos(x_1)\cos(x_2)\right\|^{-1}$	[-11 11]	(-9.65, 9.65)	-0.963
Rastrigin *(F4)*	$f(x) = 10d + \sum_{i=1}^{d}[x_i^2 - 10\cos(2\pi x_i)]$	[-5.12 5.12]	(0, 0)	0
Rosenbrock *(F5)*	$f(x) = \sum_{i=1}^{d-1}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-2.3 2.3]	(1, 1)	0

A. *The Effect of Spiral Angle*

The spiral angle is one of the important parameters of SOA. The effect of the spiral angle parameter is shown in Fig. 4 for two benchmark functions. As can be seen from these figures, the variation in the spiral angle affects to the optimization result. For that reason, the proposed ASOA versions include the adaptive based spiral angle structure.

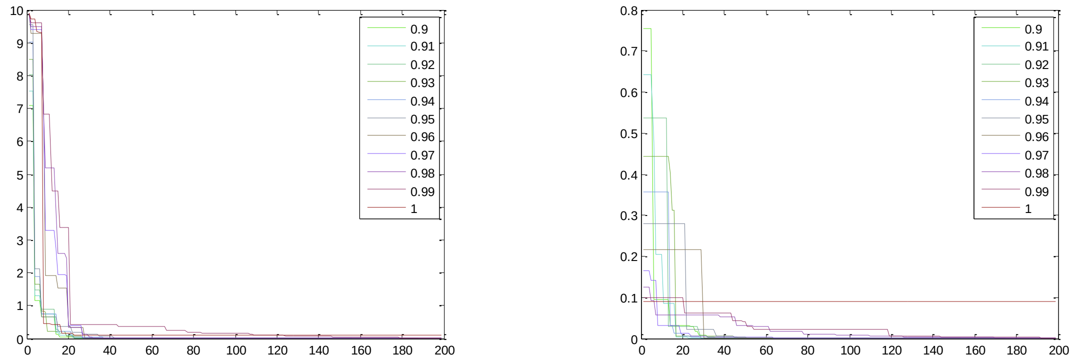


Ackley *(F1)* Rastrigin *(F4)*

Figure 4. Results of *F1* and *F4* functions for different spiral angel values.

B. *The Effect of Spiral Radius*

The other important parameter of SOA is spiral radius. In Fig. 5, the optimization results for the different spiral radius values are presented for other two benchmark functions.



Himmelblau (F2) Rosenbrock *(F5)*

Figure 5. Results of *F2* and *F5* functions for different spiral radius values.

C. The Diversity

Diversity is an important concept in heuristic algorithms because of obtaining new candidates for solution from a homogeneous population distribution. ASOA versions consist of the adaptive structure for this purpose. In addition to this, value to reach (VTR) was added into the all algorithms, so being the same individuals in the population was avoid. In Fig. 6, the distributions of the individuals in the population for the different iterations are shown.

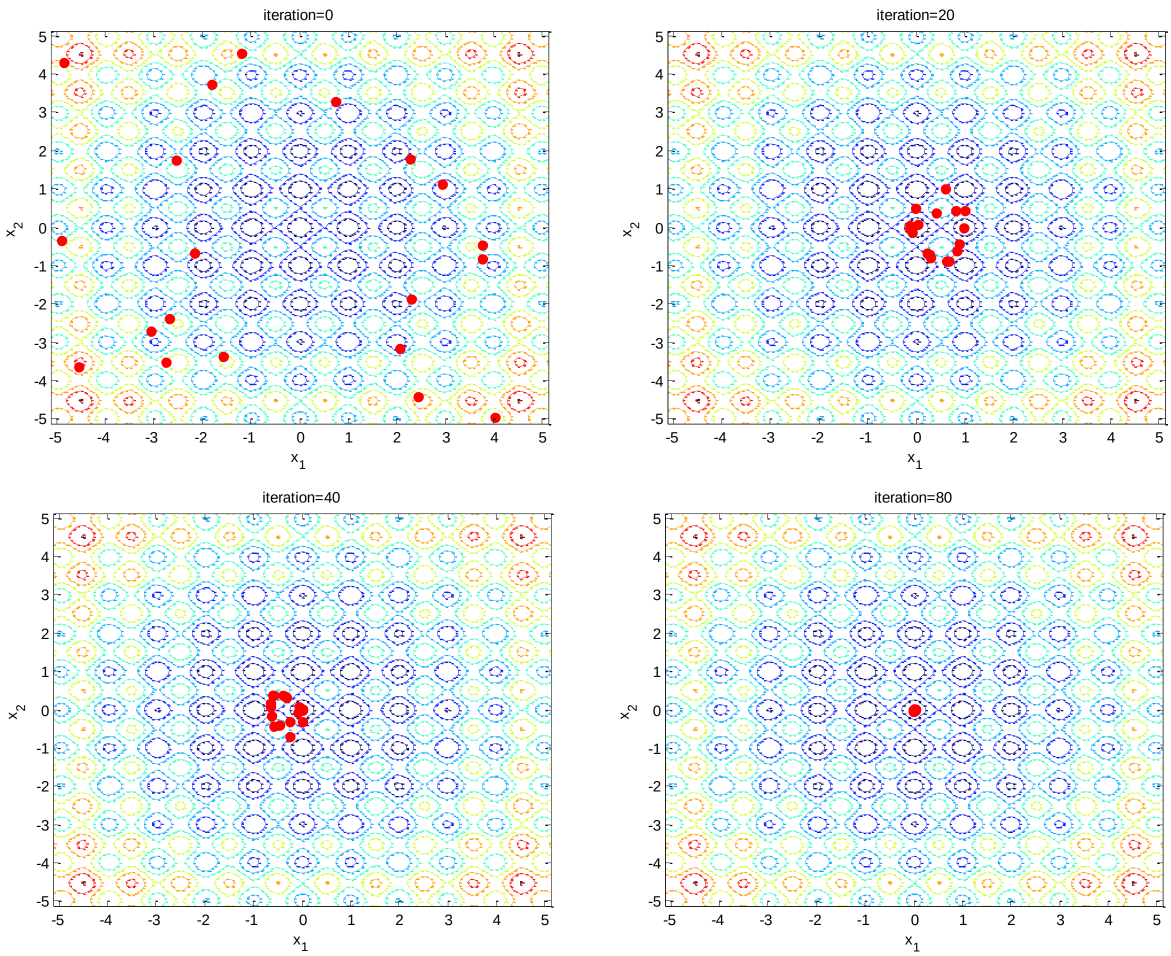


Figure 6. Population diversity for F4 function.

D. The Comparison of ASOAs and SOA

In this section, ASOA versions were compared with classic SOA to evaluate their performances. Table 2 summarizes the average results of 50 independent runs of the proposed adaptive based SOA algorithms and classic SOA according to the mean best and standard deviation values. In this table, mean best indicates the average of minimum values obtained by ASOA versions and SOA. This indicator represents with the standard deviation (std dev) to evaluate the performances of the algorithms. For five benchmark functions, $ASOA_2$ has got the best performance as to mean best indicator. $ASOA_1$ is only successful for one function (F1). In terms of the number of function evaluations (NFE) and CPU-time, the optimization results with 50 independent runs for all algorithms are given in Table 3. According to the NFE and CPU-time indicators, $ASOA_3$ is faster than the other algorithms, but this result does not show the guaranty solution for optimization process.

Table 2. Experimental results (Mean Best & Std Dev.) with 50 independent runs of SOA and ASOA versions

	Mean Best (Std Dev)			
Function	SOA	$ASOA_1$	$ASOA_2$	$ASOA_3$
F1	2.77e-4 (1.48e-4)	**2.45e-5 (7.63e-5)**	3.32e-4 (2.73e-4)	1.76e-1 (1.25e-0)
F2	4.14e-9 (9.48e-9)	8.72e-9 (2.14e-8)	**4.08e-9 (1.18e-8)**	1.57e-8 (8.02e-8)
F3	-9.54e-1 (1.45e-2)	-9.53e-1 (1.47e-2)	**-9.60e-1 (1.03e-2)**	-9.57e-1 (1.27e-2)
F4	5.77e-1 (8.78e-1)	6.77e-1 (9.31e-1)	**3.58e-1 (7.98e-1)**	8.76e-1 (2.32e-0)
F5	3.65e-2 (1.82e-1)	1.48e-1 (8.08e-1)	**2.41e-2 (6.47e-2)**	3.00e-2 (8.09e-1)

Table 3. Experimental results (NFE^{50} & $CPU\text{-}time^{50}$) with 50 independent runs of SOA and ASOA versions

	NFE^{50} ($CPU\text{-}time^{50}$ sec)[a]			
Function	SOA	$ASOA_1$	$ASOA_2$	$ASOA_3$
F1	4000.0 (**0.265**)	3972.4 (0.268)	4000.0 (0.270)	**3939.6** (0.272)
F2	3800.0 (0.224)	**2502.4** (0.154)	3828.0 (0.232)	2668.4 (**0.153**)
F3	2289.6 (0.136)	1772.8 (0.111)	2260.0 (0.140)	**1651.2** (**0.101**)
F4	4000.0 (0.244)	3172.4 (0.199)	4000.0 (0.254)	**2958.8** (**0.176**)
F5	3846.4 (0.232)	2896.0 (0.172)	3950.4 (0.229)	**2769.2** (**0.164**)

[a] NFE^n: Number of function evaluations, $CPU\text{-}time^n$: time taken by CPU per execution (average of 'n' executions)

In Fig. 7, the average results with 50 independent runs of the proposed adaptive based SOA algorithms and classic SOA are presented with each other for F3 and F4 functions. In the results of F3 function, it is shown that $ASOA_2$ has the best minimum values than the others. Although $ASOA_1$ and $ASOA_3$ have the short iteration values, they stay at the local minimal points. For F4 benchmark function, there is no clear difference between the algorithms as can be understood from this figure.

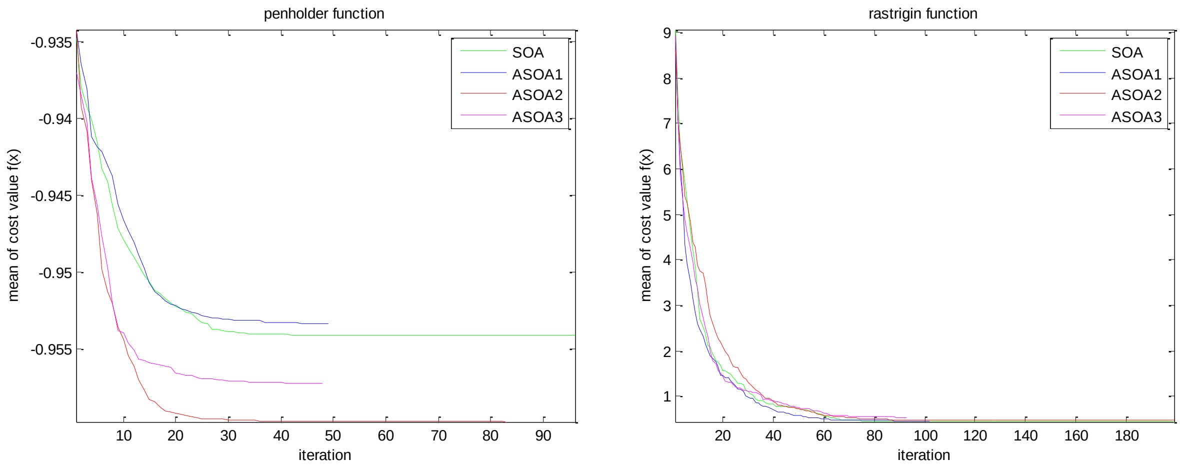


Figure 7. Results of SOA and $ASOA_1$, $ASOA_2$, $ASOA_3$ algorithms for *F3*and *F4* functions.

V. CONCLUSION

In this paper, self-adaptive based spiral optimization algorithms are proposed. There are three different versions regarding adaptive structure. This paper includes the effects of the SOA's parameters, diversity of the ASOA versions and the comparison between ASOA versions and classic SOA. The results show that the self-adaptive concept implemented into the SOA, has successful performance in terms of the mean best (standard deviation) and NFE (CPU-time) indicators. In the future works, $ASOA_2$ that is the best algorithm than the others, will apply to the optimization process such as scheduling, economics, chemical process etc.

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